CS 70 Discrete Mathematics and Probability Theory
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DIS 14A

1 Working with the Law of Large Numbers

(a) A fair coin is tossed and you win a prize if there are more than 60% heads. Which is better: 10 tosses or 100 tosses? Explain.

(b) A fair coin is tossed and you win a prize if there are more than 40% heads. Which is better: 10 tosses or 100 tosses? Explain.

(c) A coin is tossed and you win a prize if there are between 40% and 60% heads. Which is better: 10 tosses or 100 tosses? Explain.

(d) A coin is tossed and you win a prize if there are exactly 50% heads. Which is better: 10 tosses or 100 tosses? Explain.

2 Uniform Probability Space

Let $\Omega = \{1,2,3,4,5,6\}$ be a uniform probability space. Let also $X(\omega)$ and $Y(\omega)$, for $\omega \in \Omega$, be the random variables defined in the table:

(a) Fill in the blank entries of the table. In the column to the far right, fill in the expected value of the random variable.

(b) Are the variables correlated or uncorrelated? Are the variables independent or dependent?

(c) Calculate $\mathbb{E}[(Y - L[Y \mid X])^2]$.

Table 1: All the rows in the table correspond to random variables.

ω	1	2	3	4	5	6	$\mathbb{E}[\cdot]$
$X(\omega)$	0	0	1	1	2	2	
$Y(\omega)$	0	2	3	5	2	0	
$X^2(\omega)$							
$Y^2(\omega)$							
$XY(\omega)$							
$L[Y \mid X](\omega)$							

3 LLSE

We have two bags of balls. The fractions of red balls and blue balls in bag A are $2/3$ and $1/3$ respectively. The fractions of red balls and blue balls in bag B are $1/2$ and $1/2$ respectively. Someone gives you one of the bags (unmarked) uniformly at random. You then draw 6 balls from that same bag with replacement. Let X_i be the indicator random variable that ball i is red. Now, let us define $X = \sum_{1 \le i \le 3} X_i$ and $Y = \sum_{4 \le i \le 6} X_i$. Find $L(Y \mid X)$. *Hint*: Recall that

$$L(Y \mid X) = \mathbb{E}(Y) + \frac{\operatorname{cov}(X,Y)}{\operatorname{var}(X)}\big(X - \mathbb{E}(X)\big).$$

Also remember that covariance is bilinear.