Climb an infinite ladder?

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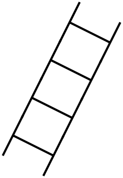
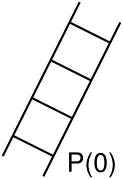


Climb an infinite ladder?

$P(0)$



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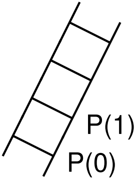
$$\forall k, P(k) \implies P(k+1)$$



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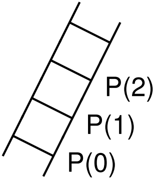
$$P(0) \implies P(1) \implies P(2)$$



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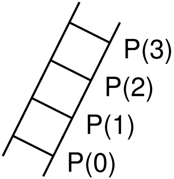

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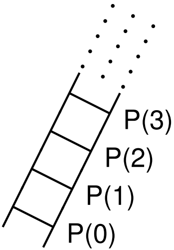

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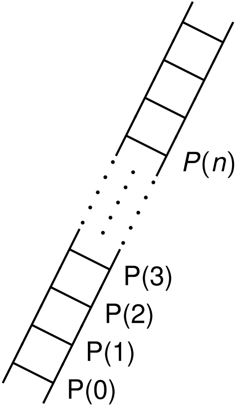

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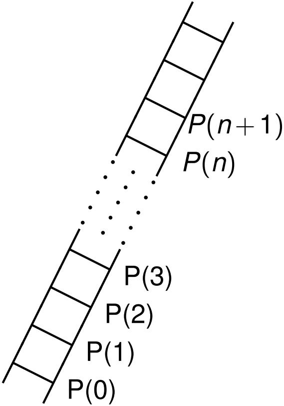

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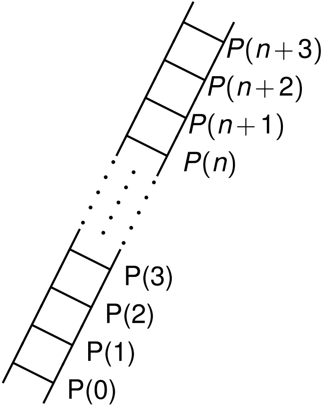

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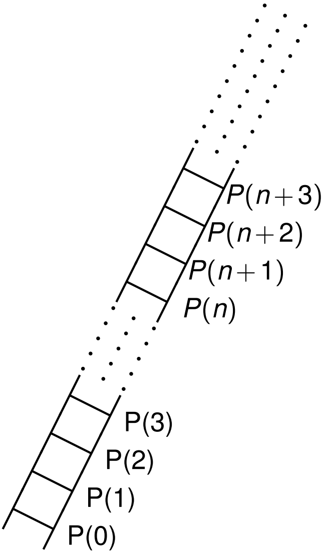

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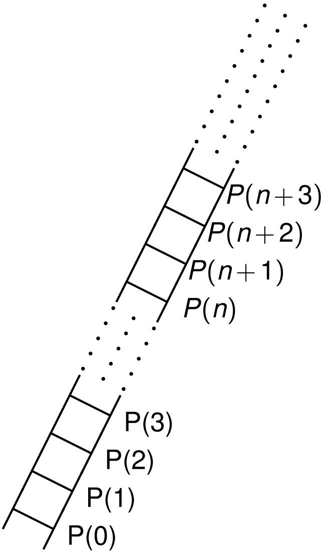

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Your favorite example of forever..

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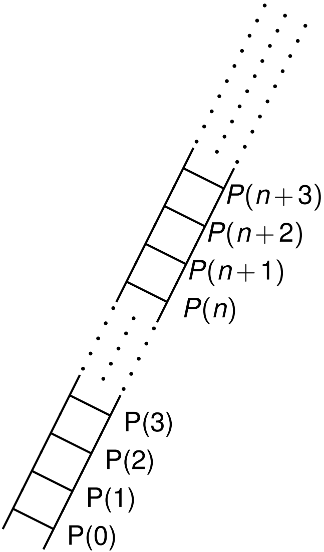

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Your favorite example of forever..or the natural numbers...

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Island with 100 possibly blue-eyed and green-eyed inhabitants.

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Thm: If there are n villagers with green eyes they leave on day n.

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Wait! Visitor added no information.

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If 66 villagers out of the 100 had green eyes, what would happen?

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Debate is tonight. Do vote.

Strong Induction and Recursion.

Thm: For every natural number $n \geq 12$, $n = 4x + 5y$.

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     (x',y') = find-x-y(n-4)
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     (x',y') = find-x-y(n-4)
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Prove: Given n, returns (x, y) where $n = 4x + 5y$, for $n \geq 12$.

Base cases: P(12) , P(13) , P(14) , P(15).

Strong Induction and Recursion.

Thm: For every natural number $n \geq 12$, $n = 4x + 5y$.

