DIS 10B

1 Variance

This problem will give you practice using the "standard method" to compute the variance of a sum of random variables that are not pairwise independent (so you cannot use "linearity" of variance). Recall that $\text{var}(X) = \mathbb{E}[X^2] - \mathbb{E}[X]^2$.

(a) A building has n floors numbered $1, 2, \ldots, n$, plus a ground floor G. At the ground floor, m people get on the elevator together, and each person gets off at one of the n floors uniformly at random (independently of everybody else). What is the *variance* of the number of floors the elevator *does not* stop at? (In fact, the variance of the number of floors the elevator *does* stop at must be the same, but the former is a little easier to compute.)

(b) A group of three friends has n books they would all like to read. Each friend (independently of the other two) picks a random permutation of the books and reads them in that order, one book per week (for n consecutive weeks). Let X be the number of weeks in which all three friends are reading the same book. Compute $\text{var}(X)$.

2 Family Planning

Mr. and Mrs. Brown decide to continue having children until they either have their first girl or until they have three children. Assume that each child is equally likely to be a boy or a girl, independent of all other children, and that there are no multiple births. Let G denote the numbers of girls that the Browns have. Let C be the total number of children they have.

(a) Determine the sample space, along with the probability of each sample point.

(b) Compute the joint distribution of G and C. Fill in the table below.

	$C=1$	$C=2$	$C=3$
$G=0$			
$G=1$			

(c) Use the joint distribution to compute the marginal distributions of G and C and confirm that the values are as you'd expect. Fill in the tables below.

$\mathbb{P}(G=0)$	
$\mathbb{P}(G=1)$	

$\mathbb{P}(C=1)$	$\mathbb{P}(C=2)$	$\mathbb{P}(C=3)$

(d) Are G and C independent?

(e) What is the expected number of girls the Browns will have? What is the expected number of children that the Browns will have?

3 Binomial Conditioning

Let $n \in \mathbb{Z}_+$ and $p, q \in [0,1]$. Let $X \sim \text{Binomial}(n,p)$ and suppose that conditioned on $X = x$, $Y \sim \text{Binomial}(x,q)$. What is the unconditional distribution of Y?