CS 70 Discrete Mathematics and Probability Theory
Spring 2020

Quiz 0

1. [True or False] Mark each of the following "True" if it is a valid logical equivalence, or "False" otherwise.

(a) ☐ $P \implies Q \equiv P \vee \neg Q$

(b) ☐ $P \implies Q \equiv (\neg P \implies \neg Q)$

(c) ☐ $P \implies Q \equiv (Q \wedge P) \vee \neg P$

Solution:

(a) **False.** $P \implies Q$ is equivalent to $\neg P \vee Q$, and is *not* equivalent to $P \vee \neg Q$.

(b) **False.** This is the inverse of the implication, which, unlike the contrapositive $\neg Q \implies \neg P$, is not equivalent.

(c) **True.** You can verify this with a truth table.

2. [True or False] Let $P(x)$ and $Q(x)$ be a propositions about an integer x, and suppose you want to prove the theorem $\forall x, \ (P(x) \implies Q(x))$. Mark each of the following proof strategies "True" if it would be a valid way to proceed with such a proof, or "False" otherwise.

(a) ☐ Find an x such that $Q(x)$ is true or $P(x)$ is false.

(b) ☐ Show that, for every x, if $Q(x)$ is false then $P(x)$ is false.

(c) ☐ Assume that there exists an x such that $P(x)$ is false and $Q(x)$ is false and derive a contradiction.

(d) ☐ Assume that there exists an x such that $P(x)$ is true and $Q(x)$ is false and derive a contradiction.

Solution:

(a) **False.** This tells you nothing about *all* x. This would show $\exists x, \ (\neg P(x) \vee Q(x))$ (note the "exists" instead of "for all").

(b) **True.** This proves the contrapositive, which is equivalent to the original implication.

(c) **False.** The negation of $\forall x, \ (P(x) \implies Q(x))$ is $\exists x, \ (P(x) \wedge \neg Q(x))$ (why?). So if we are to do a proof by contradiction, this is what we should assume.

(d) **True.** See previous part.

3. [Proof] Suppose you have a rectangular array of pebbles, where each pebble is either red or blue. Suppose that for every way of choosing one pebble from each column, there exists a red pebble among the chosen ones. Prove that there must exist an all-red column.

Solution: We give a proof by contraposition. Suppose there does not exist an all-red column. This means that, in each column, we can find a blue pebble. Therefore, if we take one blue pebble from each column, we have a way of choosing one pebble from each column without any red pebbles. This is the negation of the original hypothesis, so we are done.