Instead of proof, let's write some code!

```
def find-x-y(n):
   if (n==12) return (3,0)
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Slight differences: showed for all $n \geq 16$ that $\wedge_{i=4}^{n-1} P(i) \implies P(n)$.

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The induction principle works on the natural numbers.

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Proves statements of form: $\forall n \in \mathbb{N}, P(n)$.

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Can you do induction over other things? Yes.

Any set where any subset of the set has a smallest element.

In some sense, the natural numbers.

Stable Matching Problem

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- n candidates and n jobs.

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- n candidates and n jobs.
- Each job has a ranked preference list of candidates.

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Stable Matching Problem

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How should they be matched?

- Maximize total satisfaction.

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How should they be matched?

- Maximize total satisfaction.
- Maximize number of first choices.
- Maximize worse off.
- Minimize difference between preference ranks.

The best laid plans..

Consider the pairs..

- Cal Bears and the Pac-12
- Wake Forest and the ACC

The best laid plans..

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Cal Bears prefers the ACC

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The ACC prefers Cal Bears.

Uh..oh.

The best laid plans..

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- Cal Bears and the Pac-12
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The ACC prefers Cal Bears.

Uh..oh. Sad Pac-12, (and Wake Forest.)

So..

Produce a matching where there are no crazy moves!

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Definition: A **matching** is disjoint set of n job-candidate pairs.

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Produce a matching where there are no crazy moves!

Definition: A **matching** is disjoint set of *n* job-candidate pairs.

Example: A matching
$S = \{(CalBears, Pac-12); (WakeForest, ACC)\}$.

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b and g^* prefer each other to their partners in S

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Example: Cal Bears and the ACC are a rogue couple in S.

Not a great example of stable matching, but interesting exercise in "selfish" incentives.

A stable matching??

Given a set of preferences.

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Is there a stable matching?

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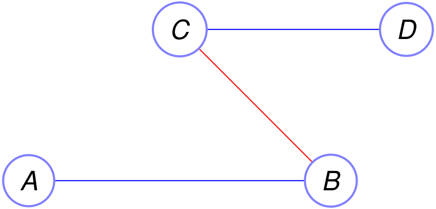
Consider a single type version: stable roommates.

A	B	C	D
B	C	A	D
C	A	B	D
D	A	B	C



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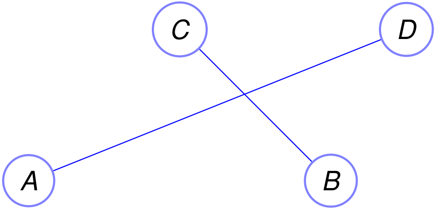

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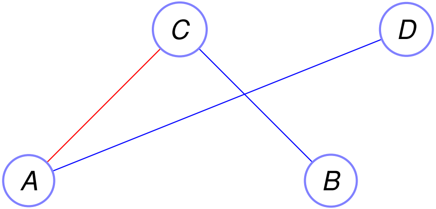

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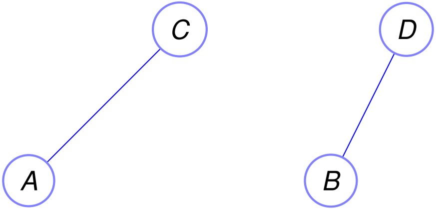

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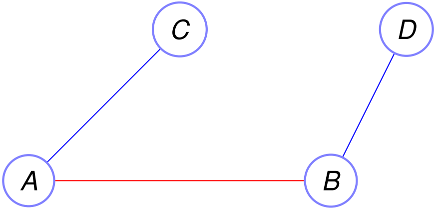

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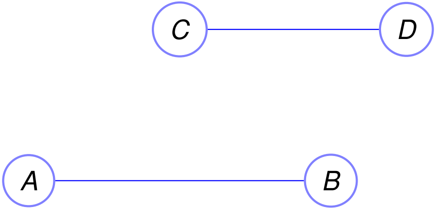

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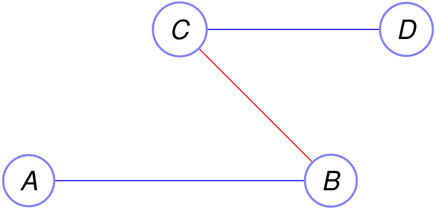

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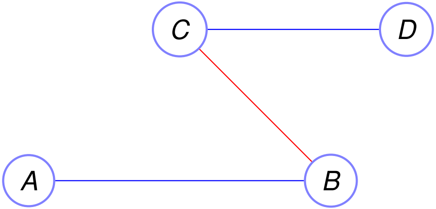

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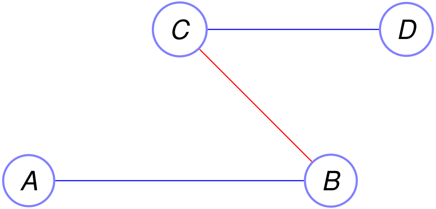

The Propose and Reject Algorithm.

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Each Day:

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1. Each job **proposes** to its favorite candidate on its list.

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Stop when each job gets exactly one proposal.

Example.

Jobs			
A	1	2	3
B	1	2	3
C	2	1	3

Candidates			
1	C	A	B
2	A	B	C
3	A	C	B

Example.

Jobs			
A	1	2	3
B	1	2	3
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Candidates			
1	C	A	B
2	A	B	C
3	A	C	B

	Day 1	Day 2	Day 3	Day 4	Day 5
1					
2					
3					

Example.

Jobs			
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Candidates			
1	C	A	B
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3	A	C	B

	Day 1	Day 2	Day 3	Day 4	Day 5
1	A, B				
2	C				
3					

Example.

Jobs			
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1	C	A	B
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	Day 1	Day 2	Day 3	Day 4	Day 5
1	A, B				
2	C				
3					

Example.

Jobs			
A	1	2	3
B	~~1~~ X	2	3
C	2	1	3

Candidates			
1	C	A	B
2	A	B	C
3	A	C	B

	Day 1	Day 2	Day 3	Day 4	Day 5
1	A, ~~B~~ X	A			
2	C	B, C			
3					

Example.

Jobs			
A	1	2	3
B	~~1~~	2	3
C	~~2~~	1	3

Candidates			
1	C	A	B
2	A	B	C
3	A	C	B

	Day 1	Day 2	Day 3	Day 4	Day 5
1	A, ~~B~~	A			
2	C	B, ~~C~~			
3					

Example.

Jobs			
A	1	2	3
B	~~1~~	2	3
C	~~2~~	1	3

Candidates			
1	C	A	B
2	A	B	C
3	A	C	B

	Day 1	Day 2	Day 3	Day 4	Day 5
