CS 70 Discrete Mathematics and Probability Theory
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DIS 4A

1 Hello World!

Determine the computability of the following tasks. If it's not computable, write a reduction or self-reference proof. If it is, write the program.

(a) You want to determine whether a program P on input x prints "Hello World!". Is there a computer program that can perform this task? Justify your answer.

(b) You want to determine whether a program P prints "Hello World!" before running the kth line in the program. Is there a computer program that can perform this task? Justify your answer.

(c) You want to determine whether a program P prints "Hello World!" in the first k steps of its execution. Is there a computer program that can perform this task? Justify your answer.

2 Code Reachability

Consider triplets (M, x, L) where

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M is a Java program
x is some input
L is an integer
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and the question of: if we execute $M(x)$, do we ever hit line L?

Prove this problem is undecidable.

3 Kolmogorov Complexity

Compression of a bit string x of length n involves creating a program shorter than n bits that returns x. The Kolmogorov complexity of a string $K(x)$ is the length of shortest program that returns x (i.e. the length of a maximally compressed version of x).

(a) Explain why "the smallest positive integer not definable in under 100 characters" is paradoxical.

(b) Prove that for any length n, there must be at least one bit string that cannot be compressed.

(c) Imagine you had the program K, which outputs the Kolmogorov complexity of string. Design a program P that when given integer n outputs the bit string of length n with the highest Kolmogorov complexity. If there are multiple strings with the highest complexity, output the lexicographically first (i.e. the one that would come first in a dictionary).

(d) Suppose the program P you just wrote can be written in m bits. Show that P and by extension, K, cannot exist, for a sufficiently large input n.

(e) Consider the program I, which can be written in m bits, that when given any input string x returns true if x is incompressible and returns false otherwise. Show that such a program cannot exist.