

1 Aces

Consider a standard 52-card deck of cards:

- (a) Find the probability of getting an ace or a red card, when drawing a single card.
- (b) Find the probability of getting an ace or a spade, but not both, when drawing a single card.
- (c) Find the probability of getting the ace of diamonds when drawing a 5 card hand.
- (d) Find the probability of getting exactly 2 aces when drawing a 5 card hand.
- (e) Find the probability of getting at least 1 ace when drawing a 5 card hand.
- (f) Find the probability of getting at least 1 ace or at least 1 heart when drawing a 5 card hand.

2 Balls and Bins, All Day Every Day

You throw n balls into n bins uniformly at random, where n is a positive *even* integer.

- (a) What is the probability that exactly k balls land in the first bin, where k is an integer $0 \leq k \leq n$?
- (b) What is the probability p that at least half of the balls land in the first bin? (You may leave your answer as a summation.)
- (c) Using the union bound, give a simple upper bound, in terms of p , on the probability that some bin contains at least half of the balls.

- (d) What is the probability, in terms of p , that at least half of the balls land in the first bin, or at least half of the balls land in the second bin?
- (e) After you throw the balls into the bins, you walk over to the bin which contains the first ball you threw, and you randomly pick a ball from this bin. What is the probability that you pick up the first ball you threw? (Again, leave your answer as a summation.)

3 Pairs of Beads

Sinho has a set of $2n$ beads ($n \geq 2$) of n different colors, such that there are two beads of each color. He wants to give out pairs of beads as gifts to all the other $n - 1$ TAs, and plans on keeping the final pair for himself (since he is, after all, also a TA). To do so, he first chooses two beads at random to give to the first TA he sees. Then he chooses two beads at random from those remaining to give to the second TA he sees. He continues giving each TA he sees two beads chosen at random from his remaining beads until he has seen all $n - 1$ TAs, leaving him with just the two beads he plans to keep for himself. Prove that the probability that any other TA (*not* including Sinho himself) gets two beads of the same color is no more than $\frac{1}{2}$.