1	A, ~~B~~	A	A , C		
2	C	B, ~~C~~	B		
3					

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A	~~1~~	2	3
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3					

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2	C	B, ~~C~~	B	A,B	
3					

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C	~~2~~	1	3

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	Day 1	Day 2	Day 3	Day 4	Day 5
1	A, ~~B~~	A	~~A~~ , C	C	
2	C	B, ~~C~~	B	A, ~~B~~	
3					

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C	~~2~~	1	3

Candidates			
1	C	A	B
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2	C	B, ~~C~~	B	A, ~~B~~	A
3					B

Example.

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2	A	B	C
3	A	C	B

	Day 1	Day 2	Day 3	Day 4	Day 5
1	A, ~~B~~	A	~~A~~ , C	C	C
2	C	B, ~~C~~	B	A, ~~B~~	A
3					B

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Does this terminate?

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Terminates in $\leq n^2$ steps!

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(B) When the economy is good.

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Economics: Study of choice. Freedom of choice.

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Poll: The argument for termination ...

(A) Implies: no unmatched job at end.

(B) Uses Improvement Lemma: every candidate matched.

(C) From Algorithm: unmatched job would ask everyone.

(D) Implies: every one gets their favorite job.

Poll: The argument for termination ...

(A) Implies: no unmatched job at end.

(B) Uses Improvement Lemma: every candidate matched.

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(D) Implies: every one gets their favorite job.

(D) is false.

Matching is Stable.

Lemma: There is no rogue couple for the matching formed by traditional marriage algorithm.

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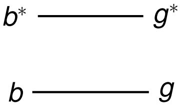

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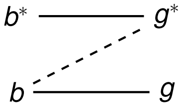

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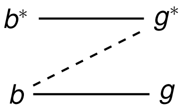

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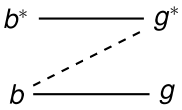

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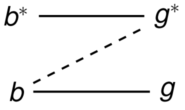

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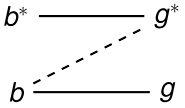

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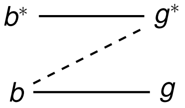

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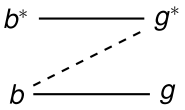

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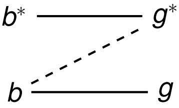

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Poll

Proof of Job Propose and Reject produces stable pairing uses?

(A) Contradiction.

(B) Uses the improvement lemma.

(C) Induction.

(D) Direct.

(E) The algorithm description.

Poll

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Poll

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(A), (B), (C), (E).(Maybe (D) internally. Semantics.)

Good for jobs? candidates?

Is the Job-Proposes better for jobs?

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B:	1,2	2:	B,A

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Consider pairing: $(A,1),(B,2)$.

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Pairing T: $(A,2),(B,1)$.

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Which is optimal for 1?

Understanding Optimality: by example.

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B:	1,2	2:	B,A

Consider pairing: $(A,1),(B,2)$.

Stable? Yes.

Optimal for B?

Notice: only one stable pairing.

If (A,2) are pair, (A,1) is rogue couple.

So this is the best B can do in a stable pairing.

So optimal for B.

Also optimal for A, 1 and 2. Also pessimal for A,B,1 and 2.

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Pairing S: $(A,1),(B,2)$. Stable? Yes.

Pairing T: $(A,2),(B,1)$. Also Stable.

Which is optimal for A? S Which is optimal for B? S

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Pessimality?

Job Propose and Candidate Reject is optimal!

For jobs?

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For jobs? For candidates?

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Theorem: Job Propose and Reject produces a job-optimal pairing.

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Structural statement: Job optimality

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Quick Questions.

How does one make it better for candidates?

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Propose and Reject - stable matching algorithm. One side proposes.

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Residency Matching..

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The method was used to match residents to hospitals.

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Takeaways.

Analysis of cool algorithm with interesting goal: stability.

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