

1 Variance

This problem will give you practice using the "standard method" to compute the variance of a sum of random variables that are not pairwise independent (so you cannot use "linearity" of variance).

- (a) A building has n floors numbered $1, 2, \dots, n$, plus a ground floor G. At the ground floor, m people get on the elevator together, and each person gets off at one of the n floors uniformly at random (independently of everybody else). What is the *variance* of the number of floors the elevator *does not* stop at? (In fact, the variance of the number of floors the elevator *does* stop at must be the same, but the former is a little easier to compute.)

- (b) A group of three friends has n books they would all like to read. Each friend (independently of the other two) picks a random permutation of the books and reads them in that order, one book per week (for n consecutive weeks). Let X be the number of weeks in which all three friends are reading the same book. Compute $\text{var}(X)$.

2 Correlation and Independence

- (a) What does it mean for two random variables to be uncorrelated?

- (b) What does it mean for two random variables to be independent?

- (c) Are all uncorrelated variables independent? Are all independent variables uncorrelated? If your answer is yes, justify your answer; if your answer is no, give a counterexample.

3 Covariance

We have a bag of 5 red and 5 blue balls. We take two balls from the bag without replacement. Let X_1 and X_2 be indicator random variables for the first and second ball being red. What is $\text{cov}(X_1, X_2)$?