

## 1 Linearity

Solve each of the following problems using linearity of expectation. Explain your methods clearly.

- (a) In an arcade, you play game  $A$  10 times and game  $B$  20 times. Each time you play game  $A$ , you win with probability  $1/3$  (independently of the other times), and if you win you get 3 tickets (redeemable for prizes), and if you lose you get 0 tickets. Game  $B$  is similar, but you win with probability  $1/5$ , and if you win you get 4 tickets. What is the expected total number of tickets you receive?
- (b) A monkey types at a 26-letter keyboard with one key corresponding to each of the lower-case English letters. Each keystroke is chosen independently and uniformly at random from the 26 possibilities. If the monkey types 1 million letters, what is the expected number of times the sequence “book” appears?

## 2 Joint Distributions

- (a) Give an example of discrete random variables  $X$  and  $Y$  with the property that  $\mathbb{E}[XY] \neq \mathbb{E}[X]\mathbb{E}[Y]$ . You should specify the joint distribution of  $X$  and  $Y$ .
- (b) Give an example of discrete random variables  $X$  and  $Y$  that (i) are *not independent* and (ii) have the property that  $\mathbb{E}[XY] = 0$ ,  $\mathbb{E}[X] = 0$ , and  $\mathbb{E}[Y] = 0$ . Again you should specify the joint distribution of  $X$  and  $Y$ .

### 3 Ball in Bins

You are throwing  $k$  balls into  $n$  bins. Let  $X_i$  be the number of balls thrown into bin  $i$ .

(a) What is  $\mathbb{E}[X_i]$ ?

(b) What is the expected number of empty bins?

(c) Define a collision to occur when two balls land in the same bin (if there are  $n$  balls in a bin, count that as  $n - 1$  collisions). What is the expected number of collisions?