Lecture Outline

Strengthening Induction Hypothesis.

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Strong Induction

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Strengthening Induction Hypothesis.

Strong Induction

Well ordered principle.

Strengthening Induction Hypothesis.

Theorem: The sum of the first *n* odd numbers is a perfect square.

Strengthening Induction Hypothesis.

Theorem: The sum of the first n odd numbers is a perfect square.

Theorem: The sum of the first n odd numbers is n^2.

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kth odd number is $2k-1$ for $k \geq 1$.

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Base Case 1 (1st odd number) is 1^2.

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Base Case: 1 (1st odd number) is 1^2.

Induction Hypothesis: Sum of first k odds is perfect square $a^2 = k^2$.

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Induction Step To prove that sum of first $k+1$ odds is $(k+1)^2$.

Strengthening Induction Hypothesis.

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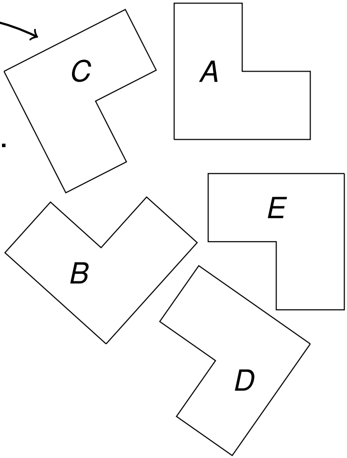
1. The $(k+1)$st odd number is $2(k+1)-1 = 2k+1$.
2. Sum of the first $k+1$ odds is $a^2 + 2k + 1 = k^2 + 2k + 1$
3. $k^2 + 2k + 1 = (k+1)^2$
   ... P(k+1)!

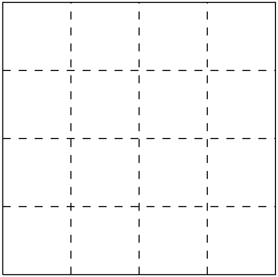
□

Tiling Cory Hall Courtyard.

Use these *L*-tiles.



To Tile this 4×4 courtyard.



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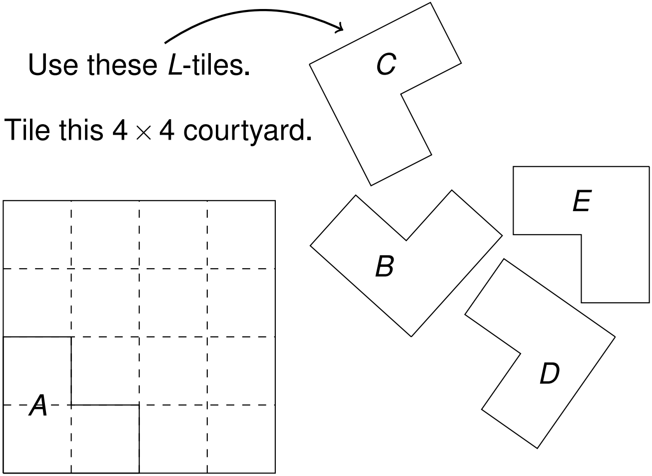

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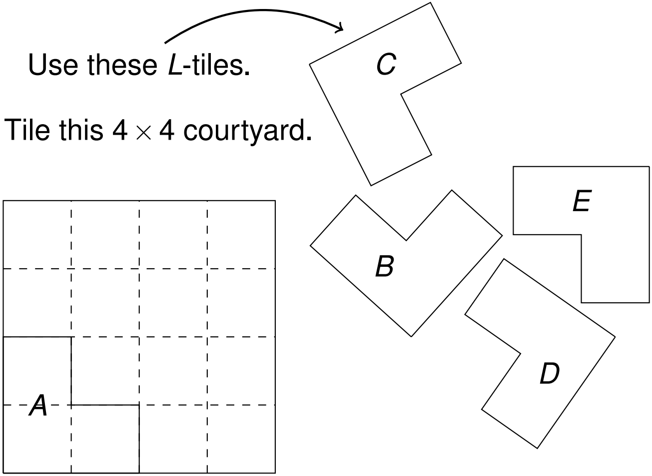

