70: Discrete Math and Probability Theory

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Programming

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Programming + Data Structures/Algorithms

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Programming + Data Structures/Algorithms + Microprocessors

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$\equiv$ Superpower!

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Probability!

Instructors

Instructor: Sanjit Seshia.

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Professor of EECS (office: 566 Cory)

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Research: Formal Methods (a.k.a. Computational Proof Methods)

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Taught: CS 70, EECS 149/249A, CS 172, EECS 144/244, EECS 219C, EECS149.1x on edX, ...

Instructors

- Alistair Sinclair
- Professor of CS (office 677 Soda)

- @ Berkeley since pre-history (1994)
- Originally from the UK: undergrad @ Cambridge, PhD @ Edinburgh

- Research: CS Theory, esp. algorithms, randomness, statistical physics, stochastic processes…
- Teaching: CS70, CS170, CS172, CS174 + various grad classes

Admin

Course Webpage: **http://eecs70.org/**

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Questions/Announcements

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homework/no-homework option continues

Learning and Teaching

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Learn by Doing (Mathematical Modeling/Problem Solving)

CS70: Lecture 1. Outline.

Today: Note 1.

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The language of proofs!

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The language of proofs! Mathematical Logic!

1. Propositions.
2. Propositional Forms.
3. Implication.
4. Truth Tables
5. Quantifiers
6. More De Morgan's Laws

Propositions: Statements that are true or false.

$\sqrt{2}$ is irrational
2+2 = 4
2+2 = 3
826th digit of pi is 4
Stephen Curry is a good basketball player
All evens > 2 are unique sums of 2 primes
$4+5$
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Propositional Forms.

Put propositions together to make another...

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Propositional Forms: quick check!

$P =$ "$\sqrt{2}$ is rational"

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Put them together..

Propositions:

P_1 - Person 1 rides the bus.

Put them together..

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P_2 - Person 2 rides the bus.

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Suppose we can't have either of the following happen; That either person 1 or person 2 ride the bus and person 3 or 4 ride the bus. Or that person 2 or person 3 ride the bus and that either person 4 ride the bus or person 5 doesn't.

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$\neg(((P_1 \vee P_2) \wedge (P_3 \vee P_4)) \vee ((P_2 \vee P_3) \wedge (P_4 \vee \neg P_5)))$

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Who can ride the bus?

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Suppose we can't have either of the following happen; That either person 1 or person 2 ride the bus and person 3 or 4 ride the bus. Or that person 2 or person 3 ride the bus and that either person 4 ride the bus or person 5 doesn't.

Propositional Form:

$\neg(((P_1 \vee P_2) \wedge (P_3 \vee P_4)) \vee ((P_2 \vee P_3) \wedge (P_4 \vee \neg P_5)))$

Who can ride the bus?

What combinations of people can ride the bus?

Put them together..

Propositions:
P_1 - Person 1 rides the bus.
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What combinations of people can ride the bus?

This seems ...complicated.

We need a way to keep track!

Truth Tables for Propositional Forms.

P	Q	$P \wedge Q$
T	T	T
T	F	
F	T	
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One use for truth tables: Logical Equivalence of propositional forms!

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DeMorgan's Law's for Negation: distribute and flip!

$\neg(P \wedge Q)$

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Example: Statement: If you stand in the rain, then you'll get wet.

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Can conclude: "you'll get wet."

Non-Consequences/consequences of Implication

The statement "$P \implies Q$"

Non-Consequences/consequences of Implication

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only is False if P is True and Q is False .

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Some Fun: use propositional formulas to describe implication?

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Some Fun: use propositional formulas to describe implication?
$((P \implies Q) \wedge P) \implies Q$.

Implication and English.

$P \implies Q$

- If P, then Q.

Implication and English.

$P \implies Q$

- If P, then Q.
- Q if P.

Implication and English.

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Implication and English.

$P \implies Q$

- If P, then Q.
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- P only if Q.

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$P \implies Q$

- If P, then Q.
- Q if P.
- P only if Q.
- P is sufficient for Q.

Implication and English.

$P \implies Q$

- If P, then Q.
- Q if P.
- P only if Q.
- P is sufficient for Q.
- Q is necessary for P.

Truth Table: implication.

P	Q	$P \implies Q$
T	T	T
T	F	
F	T	
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Truth Table: implication.

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P	Q	$\neg P \vee Q$
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$\neg P \lor Q \quad \equiv P \implies Q.$

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$\neg P \vee Q \quad \equiv P \implies Q$.

These two propositional forms are logically equivalent!

Contrapositive, Converse

- Contrapositive of $P \implies Q$ is $\neg Q \implies \neg P$.

Contrapositive, Converse

- Contrapositive of $P \implies Q$ is $\neg Q \implies \neg P$.
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- **Definition:** If $P \implies Q$ and $Q \implies P$ is P if and only if Q or $P \iff Q$.
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Next: Statements about boolean valued functions!!

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- See note 0 for more!

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English: there is an x in S where $P(x)$ does not hold.
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Next Time: proofs!