# CS 70 Discrete Mathematics and Probability Theory Spring 2024 Seshia, Sinclair

# 1 Natural Induction on Inequality

Note 3 Prove that if  $n \in \mathbb{N}$  and x > 0, then  $(1+x)^n \ge 1 + nx$ .

# 2 Make It Stronger

Suppose that the sequence  $a_1, a_2, ...$  is defined by  $a_1 = 1$  and  $a_{n+1} = 3a_n^2$  for  $n \ge 1$ . We want to prove that  $a_n \le 3^{(2^n)}$ 

for every positive integer n.

Note 3

(a) Suppose that we want to prove this statement using induction. Can we let our inductive hypothesis be simply  $a_n \le 3^{(2^n)}$ ? Attempt an induction proof with this hypothesis to show why this does not work.

(b) Try to instead prove the statement  $a_n \le 3^{(2^n-1)}$  using induction.

(c) Why does the hypothesis in part (b) imply the overall claim?

# 3 Binary Numbers

Note 3

Prove that every positive integer n can be written in binary. In other words, prove that for any positive integer n, we can write

$$n = c_k \cdot 2^k + c_{k-1} \cdot 2^{k-1} + \dots + c_1 \cdot 2^1 + c_0 \cdot 2^0$$
,

for some  $k \in \mathbb{N}$  and  $c_i \in \{0,1\}$  for all  $i \leq k$ .

#### 4 Fibonacci for Home

Note 3

Recall, the Fibonacci numbers, defined recursively as

$$F_1 = 1$$
,  $F_2 = 1$ , and  $F_n = F_{n-2} + F_{n-1}$ .

Prove that every third Fibonacci number is even. For example,  $F_3 = 2$  is even and  $F_6 = 8$  is even.