1 Uniform Distribution

You have two fidget spinners, each having a circumference of 10. You mark one point on each spinner as a needle and place each of them at the center of a circle with values in the range [0, 10) marked on the circumference. If you spin both (independently) and let $X$ be the position of the first spinner’s mark and $Y$ be the position of the second spinner’s mark, what is the probability that $X \geq 5$, given that $Y \geq X$?
2 Continuous Joint Densities

The joint probability density function of two random variables \( X \) and \( Y \) is given by \( f(x,y) = C_{xy} \) for \( 0 \leq x \leq 1, 0 \leq y \leq 2 \), and 0 otherwise (for a constant \( C \)).

(a) Find the constant \( C \) that ensures that \( f(x,y) \) is indeed a probability density function.

(b) Find \( f_X(x) \), the marginal distribution of \( X \).

(c) Find the conditional distribution of \( Y \) given \( X = x \).

(d) Are \( X \) and \( Y \) independent?
3 Joint Distributions

(a) Give an example of discrete random variables $X$ and $Y$ with the property that $E[XY] \neq E[X]E[Y]$. You should specify the joint distribution of $X$ and $Y$.

(b) Give an example of discrete random variables $X$ and $Y$ that (i) are not independent and (ii) have the property that $E[XY] = 0$, $E[X] = 0$, and $E[Y] = 0$. Again you should specify the joint distribution of $X$ and $Y$. 