# CS70: On to probability.

Modeling Uncertainty: Probability Space

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**Modeling Uncertainty: Probability Space**

1. Key Points
2. Random Experiments
3. Probability Space

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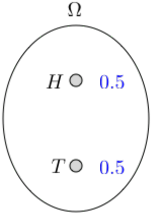
Physical Experiment




Probability Model

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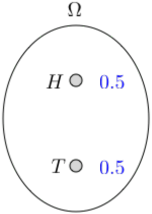


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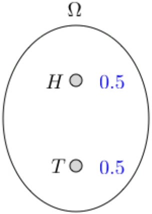


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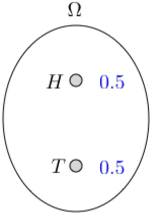


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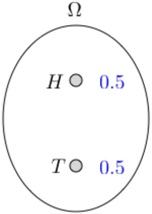


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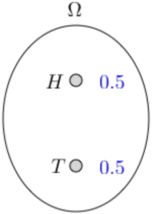


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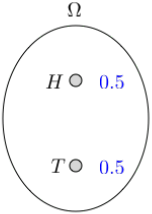


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- Tautology? No: Statistical regularity!

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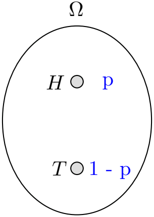


Probability Model

# Flip Two Fair Coins

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# Flip Glued Coins

Flips two coins glued together side by side:

Glued coins

50%

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- Possible outcomes: $\{HT, TH\}$.
- Likelihoods: $HT : 0.5, TH : 0.5$.
- Note: Coins are glued so that they show different faces.

# Flip two Attached Coins

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- Note: Coins are attached so that they tend to show the same face, unless the spring twists enough.

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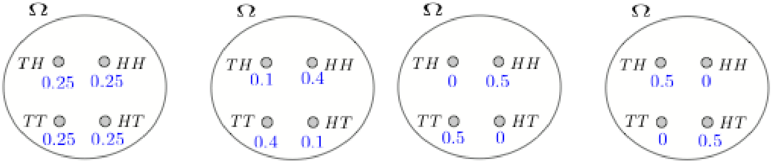


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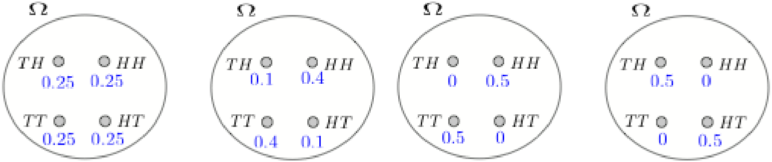


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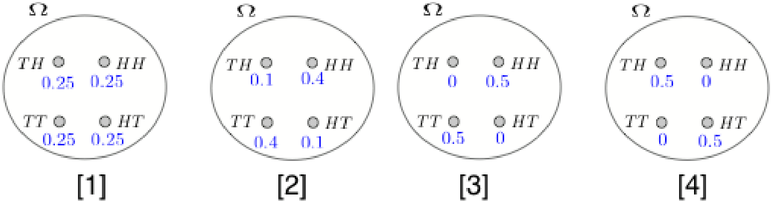


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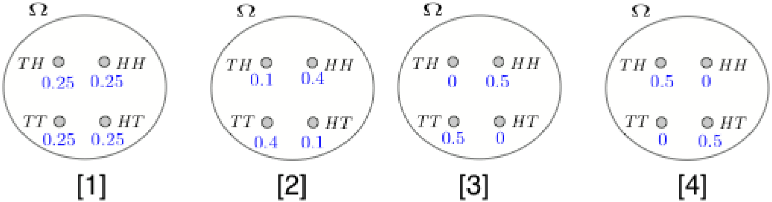


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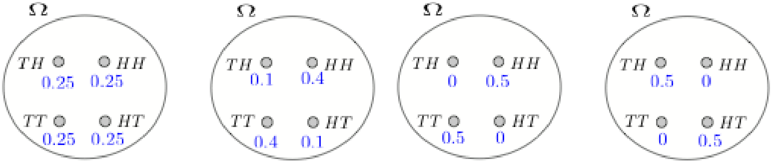


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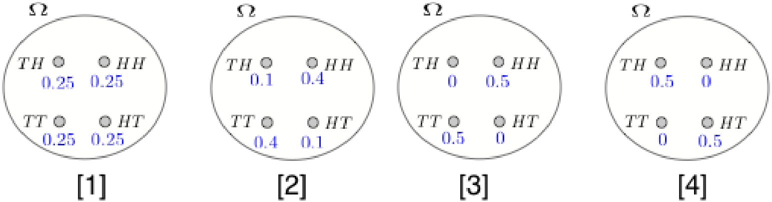


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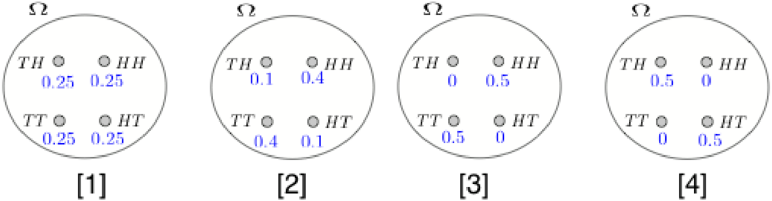


