1 Head Count

Consider a coin with \( P(\text{Heads}) = \frac{2}{5} \). Suppose you flip the coin 20 times, and define \( X \) to be the number of heads.

(a) Name the distribution of \( X \) and what its parameters are.

(b) What is \( P(X = 7) \)?

(c) What is \( P(X \geq 1) \)? Hint: You should be able to do this without a summation.

(d) What is \( P(12 \leq X \leq 14) \)?

2 Exploring the Geometric Distribution

Suppose \( X \sim \text{Geometric}(p) \) and \( Y \sim \text{Geometric}(q) \) are independent. Find the distribution of \( \min\{X, Y\} \) and justify your answer.

3 The Memoryless Property

Let \( X \) be a discrete random variable which takes on values in \( \mathbb{Z}_+ \). Suppose that for all \( m, n \in \mathbb{N} \), we have \( P(X > m + n \mid X > n) = P(X > m) \). Prove that \( X \) is a geometric distribution. Hint: In order to prove that \( X \) is geometric, it suffices to prove that there exists a \( p \in [0, 1] \) such that \( P(X > i) = (1 - p)^i \) for all \( i > 0 \).
4 Cookie Jars

You have two jars of cookies, each of which starts with $n$ cookies initially. Every day, when you come home, you pick one of the two jars randomly (each jar is chosen with probability $1/2$) and eat one cookie from that jar. One day, you come home and reach inside one of the jars of cookies, but you find that is empty! Let $X$ be the random variable representing the number of remaining cookies in non-empty jar at that time. What is the distribution of $X$?