1 DeMorgan's Laws

Use truth tables to show that $\neg(A \vee B) \equiv \neg A \wedge \neg B$ and $\neg(A \wedge B) \equiv \neg A \vee \neg B$. These two equivalences are known as DeMorgan's Laws.

2 XOR

The truth table of XOR (denoted by $\oplus$) is as follows.

A	B	A $\oplus$ B
F	F	F
F	T	T
T	F	T
T	T	F

1. Express XOR using only $(\wedge, \vee, \neg)$ and parentheses.

2. Does $(A \oplus B)$ imply $(A \vee B)$? Explain briefly.

3. Does $(A \vee B)$ imply $(A \oplus B)$? Explain briefly.

3 Numbers of Friends

Prove that if there are $n \geq 2$ people at a party, then at least 2 of them have the same number of friends at the party. Assume that friendships are always reciprocated: that is, if Alice is friends with Bob, then Bob is also friends with Alice.
(Hint: The Pigeonhole Principle states that if n items are placed in m containers, where $n > m$, at least one container must contain more than one item. You may use this without proof.)

4 Proof Practice

(a) Prove that $\forall n \in \mathbb{N}$, if n is odd, then $n^2 + 1$ is even.

(b) Prove that $\forall x, y \in \mathbb{R}$, $\min(x, y) = (x + y - |x - y|)/2$.

(c) Prove that $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.

(d) Suppose $A \subseteq B$. Prove $\mathscr{P}(A) \subseteq \mathscr{P}(B)$.