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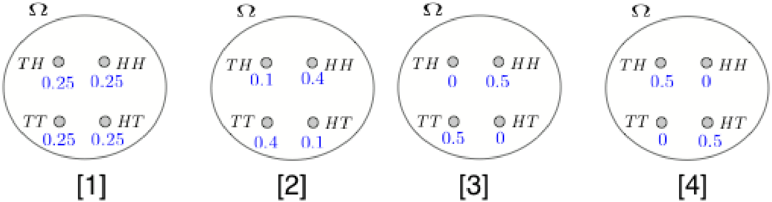


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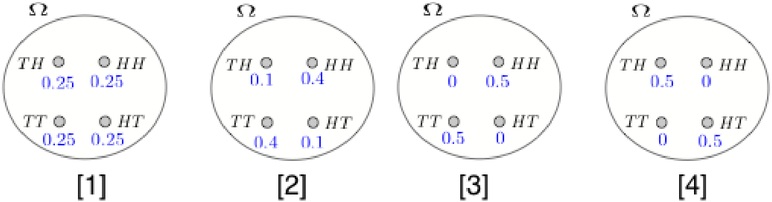


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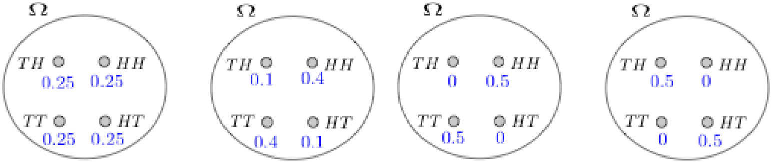


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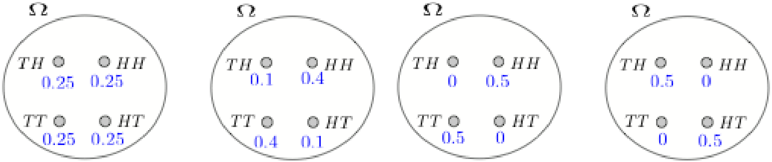


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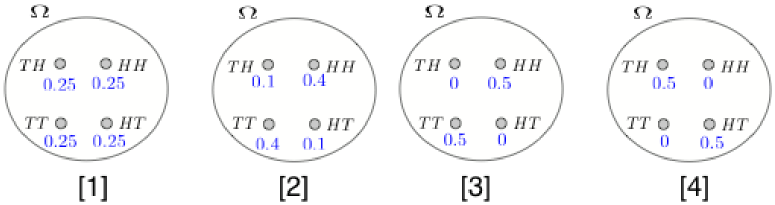


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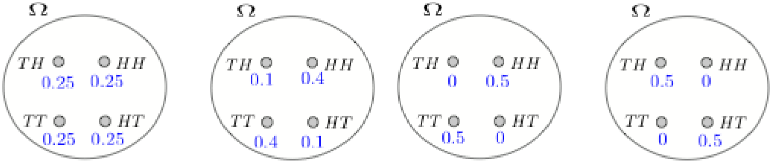


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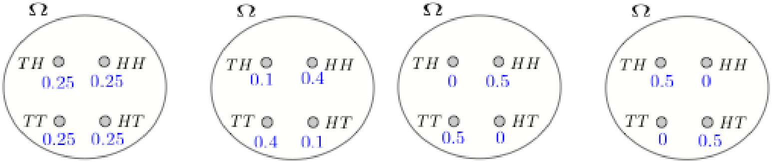


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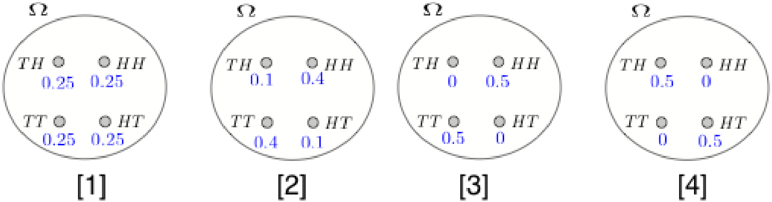


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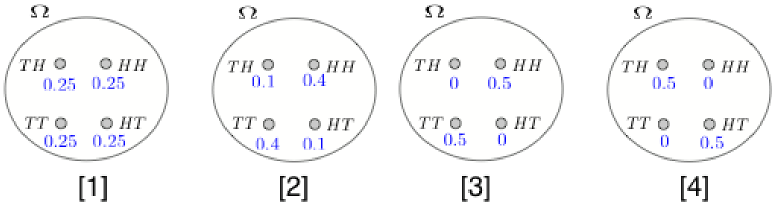


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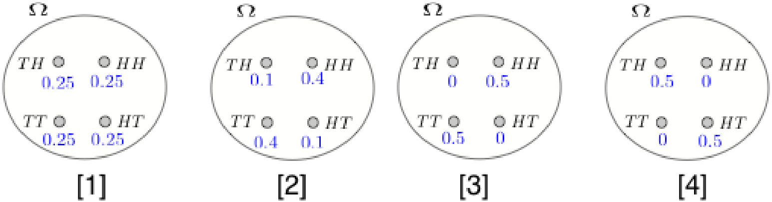


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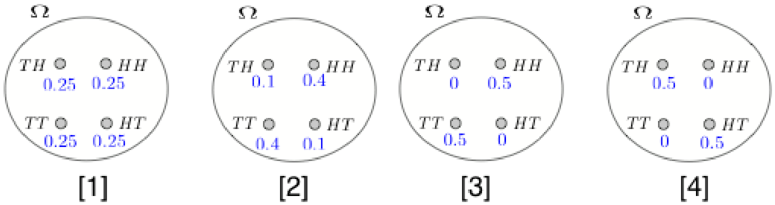


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Flip two fair coins.

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No to (A), (B), and (D).

