

## 1 Balls and Bins

You have  $n$  empty bins and you throw balls into them one by one randomly. A collision is when a ball is thrown into a bin which already has another ball.

- (a) What is the probability that the first ball thrown will cause the first collision?
- (b) What is the probability that the second ball thrown will cause the first collision?
- (c) What is the probability that, given the first two balls are not in collision, the third ball thrown will cause the first collision?
- (d) What is the probability that the third ball thrown will cause the first collision?
- (e) What is the probability that, given the first  $m - 1$  balls are not in collision, the  $m^{\text{th}}$  ball thrown will cause the first collision?
- (f) What is the probability that the  $m^{\text{th}}$  ball thrown will cause the first collision?

## 2 To Be Fair

Suppose you have a biased coin with  $\mathbb{P}(\text{heads}) \neq 0.5$ . How could you use this coin to simulate a fair coin? (*Hint*: Think about pairs of tosses.)

### 3 Poisoned Smarties

Supposed there are 3 men who are all owners of their own Smarties factories. Burr Kelly, being the brightest and most innovative of the men, produces considerably more Smarties than his competitors and has a commanding 45% of the market share. Yousef See, who inherited his riches, lags behind Burr and produces 35% of the world's Smarties. Finally Stan Furd, brings up the rear with a measly 20%. However, a recent string of Smarties related food poisoning has forced the FDA investigate these factories to find the root of the problem. Through his investigations, the inspector found that one Smarty out of every 100 at Kelly's factory was poisonous. At See's factory, 1.5% of Smarties produced were poisonous. And at Furd's factory, the probability a Smarty was poisonous was 0.02.

- (a) What is the probability that a randomly selected Smarty will be safe to eat?
- (b) If we know that a certain Smarty didn't come from Burr Kelly's factory, what is the probability that this Smarty is poisonous?
- (c) Given this information, if a randomly selected Smarty is poisonous, what is the probability it came from Stan Furd's Smarties Factory?

### 4 Rain and Wind

The local weather channel just released a statistic for the months of November and December. It said that the probability that it would rain on a windy day is 0.3 and the probability that it would rain on a non-windy day is 0.8. The probability of a day being windy is 0.2. As a student in EECS 70, you are curious to play around with these numbers. Find the probability that:

- (a) A given day is both windy and rainy.
- (b) A given day is rainy.
- (c) For a given pair of days, exactly one of the two days is rainy.
- (d) A given day that is non-rainy is also non-windy.