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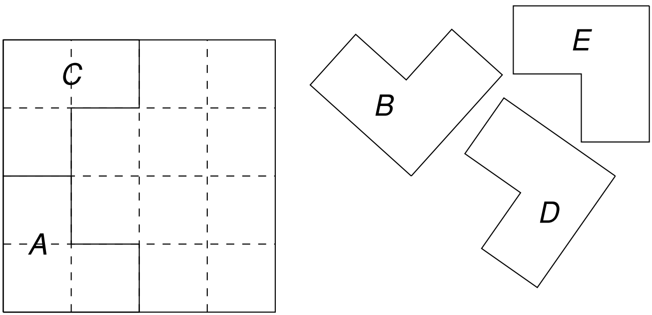

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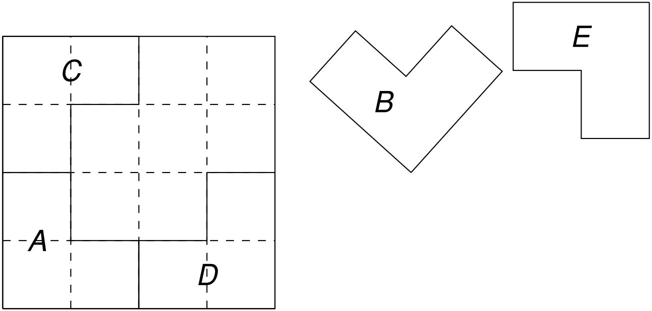

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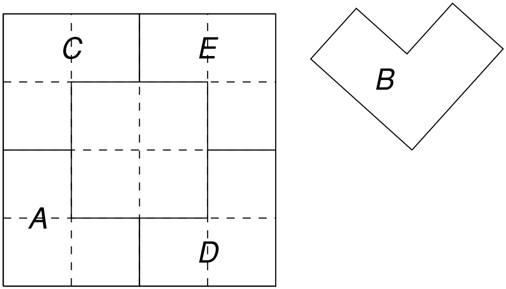

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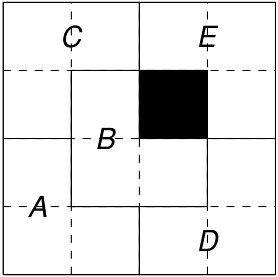

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Use these L-tiles.

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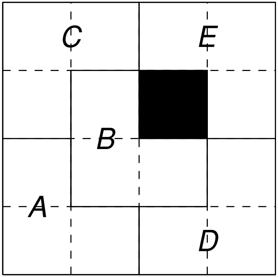

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Tiling Cory Hall Courtyard.

Use these L-tiles.

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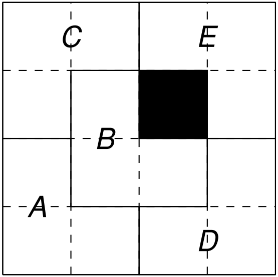


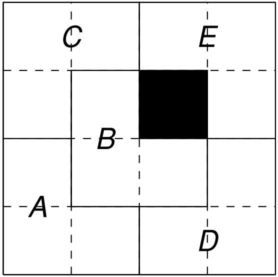
Alright!

Tiled 4×4 square with 2×2 L-tiles.

Tiling Cory Hall Courtyard.

Use these L-tiles.

To Tile this 4×4 courtyard.



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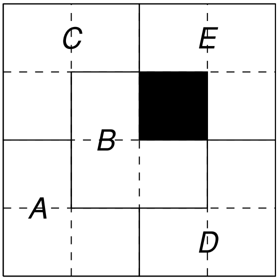
Tiled 4×4 square with 2×2 L-tiles.

with a center hole.

Tiling Cory Hall Courtyard.

Use these L-tiles.

To Tile this 4×4 courtyard.



Alright!

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with a center hole.

Can we tile any $2^n \times 2^n$ with L-tiles (with a hole)

Tiling Cory Hall Courtyard.

Use these L-tiles.

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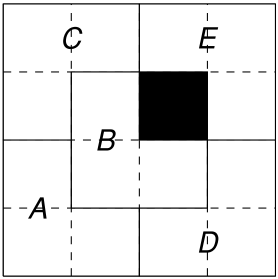

Alright!

Tiled 4×4 square with 2×2 L-tiles.

with a center hole.

Can we tile any $2^n \times 2^n$ with L-tiles (with a hole) for every n!

Hole have to be there? Maybe just one?

Theorem: Any tiling of $2^n \times 2^n$ square has to have one hole.

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Theorem: Any tiling of $2^n \times 2^n$ square has to have one hole.

Proof: Each tile covers 3 squares. The remainder of 2^{2n} divided by 3 is 1.