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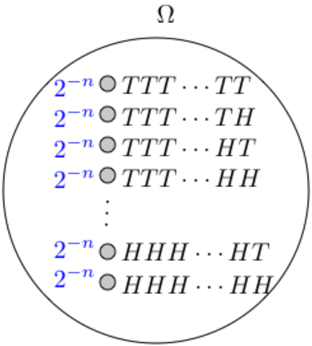

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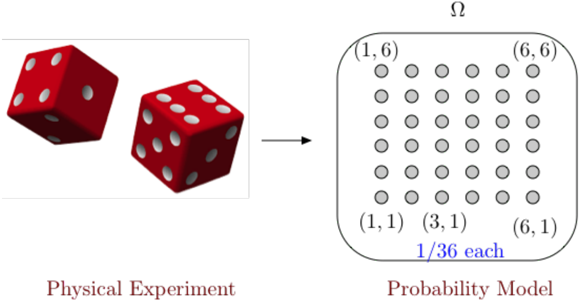

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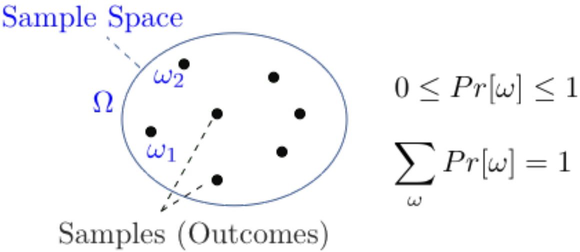

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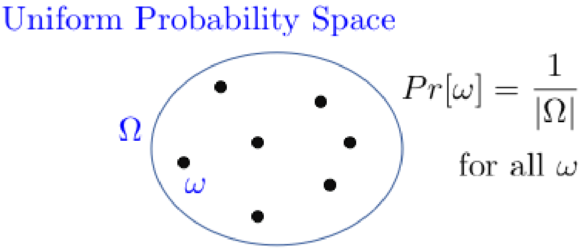

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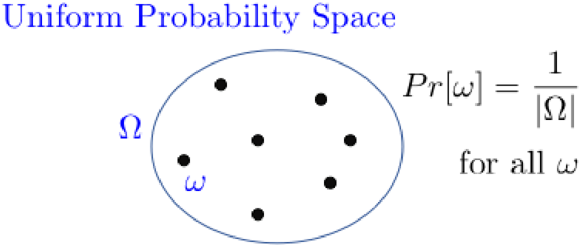


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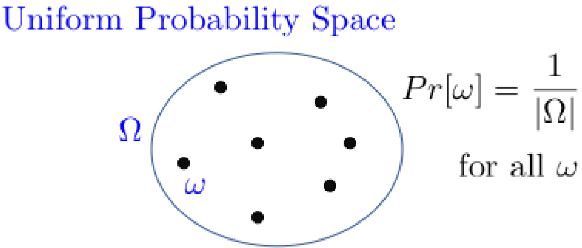


Examples:

- Flipping two fair coins, dealing a poker hand are uniform probability spaces.
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Simplest physical model of a uniform probability space:

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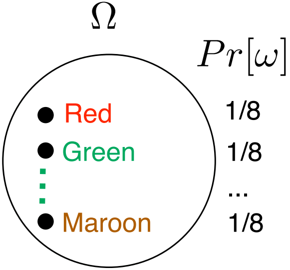
Physical experiment




Probability model

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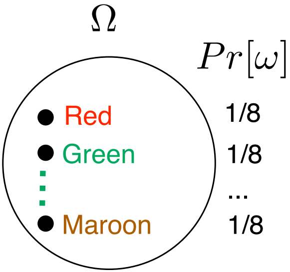


Probability model

A bag of identical balls, except for their color (or a label).

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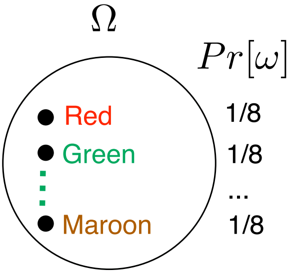


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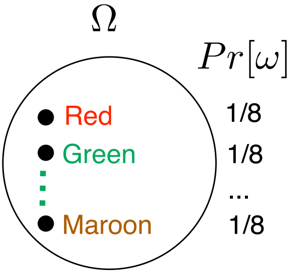


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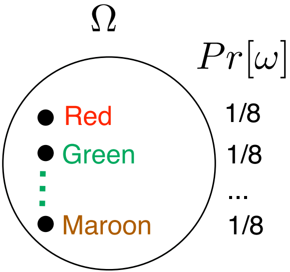


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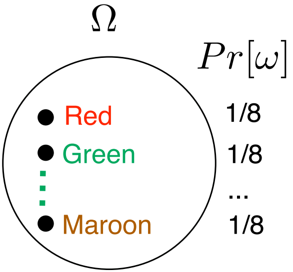


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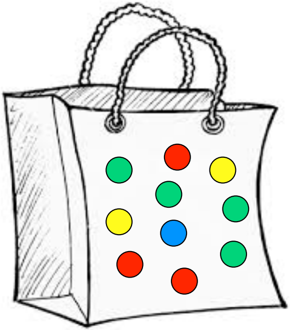

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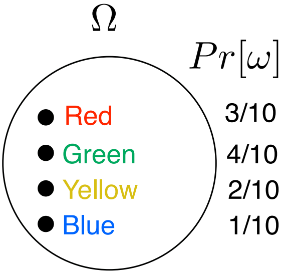


