CS 70 Discrete Mathematics and Probability Theory
Spring 2020

DIS 1A

1 Induction

Prove the following using induction:

(a) For all integers $n > 2$, $2^n > 2n+1$.

(b) For all positive integers n, $1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$.

(c) For all positive integers n, $\frac{5}{4} \cdot 8^n + 3^{3n-1}$ is divisible by 19.

2 Make It Stronger

Let $x \geq 1$ be a real number. Use induction to prove that for all positive integers n, all of the entries in the matrix

$$\begin{pmatrix} 1 & x \\ 0 & 1 \end{pmatrix}^n$$

are $\leq xn$. (Hint 1: Find a way to strengthen the inductive hypothesis! Hint 2: Try writing out the first few powers.)

3 Binary Numbers

Prove that every positive integer n can be written in binary. In other words, prove that we can write

$$n = c_k \cdot 2^k + c_{k-1} \cdot 2^{k-1} + \cdots + c_1 \cdot 2^1 + c_0 \cdot 2^0,$$

where $k \in \mathbb{N}$ and $c_k \in \{0, 1\}$.