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$$2^{2(k+1)}$$

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$$2^{2(k+1)} = 2^{2k} * 2^2$$

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\begin{aligned}
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&= 4 * 2^{2k}
\end{aligned}
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$$\begin{aligned} 2^{2(k+1)} &= 2^{2k} * 2^2 \\ &= 4 * 2^{2k} \\ &= 4 * (3a + 1) \\ &= 12a + 3 + 1 \\ &= 3(4a + 1) + 1 \end{aligned}$$

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a integer $\Longrightarrow$ $(4a + 1)$ is an integer.

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a integer $\implies (4a + 1)$ is an integer. □

Hole in center?

Theorem: Can tile the $2^n \times 2^n$ square to leave a hole adjacent to the center.

Proof:

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The hole is adjacent to the center of the 2×2 square.

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Induction Hypothesis:

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Induction Hypothesis:
Any $2^n \times 2^n$ square can be tiled with a hole at the center.

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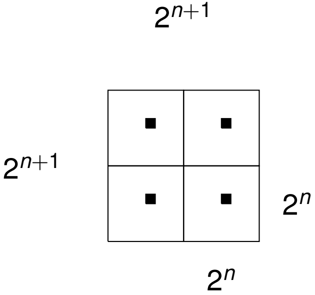

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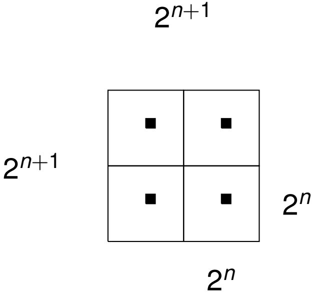

What to do now???

Hole can be anywhere!

Theorem: Can tile the $2^n \times 2^n$ to leave a hole adjacent *anywhere.*

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Induction Hypothesis:
"Any $2^n \times 2^n$ square can be tiled with a hole **anywhere.**"
Consider $2^{n+1} \times 2^{n+1}$ square.

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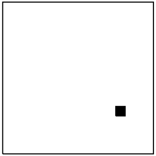

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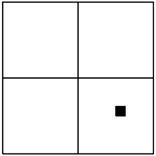

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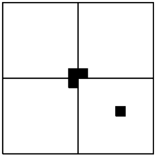

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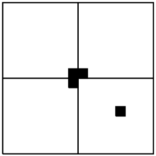

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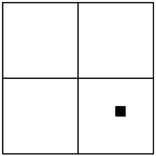

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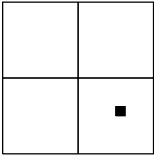

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Strong Induction: Example

Theorem: Every natural number $n > 1$ is either a prime or can be written as a product of primes.

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$P(n)$ says nothing about a, b!

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Theorem: Every natural number $n > 1$ is either a prime or can be written as a product of primes.

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The **Well ordering principle** states that for any subset of the natural numbers there is a smallest element.

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Slight differences: showed for all $n \geq 16$ that $\wedge_{i=4}^{n-1} P(i) \implies P(n)$.

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New Base Case: $P(2)$: there are two horses with same color.

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Fix base case.

Horses of the same color...

Theorem: All horses have the same color.

Base Case: $P(1)$ - trivially true.
New Base Case: $P(2)$: there are two horses with same color.

Induction Hypothesis: $P(k)$ - Any k horses have the same color.

Induction step $P(k+1)$?
First k have same color by $P(k)$.
Second k have same color by $P(k)$.
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As we will see, it is more subtle to catch errors in proofs of correct theorems!!

Summary: principle of induction.

Today: More induction.

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$(P(0) \wedge ((\forall k \in N)(P(k) \implies P(k+1))))$

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Statement to prove: $P(n)$ for n starting from n_0

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Base Case: Prove $P(n_0)$.
Ind. Step: Prove.

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$(P(0) \wedge ((\forall k \in N)(P(k) \implies P(k+1)))) \implies (\forall n \in N)(P(n))$

Statement to prove: $P(n)$ for n starting from n_0
Base Case: Prove $P(n_0)$.
Ind. Step: Prove. For all values, $n \geq n_0$, $P(n) \implies P(n+1)$.

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Strong Induction:

Summary: principle of induction.

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Strong Induction:
$(P(0) \wedge ((\forall n \leq kP(n)) \implies P(k+1)))$

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Also Today: strengthened induction hypothesis.

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Also Today: strengthened induction hypothesis.

Strengthen theorem statement.
Sum of first n odds is n^2.
Hole anywhere.
Not same as strong induction.

Summary: principle of induction.

Today: More induction.

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Induction $\equiv$ Recursion.