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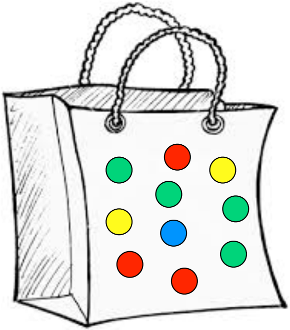

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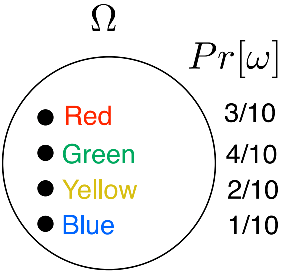


Probability model

$\Omega = \{$Red, Green, Yellow, Blue$\}$

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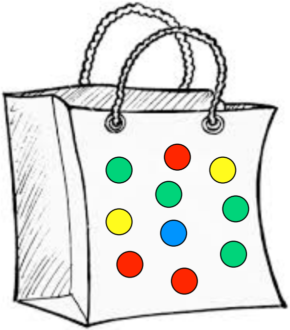

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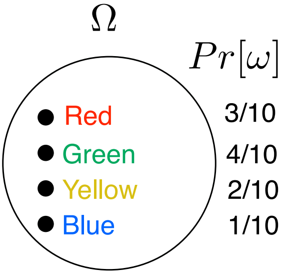


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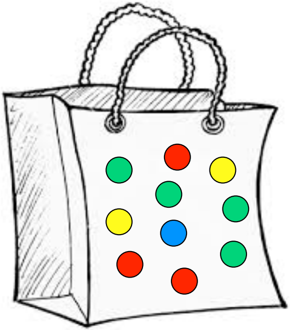

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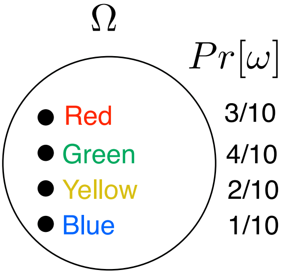


Probability model

$$\Omega = \{\text{Red, Green, Yellow, Blue}\}$$

$$Pr[\text{Red}] = \frac{3}{10},$$

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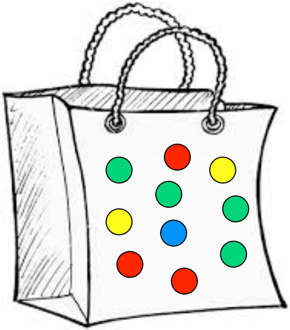

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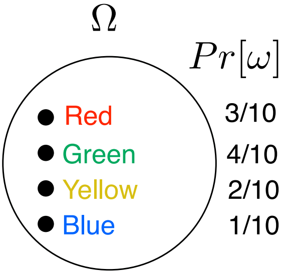


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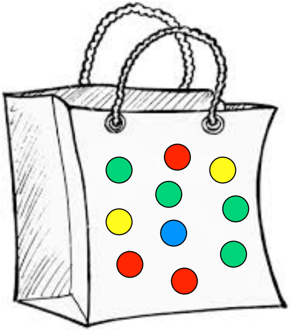

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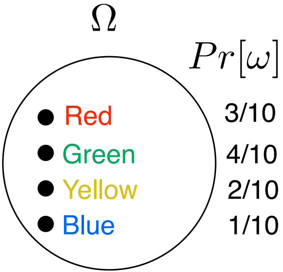


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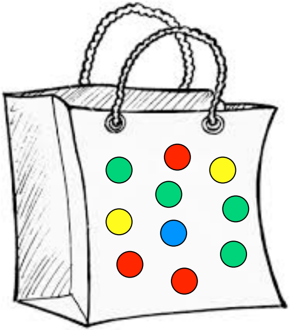

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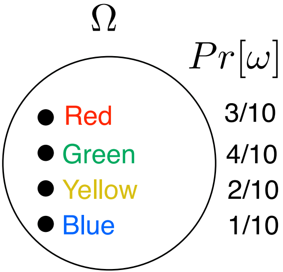


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Note: Probabilities are restricted to rational numbers: $\frac{N_k}{N}$.

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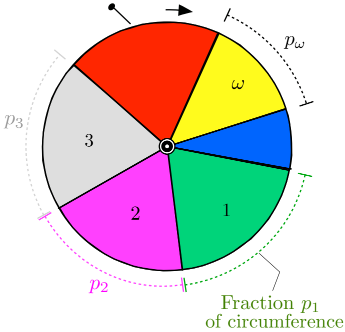


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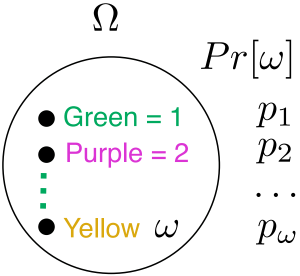


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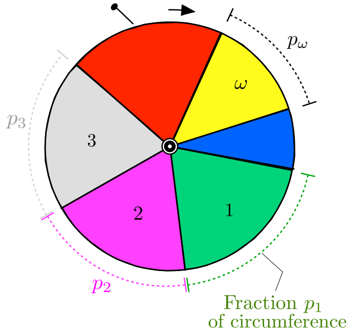


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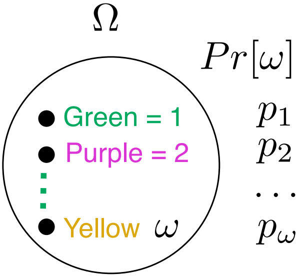


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The roulette wheel stops in sector $\omega$ with probability $p_\omega$.

