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Naive Set Theory: Any definable collection is a set.

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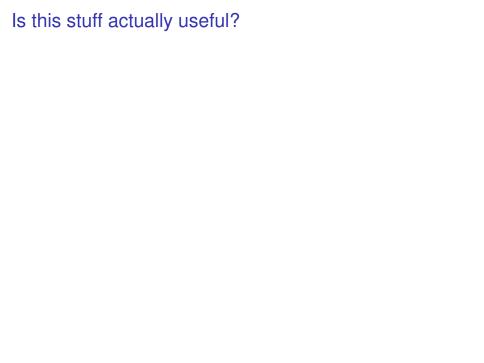
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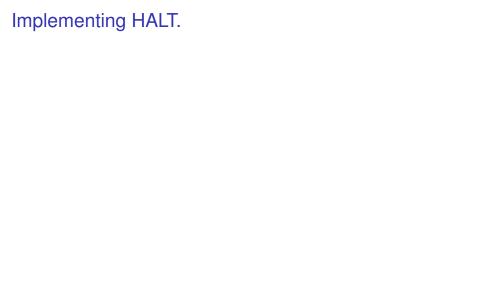
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Implementing HALT.

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How long do you wait?



Halt does not exist.

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Proof Idea: Proof by contradiction, use self-reference.

Proof:

Proof: Assume there is a program $HALT(\cdot, \cdot)$.

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Now that's a computer! (not far from today's computers)

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Along the way: "built" computers out of arithmetic. Showed that every mathematical statement corresponds to an

....natural number!!!!

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Program: Turing (or DIAGONAL) takes P.

Assume there is HALT.

DIAGONAL flips answer.

Loops if P halts, halts if P loops.

What does Turing do on turing? Doesn't loop or HALT.

HALT does not exist!

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Computation is a lens for other action in the world.