

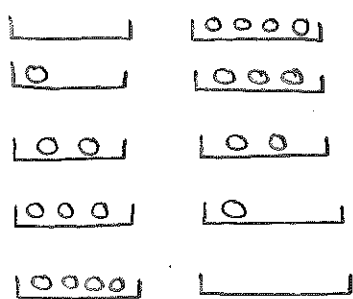
# Note on #5 Finicky Bins midterm 2 review problem

Consider a simpler problem.

Suppose there are 4 balls, 2 bins, and if  $\geq 3$  balls fall into a single bin then they will roll away.

How many different ways can 4 balls be placed across the 2 bins?

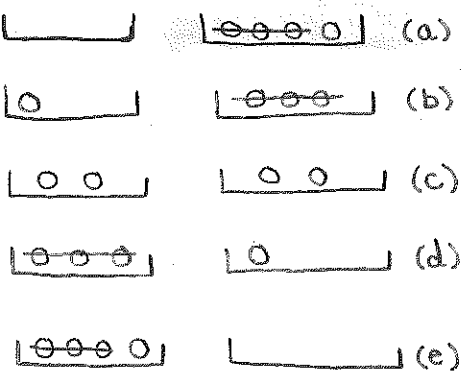
## STEP 1



If no balls were to roll away, then there are 5 different ways.

## STEP 2

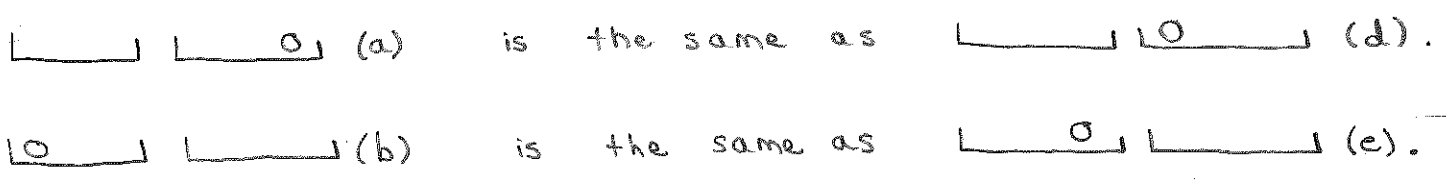
Now, let's cross out the balls that have rolled away.



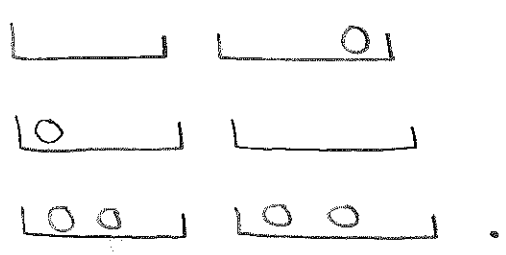
In scenario (c), no balls are crossed out. In the other 4 scenarios, some balls have been crossed out.

**STEP 3**

Some of the scenarios with crossed out balls are now the same.



In all, there are **3 different ways** the 4 balls can be arranged among the 2 bins. They are:



Explaining the solution of simpler problem:

# different ways, no balls crossed out

$$\binom{5}{1}$$

# ways to place 4 balls among 2 bins if no balls were to roll away

$$- 2 \cdot \binom{2}{1}$$

the 3 balls can be placed in one of 2 bins  
 # ways to place the remaining 1 ball among 2 bins

# different ways, some balls "crossed out"

$$+ \binom{2}{1}$$

There are  $2 \binom{2}{1} = 4$  scenarios in which balls will be lost. Once the balls fall out  $\frac{1}{2}$  of the scenarios will end up being the same.

Now consider the Finicky Bins problem as written.

Let's break down the solution.

$$\binom{10}{3} - 4 \cdot \binom{5}{3}$$

↑ # ways to place 7 balls among 4 bins if no balls were to roll away

↑ the 5 balls can be placed in one of 4 bins

↑ # ways to place the remaining 2 balls among 4 bins

# different ways, no balls lost

# ways, if no balls were to roll away

# scenarios in which balls will be lost

$$\frac{1}{4} \cdot 4 \binom{5}{3}$$

↑ to account for duplicates once the balls fall out

↑ # scenarios in which balls will be lost

# different ways, some balls lost

All together,

$$\begin{aligned} \# \text{ different ways} &= \# \text{ different ways, no balls lost} + \# \text{ different ways, some balls lost} \\ &= \binom{10}{3} - 4 \binom{5}{3} + \binom{5}{3} \end{aligned}$$