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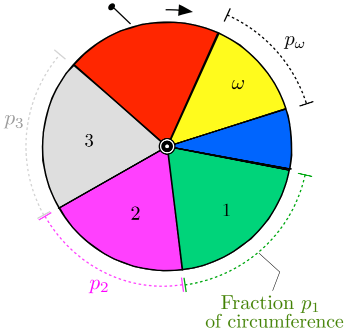


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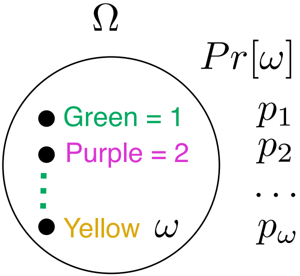


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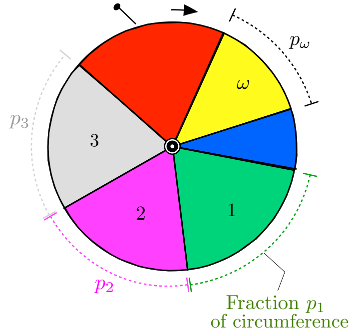


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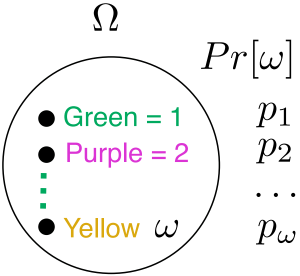


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With possibly different probabilities for each marble..

# Onwards in Probability.

Events, Conditional Probability, Independence, Bayes' Rule

# CS70: On to Events.

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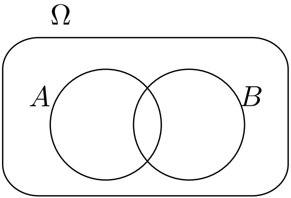


Figure: Two events

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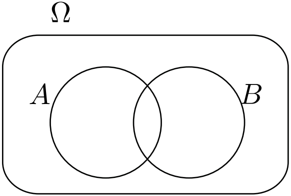


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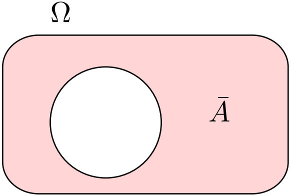


Figure: Complement (not)

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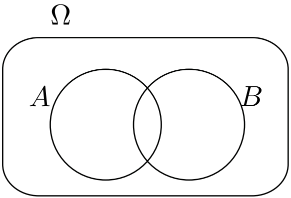


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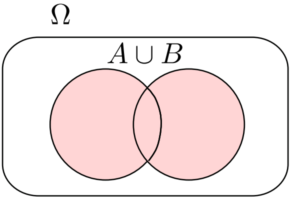


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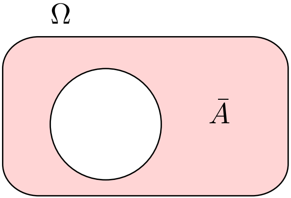


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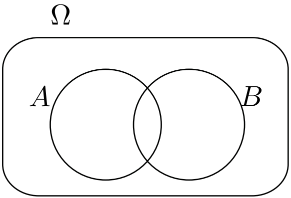


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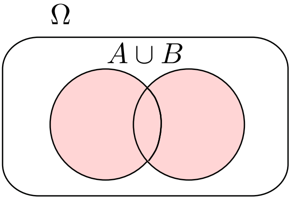


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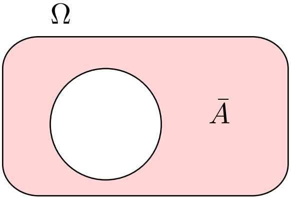


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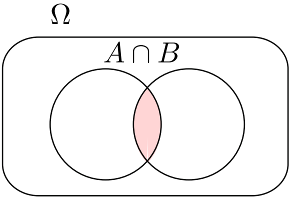


Figure: Intersection (and)

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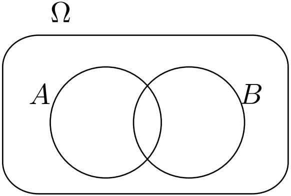


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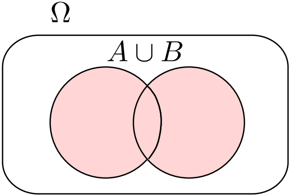


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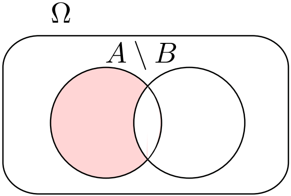



Figure: Difference (*A*, not *B*)

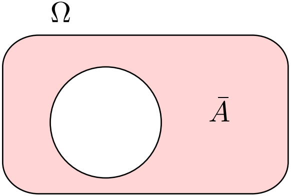



Figure: Complement (not)

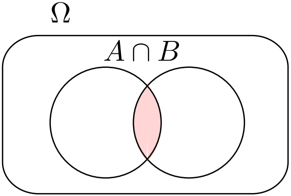



Figure: Intersection (and)

# Set notation review

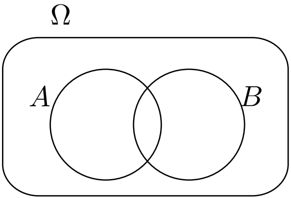



Figure: Two events

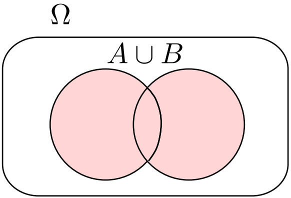



Figure: Union (or)

