CS 70 Discrete Mathematics and Probability Theory
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DIS 6A

1 Correlation and Independence

(a) What does it mean for two random variables to be uncorrelated?

(b) What does it mean for two random variables to be independent?

(c) Are all uncorrelated variables independent? Are all independent variables uncorrelated? If your answer is yes, justify your answer; if your answer is no, give a counterexample.

2 Covariance

We have a bag of 5 red and 5 blue balls. We take two balls from the bag without replacement. Let X_1 and X_2 be indicator random variables for the first and second ball being red. What is $\text{cov}(X_1, X_2)$? Recall that $\text{cov}(X_1, X_2) = \mathbb{E}[XY] - \mathbb{E}[X]\,\mathbb{E}[Y]$.

3 Tale Sum

You're listening to a local storyteller reciting a tall tale. The storyteller will vary the length of the tale based on how excitedly the crowd seems to be listening; you've determined that for all $t > 0$, the probability that the story goes on for exactly t minutes is $\frac{t}{(t+1)!}$. What is the expected length of the tale?

Hint: You can write $\frac{t}{(t+1)!}$ as $\frac{t+1}{(t+1)!} - \frac{1}{(t+1)!}$.

Hint 2: Recall the power series for e^x: $e^x = \sum_{i=0}^{\infty} \frac{x^i}{i!}$.