1 Zerg Player

A Zerg player wants to produce an army to fight against Protoss in early game, and he wants to have a small army which consumes exactly 10 supply. And he has the following choices:

- Zerglings: consumes 1 supply
- Hydralisk: consumes 2 supply
- Roach: consumes 2 supply

How many different compositions can the player's army have? Note that Zerglings are indistinguishable, as are Hydralisks and Roachs.

Solution: Let there are i 2-supply units have been made. For the rest of supply, we can fill it with zerglings.

And if there are i 2-supply unites, there are $i+1$ different compositions: 0 Hydra i Roach $10-2i$ zerglings, 1 Hydra $i-1$ Roach $10-2i$ zerglings, ..., i Hydra 0 Roach $10-2i$ zerglings.

Then we have $\sum_{i=0}^{5}(i+1) = 1+2+3+4+5+6 = 21$.

2 Strings

What is the number of strings you can construct given:

(a) n ones, and m zeroes?

(b) n_1 A's, n_2 B's and n_3 C's?

(c) $n_1, n_2, \ldots, n_k$ respectively of k different letters?

Solution:

(a) $\binom{n+m}{n}$

(b) $(n_1+n_2+n_3)!/(n_1!\cdot n_2!\cdot n_3!)$

(c) $(n_1+n_2+\cdots+n_k)!/(n_1!\cdot n_2!\cdots n_k!)$.

3 Counting Game

RPG games are all about explore different mazes. Here is a weird maze: there are 2^n rooms, where each room is the vertex on a the n-dimensional hypercube, labeled by a n bit binary string.

For each room, there are n different doors, each door corresponding to an edge on the hypercube. If you are at room i, and choose door j, then you will go to room $i \oplus 2^j$ (flips the $j+1$-th bit in number i).

(a) How many different shortest path are there from room 0 to room $2^n - 1$?

(b) How many different paths of $n+2$ steps are there to go from room 0 to room $2^n - 1$?

(c) If $n = 8$, how many different shortest pathes are there from room 0 to room 63 that pass through 3 and 19?

Solution:

(a) $n!$, the shortest path is n, and for the i-th step, there are only $n-i$ doors flips a zero to one.

(b) The player made one mistake during his trip, so suppose he made the mistake at step i, $i > 0$, so there are i different ways to make the mistake. Then he will start from a room with $n-i+1$ zeros. So the total number is $\sum_{i=1}^{n}(\binom{n}{i} * i! * i * (n-i+1)!)$.

Optional for further steps:

$\sum_{i=1}^{n}(\binom{n}{i} * i! * i * (n-i+1)!) = \sum_{i=1}^{n} \frac{n!*i!*i*(n-i+1)!}{(n-i)!i!)} = \sum_{i=1}^{n} n! * i * (n-i+1) = n! \sum_{i=1}^{n} i(n-i+1) = n! * (\sum_{i=1}^{n}(in - i^2 + i))$ where $\sum_{i=1}^{n}(in - i^2 + i) = n * \sum_{i=1}^{n} i - \sum_{i=1}^{n} i^2 + \sum_{i=1}^{n} i = \frac{n(n+1)(n+2)}{6}$

(c) From 0 to 3, 2 different pathes. From 3 to 19: notice $3 \oplus 19 = 16$ so there is only one way. From 19 to 63, there are 3 zeros in $63 \oplus 19$ so total 3! different pathes. In total $2 * 3!$ different pathes.