CS 70 Discrete Mathematics and Probability Theory
Fall 2017 Satish Rao and Kannan Ramchandran

DIS 1A

1 Perfect Square

A *perfect square* is an integer n of the form $n = m^2$ for some integer m. Prove that every odd perfect square is of the form $8k+1$ for some integer k.

2 Pigeonhole Principle

Prove the following statement: If you put $n+1$ balls into n bins, however you want, then at least one bin must contain at least two balls. This is known as the *pigeonhole principle*.

3 Numbers of Friends

Prove that if there are $n \geq 2$ people at a party, then at least 2 of them have the same number of friends at the party.

4 Induction

Prove the following using induction:

(a) For all natural numbers $n > 2$, $2^n > 2n+1$.

(b) For all positive integers n, $1^3 + 3^3 + 5^3 + \cdots + (2n-1)^3 = n^2(2n^2-1)$.

(c) For all positive natural numbers n, $(5/4)8^n + 3^{3n-1}$ is divisible by 19.