1 Working with the Law of Large Numbers

(a) A fair coin is tossed multiple times and you win a prize if there are more than 60% heads. Which number of tosses would you prefer: 10 tosses or 100 tosses? Explain.

(b) A fair coin is tossed multiple times and you win a prize if there are more than 40% heads. Which number of tosses would you prefer: 10 tosses or 100 tosses? Explain.

(c) A fair coin is tossed multiple times and you win a prize if there are between 40% and 60% heads. Which number of tosses would you prefer: 10 tosses or 100 tosses? Explain.

(d) A fair coin is tossed multiple times and you win a prize if there are exactly 50% heads. Which number of tosses would you prefer: 10 tosses or 100 tosses? Explain.
2 Vegas

On the planet Vegas, everyone carries a coin. Many people are honest and carry a fair coin (heads on one side and tails on the other), but a fraction \( p \) of them cheat and carry a trick coin with heads on both sides. You want to estimate \( p \) with the following experiment: you pick a random sample of \( n \) people and ask each one to flip his or her coin. Assume that each person is independently likely to carry a fair or a trick coin.

1. Given the results of your experiment, how should you estimate \( p \)?
   
   \((\text{Hint: Construct an (unbiased) estimator for } p \text{ such that } E[\hat{p}] = p.\)\)

2. How many people do you need to ask to be 95% sure that your answer is off by at most 0.05?

3 Dice

In this problem, let \( X_1, X_2, \ldots, X_n \) each denote the outcomes of standard six-sided dice rolls. Let \( A \) denote the average of the outcomes \( \left( \sum_{i=1}^{n} X_i \right) / n \).

(a) For \( n = 100 \), find some \( a \) and \( b \) such that \( A \) is in the interval \([a, b]\) with probability at least 90% (Don’t use trivial intervals like \([1, 6]\)).

(b) For \( n = 30 \), find a lower bound on \( \text{Pr}[3 \leq A \leq 4] \).

(c) Find the minimum \( n \) for which you can guarantee that \( A \) is within the range \([3, 4]\) with at least 99% probability.