

1 Continuous Computations

Let X be a continuous random variable whose pdf is cx^3 (for some constant c) in the range $0 \leq x \leq 1$, and is 0 outside this range.

(a) Find c .

(b) Find $\mathbb{P}[1/3 \leq X \leq 2/3 \mid X \leq 1/2]$.

(c) Find $\mathbb{E}(X)$.

(d) Find $\text{var}(X)$.

2 Uniform Distribution

You have two spinners, each having a circumference of 10, with values in the range $[0, 10)$. If you spin both (independently) and let X be the position of the first spinner and Y be the position of the second spinner, what is the probability that $X \geq 5$, given that $Y \geq X$?

3 Exponential Distributions: Lightbulbs

A brand new lightbulb has just been installed in our classroom, and you know the life span of a lightbulb is exponentially distributed with a mean of 50 days.

- (a) Suppose an electrician is scheduled to check on the lightbulb in 30 days and replace it if it is broken. What is the probability that the electrician will find the bulb broken?

- (b) Suppose the electrician finds the bulb broken and replaces it with a new one. What is the probability that the new bulb will last at least 30 days?

- (c) Suppose the electrician finds the bulb in working condition and leaves. What is the probability that the bulb will last at least another 30 days?

4 Lunch Meeting

Alice and Bob agree to try to meet for lunch between 12 PM and 1 PM at their favorite sushi restaurant. Being extremely busy, they are unable to specify their arrival times exactly, and can say only that each of them will arrive (independently) at a time that is uniformly distributed within the hour. In order to avoid wasting precious time, if the other person is not there when they arrive they agree to wait exactly fifteen minutes before leaving. What is the probability that they will actually meet for lunch?