CS 70 Discrete Mathematics and Probability Theory Spring 2024 Seshia, Sinclair DIS 8B

1 Probability Potpourri

Provide brief justification for each part.

Note 13 Note 14

(a) For two events *A* and *B* in any probability space, show that $\mathbb{P}[A \setminus B] \ge \mathbb{P}[A] - \mathbb{P}[B]$.

(b) Suppose $\mathbb{P}[D \mid C] = \mathbb{P}[D \mid \overline{C}]$, where \overline{C} is the complement of *C*. Prove that *D* is independent of *C*.

(c) If A and B are disjoint, does that imply they're independent?

2 Easter Eggs

Note 14

You made the trek to Soda for a Spring Break-themed homework party, and every attendee gets to leave with a party favor. You're given a bag with 20 chocolate eggs and 40 (empty) plastic eggs. You pick 5 eggs (uniformly) without replacement.

(a) What is the probability that the first egg you drew was a chocolate egg?

(b) What is the probability that the second egg you drew was a chocolate egg?

(c) Given that the first egg you drew was an empty plastic one, what is the probability that the fifth egg you drew was also an empty plastic egg?

3 Balls and Bins

- Note 14 Suppose you throw *n* balls into *n* labeled bins one at a time.
 - (a) What is the probability that the first bin is empty?

(b) What is the probability that the first *k* bins are empty?

(c) Let A be the event that at least k bins are empty. Let m be the number of subsets of k bins out of the total n bins. If we assume A_i is the event that the *i*th set of k bins is empty. Then we can write A as the union of A_i's:

$$A = \bigcup_{i=1}^{m} A_i.$$

Compute *m* in terms of *n* and *k*, and use the union bound to give an upper bound on the probability $\mathbb{P}[A]$.

(d) What is the probability that the second bin is empty given that the first one is empty?

(e) Are the events that "the first bin is empty" and "the first two bins are empty" independent?

(f) Are the events that "the first bin is empty" and "the second bin is empty" independent?