1 Proof with Indicators

Let $n \in \mathbb{Z}_+$. Let $\alpha_1, \ldots, \alpha_n \in \mathbb{R}$ and let $A_1, \ldots, A_n$ be events. Prove that $\sum_{i=1}^n \sum_{j=1}^n \alpha_i \alpha_j \mathbb{P}(A_i \cap A_j) \geq 0$.

2 Binomial Conditioning

Let $n \in \mathbb{Z}_+$ and $p, q \in [0,1]$. Let $X \sim \text{Binomial}(n, p)$ and suppose that conditioned on $X = x$, $Y \sim \text{Binomial}(x, q)$. What is the unconditional distribution of Y?

3 The Memoryless Property Uniquely Characterizes the Geometric Distribution

Let X be a discrete random variable which takes on values on $\mathbb{Z}_+$. Suppose that for all $m, n \in \mathbb{N}$, we have $\mathbb{P}(X > m+n \mid X > n) = \mathbb{P}(X > m)$. Prove that X has the geometric distribution.