

1 Allen's Umbrellas

Every morning, Allen walks from his home to Soda, and every evening, Allen walks from Soda to his home. Suppose that Allen has two umbrellas in his possession, but he sometimes leaves his umbrellas behind. Specifically, before leaving from his home or Soda, he checks the weather. If it is raining outside, he will bring his umbrella (that is, if there is an umbrella where he currently is). If it is not raining outside, he will forget to bring his umbrella. Assume that the probability of rain is p .

We will model this as a Markov chain. Let $\mathcal{X} = \{0, 1, 2\}$ be the set of states, where the state i represents the number of umbrellas in his current location. Determine if the distribution of X_n converges to the invariant distribution, and compute the invariant distribution. Determine the long-term fraction of time that Allen will walk through rain with no umbrella.

2 Reflecting Random Walk

Alice starts at vertex 0 and wishes to get to vertex n . When she is at vertex 0 she has a probability of 1 of transitioning to vertex 1. For any other vertex i , there is a probability of $1/2$ of transitioning to $i+1$ and a probability of $1/2$ of transitioning to $i-1$.

- (a) What is the expected number of steps Alice takes to reach vertex n ? Write down the hitting-time equations, but do not solve them yet.

- (b) Solve the hitting-time equations. [*Hint*: Let R_i denote the expected number of steps to reach vertex n starting from vertex i . As a suggestion, try writing R_0 in terms of R_1 ; then, use this to express R_1 in terms of R_2 ; and then use this to express R_2 in terms of R_3 , and so on. See if you can notice a pattern.]

3 Predicament

Three men are on a boat with cigarettes, but they have no lighter. What do they do?

4 Which Envelope?

You have two envelopes in front of you containing cash. You know that one envelope contains twice as much money as the other envelope (the amount of money in an envelope is an integer). You are allowed to pick one envelope and see how much cash is inside, and then based on this information, you can decide to switch envelopes or stick with the envelope you already have.

Can you come up with a strategy which will allow you to pick the envelope with more money, with probability strictly greater than $1/2$?