1 Logic

Decide whether each of the following is true or false and justify your answer:

(a) $\forall x\, \big(P(x) \wedge Q(x)\big) \;\equiv\; \forall x\, P(x) \wedge \forall x\, Q(x)$

(b) $\forall x\, \big(P(x) \vee Q(x)\big) \;\equiv\; \forall x\, P(x) \vee \forall x\, Q(x)$

(c) $\exists x\, \big(P(x) \vee Q(x)\big) \;\equiv\; \exists x\, P(x) \vee \exists x\, Q(x)$

(d) $\exists x\, \big(P(x) \wedge Q(x)\big) \;\equiv\; \exists x\, P(x) \wedge \exists x\, Q(x)$

2 Contraposition

Prove the statement "if $a+b < c+d$, then $a < c$ or $b < d$".

3 Perfect Square

A *perfect square* is an integer n of the form $n = m^2$ for some integer m. Prove that every odd perfect square is of the form $8k+1$ for some integer k.

4 Numbers of Friends

Prove that if there are $n \geq 2$ people at a party, then at least 2 of them have the same number of friends at the party.
(Hint: The Pigeonhole Principle states that if n items are placed in m containers, where $n > m$, at least one container must contain more than one item. You may use this without proof.)

5 Fermat's Contradiction

Prove that $2^{1/n}$ is not rational for any integer $n \geq 3$. (*Hint*: Use Fermat's Last Theorem. It states that there exists no positive integers a, b, c s.t. $a^n + b^n = c^n$ for $n \geq 3$.)