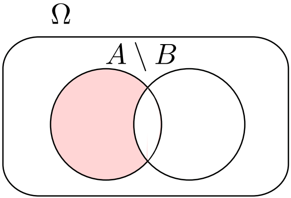



Figure: Difference (*A*, not *B*)

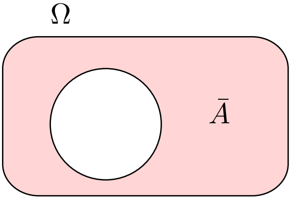
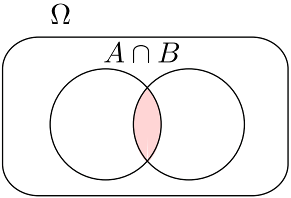



Figure: Complement (not)

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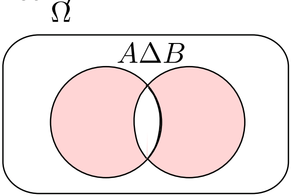


Figure: Symmetric difference (only one)

# Probability of exactly one ‘heads’ in two coin flips?

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Idea: Sum the probabilities of all the different outcomes that have exactly one ‘heads’: $HT, TH$.

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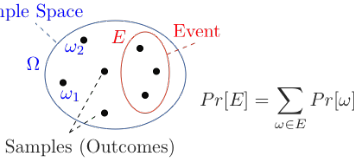

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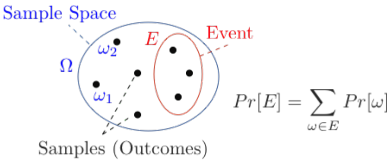


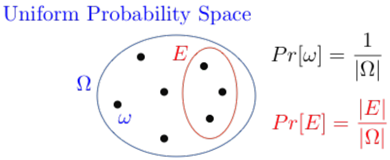

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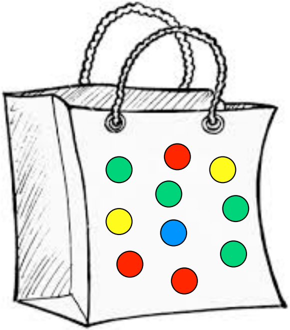
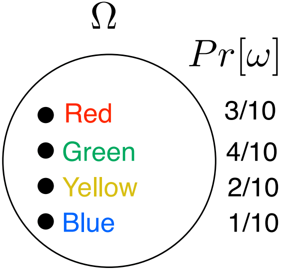


Physical experiment




Probability model

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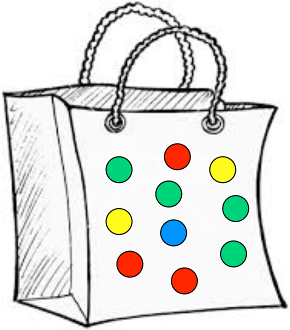

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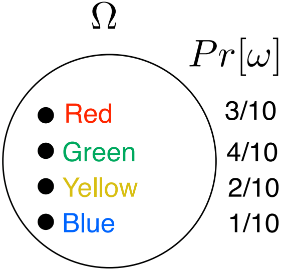


Probability model

$\Omega = \{$Red, Green, Yellow, Blue$\}$

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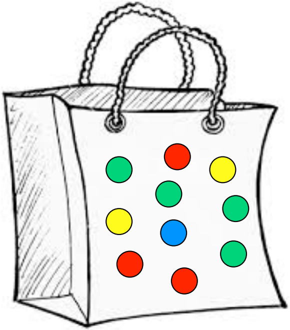

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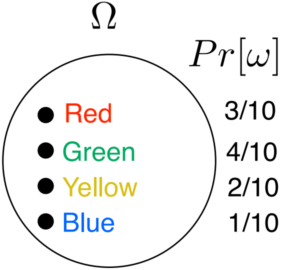


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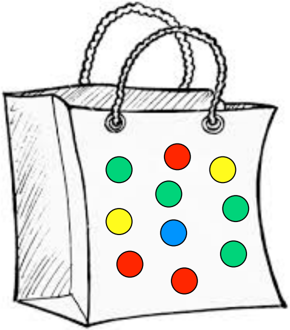

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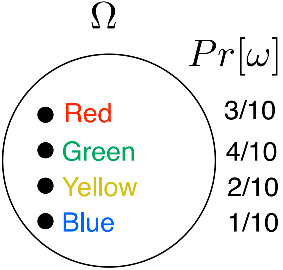


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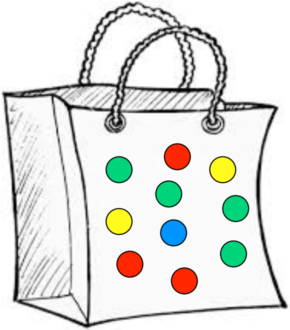

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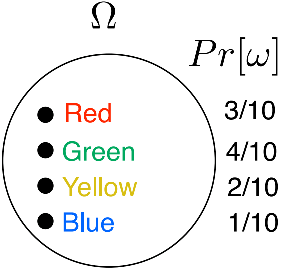


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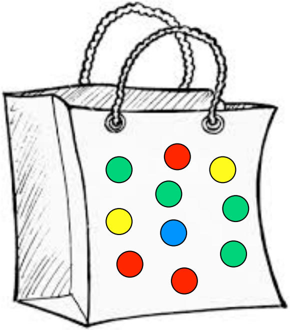

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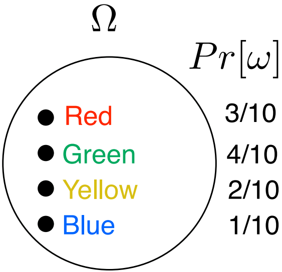


