1 Farmer's Market

Suppose you want k items from the farmer's market. Count how many ways you can do this, assuming:

(a) There are pumpkins and apples at the market.

(b) There are pumpkins, apples, oranges, and pears at the market

(c) There are n kinds of fruits at the market, and you want to end up with at least two different types of fruit.

2 Charming Star

At the end of each day, students will vote for the most charming student. There are 5 candidates and 100 voters. Each voter can only vote once, and all of their votes weigh the same. A "voting combination" is defined by how many votes each candidate receives. In this question, only the number of votes for each candidate matters; it does not matter which specific people voted for each candidate.

(a) How many possible voting combinations are there for the 5 candidates?

(b) How many possible voting combinations are there such that exactly one candidate gets more than 50 votes?

3 Story Problems

Prove the following identities by combinatorial argument:

(a) $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$

(b) $\binom{2n}{2} = 2\binom{n}{2} + n^2$

(c) $n^2 = 2\binom{n}{2} + n$

(d) $\sum_{k=1}^{n} k\binom{n}{k} = n2^{n-1}$
Hint: Consider how many ways there are to pick groups of people ("teams") and then a representative ("team leaders").

(e) $\sum_{k=j}^{n} \binom{n}{k}\binom{k}{j} = 2^{n-j}\binom{n}{j}$
Hint: Consider a generalization of the previous part.