

## 1 Warm-up

For each of the following parts, you may leave your answer as an expression.

- (a) You throw darts at a board until you hit the center area. Assume that the throws are i.i.d. and the probability of hitting the center area is  $p = 0.17$ . What is the probability that you hit the center on your eighth throw?
- (b) Let  $X \sim \text{Geometric}(0.2)$ . Calculate the expectation and variance of  $X$ .
- (c) Suppose the accidents occurring weekly on a particular stretch of a highway is Poisson distributed with average number of accidents equal to 3 cars per week. Calculate the probability that there is at least one accident this week.
- (d) Consider an experiment that consists of counting the number of  $\alpha$  particles given off in a one-second interval by one gram of radioactive material. If we know from past experience that, on average, 3.2 such  $\alpha$ -particles are given off per second, what is a good approximation to the probability that no more than 2  $\alpha$ -particles will appear in a second?

## 2 Coupon Collector Variance

It's that time of the year again - Safeway is offering its Monopoly Card promotion. Each time you visit Safeway, you are given one of  $n$  different Monopoly Cards with equal probability. You need to collect them all to redeem the grand prize.

Let  $X$  be the number of visits you have to make before you can redeem the grand prize. Show that  $\text{Var}(X) = n^2 \left( \sum_{i=1}^n i^{-2} \right) - \mathbb{E}(X)$ . [Hint: Try to break this problem down using indicators as with the coupon collector's problem. Are the indicators independent?]