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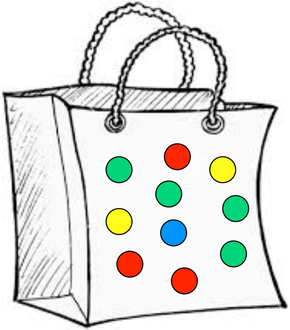

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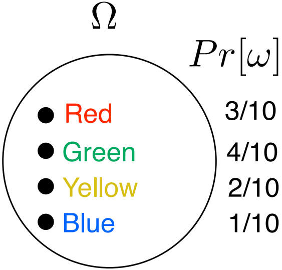


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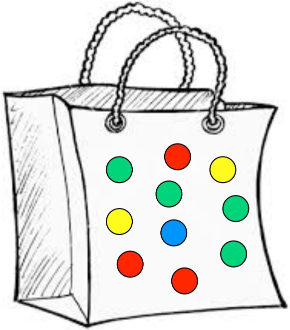

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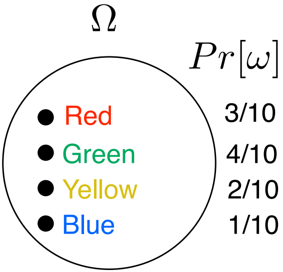


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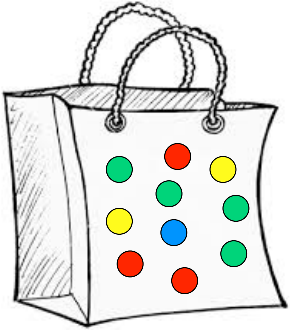

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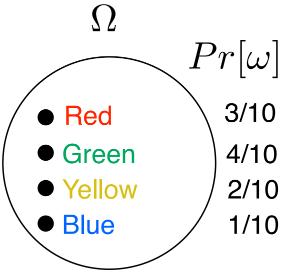


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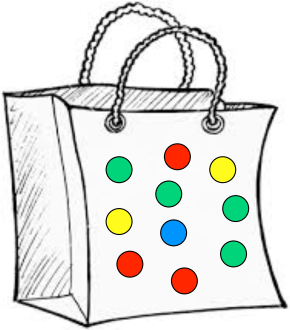

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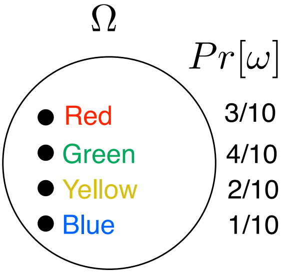


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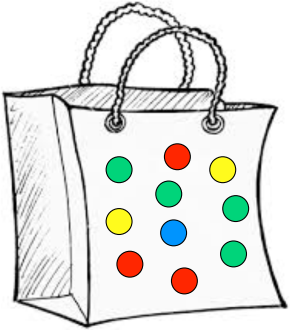

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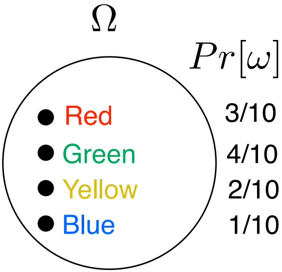


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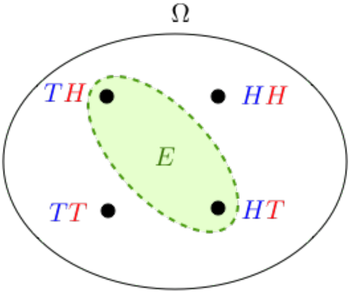

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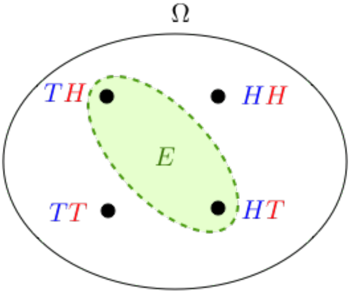


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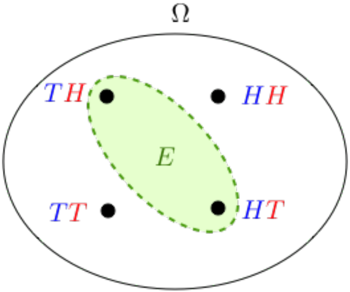


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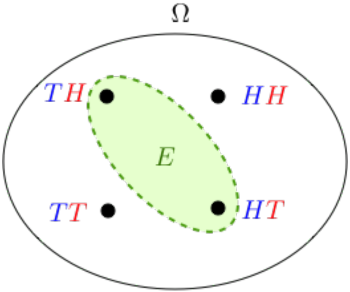


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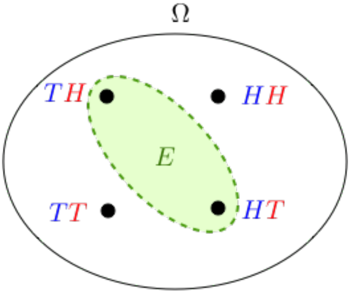


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Roll a red and a blue die.

