

## 1 Short Proofs

- (a) Prove that  $\sqrt[12]{2}$  is irrational.
- (b) Prove that  $\sum_{i=1}^n i = n(n+1)/2$ .

## 2 Infinite Primes

Prove by contradiction that there are an infinite number of primes.

## 3 Proof by?

- (a) Prove that if  $x, y \in \mathbb{Z}$ , if 10 does not divide  $xy$ , then 10 does not divide  $x$  and 10 does not divide  $y$ . In notation:  $(\forall x, y \in \mathbb{Z}) 10 \nmid xy \implies (10 \nmid x \wedge 10 \nmid y)$ . What proof technique did you use?
- (b) Prove or disprove the contrapositive.
- (c) Prove or disprove the converse.

## 4 Perfect Square

A *perfect square* is an integer  $n$  of the form  $n = m^2$  for some integer  $m$ . Prove that every odd perfect square is of the form  $8k + 1$  for some integer  $k$ .