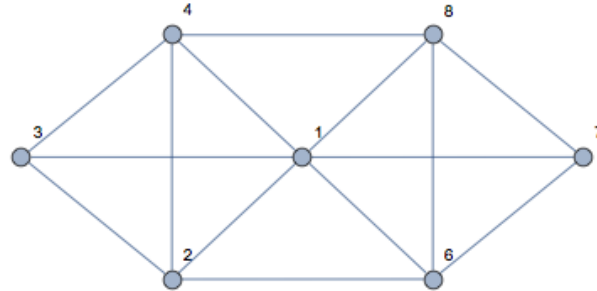


1 Eulerian Tour and Eulerian Walk



- Is there an Eulerian tour in the graph above? If no, give justification. If yes, provide an example.
- Is there an Eulerian walk in the graph above? An Eulerian walk is a walk that uses each edge exactly once. If no, give justification. If yes, provide an example.
- What is the condition that there is an Eulerian walk in an undirected graph? Briefly justify your answer.

2 Banquet Arrangement

In the words of the great Ana Lynch, “Let’s have a kiki.”

Suppose n people are attending a kiki, and each of them has at least m friends ($2 \leq m \leq n$), where friendship is mutual. Prove that we can put at least $m + 1$ of the attendants on the same round table, so that each person sits next to his or her friends on both sides.

3 Not everything is normal: Odd-Degree Vertices

Claim: Let $G = (V, E)$ be an undirected graph. The number of vertices of G that have odd degree is even.

Prove the claim above using:

- (i) Direct proof (e.g., counting the number of edges in G). *Hint: in lecture, we proved that $\sum_{v \in V} \deg v = 2|E|$.*
- (ii) Induction on $m = |E|$ (number of edges)
- (iii) Induction on $n = |V|$ (number of vertices)