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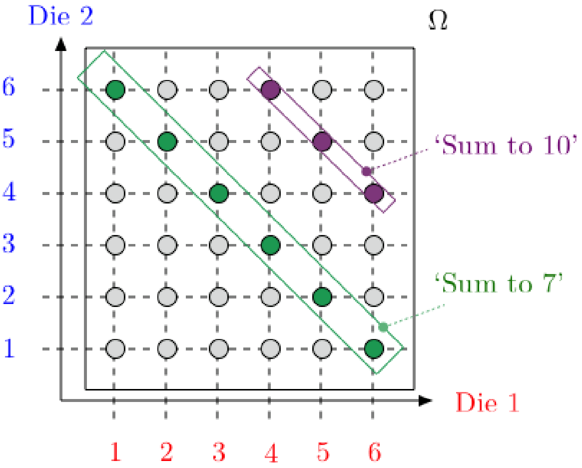


$$Pr[\text{Sum to 7}] = \frac{6}{36} \qquad Pr[\text{Sum to 10}] = \frac{3}{36}$$

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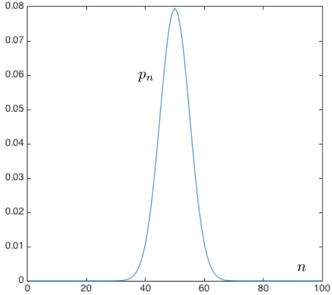

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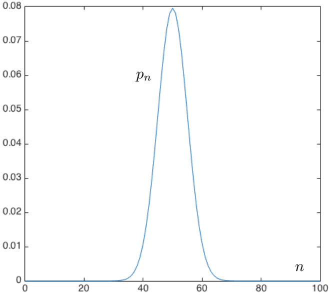


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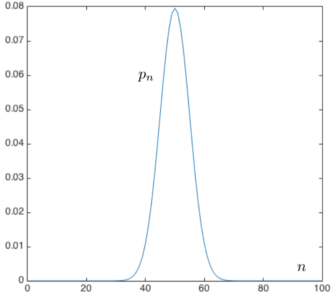


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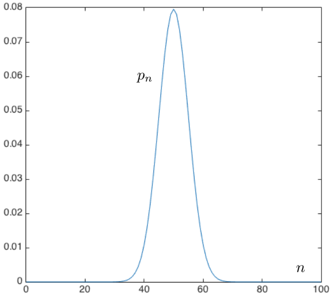


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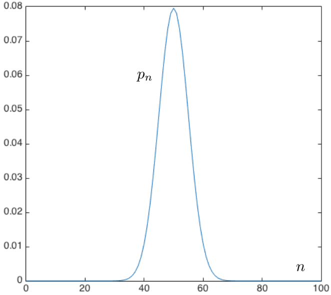


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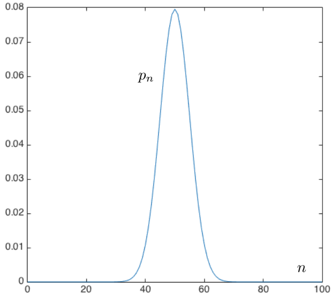


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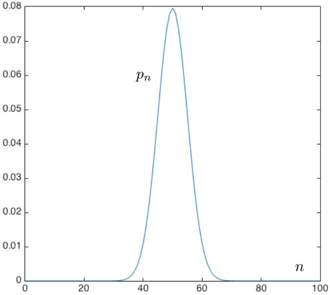


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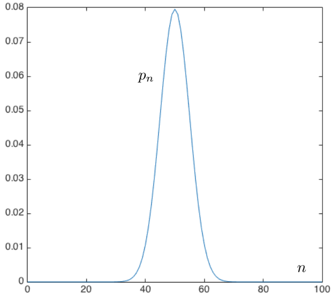


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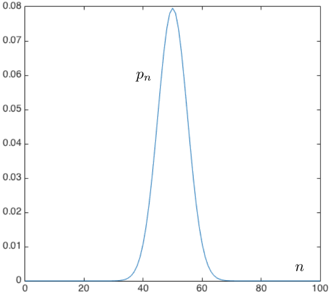


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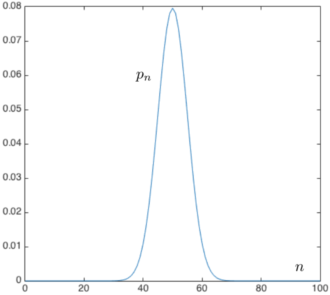


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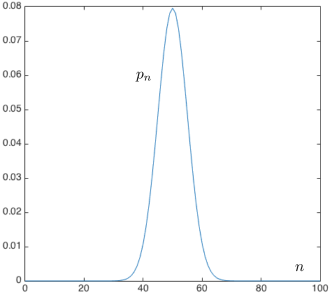


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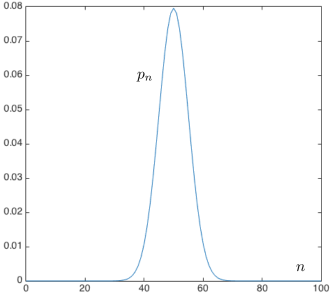


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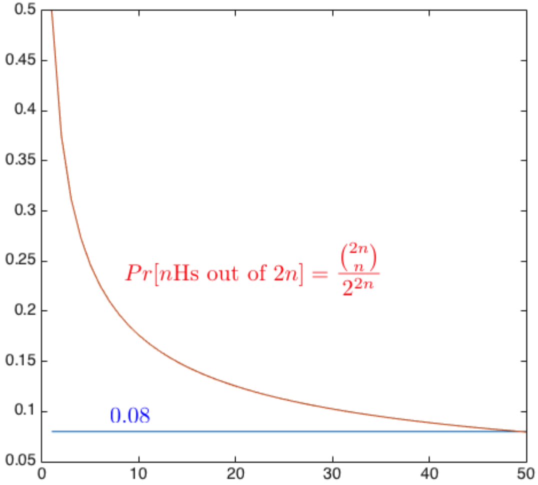

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