CS 70 Discrete Mathematics and Probability Theory
Spring 2018 Satish Rao and Babak Ayazifar

DIS 0A

1 Writing in Propositional Logic

For each of the following sentences, translate the sentence into propositional logic using the notation introduced in class, and write its negation.

(a) The square of a nonzero integer is positive.

(b) There are no integer solutions to the equation $x^2 - y^2 = 10$.

(c) There is one and only one real solution to the equation $x^3 + x + 1 = 0$.

(d) For any two distinct real numbers, we can find a rational number in between them.

2 Implication

Which of the following implications are always true, regardless of P? Give a counterexample for each false assertion (i.e. come up with a statement $P(x,y)$ that would make the implication false).

(a) $\forall x, \forall y, P(x,y) \implies \forall y, \forall x, P(x,y)$.

(b) $\exists x, \exists y, P(x,y) \implies \exists y, \exists x, P(x,y)$.

(c) $\forall x, \exists y, P(x,y) \implies \exists y, \forall x, P(x,y)$.

(d) $\exists x, \forall y, P(x,y) \implies \forall y, \exists x, P(x,y)$.

3 Logic

Decide whether each of the following is true or false and justify your answer:

(a) $\forall x\ \big(P(x) \land Q(x)\big) \equiv \forall x\ P(x) \land \forall x\ Q(x)$

(b) $\forall x\ \big(P(x) \lor Q(x)\big) \equiv \forall x\ P(x) \lor \forall x\ Q(x)$

(c) $\exists x\ \big(P(x) \lor Q(x)\big) \equiv \exists x\ P(x) \lor \exists x\ Q(x)$

(d) $\exists x\ \big(P(x) \land Q(x)\big) \equiv \exists x\ P(x) \land \exists x\ Q(x)$