1. [True or False]

(a)  For any three events \(A, B, C\), if \(P[A \cap B] > 0\) and \(P[B \cap C] > 0\), then \(P[A \cap C] > 0\).

(b)  For events \(A, B\) in a uniform probability space, the probability that neither of the events happen is \(1 - P[A] - P[B]\).

(c)  For three events \(A, B, C\) in a uniform probability space, the probability that exactly one of the events happens is \(P[A] + P[B] + P[C] - 2P[A \cap B] - 2P[A \cap C] - 2P[B \cap C] + 2P[A \cap B \cap C]\).

2. [Counting & Probability]

Consider the equation \(x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 70\), where each \(x_i\) is a non-negative integer. We choose one of these solutions uniformly at random.

(a)  What is the size of the sample space?

(b)  What is the probability that both \(x_1 \geq 30\) and \(x_2 \geq 30\)?

(c)  What it the probability that either \(x_1 \geq 30\) or \(x_2 \geq 30\)?

3. [Combinatorial Proof]

Give a combinatorial proof that \(\binom{n+k-1}{k-1} = \sum_{j=0}^{n} \binom{n-j+k-2}{k-2}\).