CS 70 Discrete Mathematics and Probability Theory
Fall 2018 Alistair Sinclair and Yun Song **DIS 0B**

1 Propositional Logic Language

For each of the following sentences, use the notation introduced in class to convert the sentence into propositional logic. Then write the statement's negation in propositional logic.

(a) The cube of a negative integer is negative.

(b) There are no integer solutions to the equation $x^2 - y^2 = 10$.

(c) There is one and only one real solution to the equation $x^3 + x + 1 = 0$.

(d) For any two distinct real numbers, we can find a rational number in between them.

2 Implication

Which of the following implications are always true, regardless of P? Give a counterexample for each false assertion (i.e. come up with a statement $P(x,y)$ that would make the implication false).

(a) $\forall x \forall y P(x,y) \implies \forall y \forall x P(x,y)$.

(b) $\exists x \exists y P(x,y) \implies \exists y \exists x P(x,y)$.

(c) $\forall x \exists y P(x,y) \implies \exists y \forall x P(x,y)$.

(d) $\exists x \forall y P(x,y) \implies \forall y \exists x P(x,y)$.

3 Logic

Decide whether each of the following is true or false and justify your answer:

(a) $\forall x\,\big(P(x) \land Q(x)\big) \;\equiv\; \forall x\,P(x) \land \forall x\,Q(x)$

(b) $\forall x\,\big(P(x) \lor Q(x)\big) \;\equiv\; \forall x\,P(x) \lor \forall x\,Q(x)$

(c) $\exists x\,\big(P(x) \lor Q(x)\big) \;\equiv\; \exists x\,P(x) \lor \exists x\,Q(x)$

(d) $\exists x\,\big(P(x) \land Q(x)\big) \;\equiv\; \exists x\,P(x) \land \exists x\,Q(x)$