

70: Discrete Math and Probability Theory

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Programming + Microprocessors

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What are your super powerful programs/processors doing?

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Logic and Proofs!

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Induction \equiv Recursion.

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What can computers do?

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Induction \equiv Recursion.

What can computers do?

Work with discrete objects.

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Discrete Math

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Discrete Math \implies immense application.

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Computers learn and interact with the world?

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E.g. machine learning, data analysis, robotics, ...

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Probability!

See note 1, for more discussion.

Satish Rao

18th year at Berkeley.

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PhD: Long time ago,

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Research: Theory

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Research: Theory (Algorithms)

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Other: 1 College kid. One Cal Grad. And another College Grad.

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Explains policies, has office hours, homework, midterm dates, etc.

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midterm 2 late! After pass/no-pass deadline!

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Questions/Announcements

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Questions/Announcements \implies piazza:

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Questions/Announcements \implies piazza:

piazza.com/berkeley/spring2017/cs70

Wason's experiment:1

Suppose we have four cards on a table:

- ▶ 1st about Alice, 2nd about Bob, 3rd Charlie, 4th Donna.

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Suppose we have four cards on a table:

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"If a person travels to Chicago, he/she flies."

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- ▶ Card contains person's **destination** on one side, and **mode of travel**.
- ▶ Consider the theory:
"If a person travels to Chicago, he/she flies."
- ▶ Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.

| |
|-----------|
| Alice |
| Baltimore |

| |
|-------|
| Bob |
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| |
|---------|
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- ▶ Which cards must you flip to test the theory?

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Answer:

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- ▶ Which cards must you flip to test the theory?

Answer: Later.

CS70: Lecture 1. Outline.

Today: Note 1.

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The language of proofs!

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Today: Note 1. Note 0 is background. Do read it.

The language of proofs!

1. Propositions.
2. Propositional Forms.
3. Implication.
4. Truth Tables
5. Quantifiers
6. More De Morgan's Laws

Propositions: Statements that are true or false.

$\sqrt{2}$ is irrational

$$2+2 = 4$$

$$2+2 = 3$$

826th digit of pi is 4

Johny Depp is a good actor

All evens > 2 are sums of 2 primes

$$4 + 5$$

$$x + x$$

Alice travelled to Chicago

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Again: “value” of a proposition is ...

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Not a Proposition

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Not a Proposition.

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Again: “value” of a proposition is ... True or False

Propositional Forms.

Put propositions together to make another...

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Conjunction (“and”): $P \wedge Q$

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Negation (“not”): $\neg P$

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Examples:

$\neg (2 + 2 = 4)$ – a proposition that is ...

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Examples:

$\neg “(2 + 2 = 4)”$ – a proposition that is ... False

“ $2 + 2 = 3$ ” \wedge “ $2 + 2 = 4$ ” – a proposition that is ...

Propositional Forms.

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“ $2 + 2 = 3$ ” \vee “ $2 + 2 = 4$ ” – a proposition that is ...

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Negation (“not”): $\neg P$

“ $\neg P$ ” is **True** when P is **False** . Else **False** .

Examples:

$\neg “(2 + 2 = 4)”$ – a proposition that is ... **False**

“ $2 + 2 = 3$ ” \wedge “ $2 + 2 = 4$ ” – a proposition that is ... **False**

“ $2 + 2 = 3$ ” \vee “ $2 + 2 = 4$ ” – a proposition that is ... **True**

Propositional Forms: quick check!

$P = \text{"}\sqrt{2} \text{ is rational"}$

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$P = \text{"}\sqrt{2} \text{ is rational"}$

$Q = \text{"826th digit of pi is 2"}$

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P is ...

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$P \wedge Q \dots$

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$P \wedge Q$... False

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$\neg P$...

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P is ...False .

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$\neg P$... True

Put them together..

Propositions:

P_1 - Person 1 rides the bus.

Put them together..

Propositions:

P_1 - Person 1 rides the bus.

P_2 - Person 2 rides the bus.

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....

Put them together..

Propositions:

P_1 - Person 1 rides the bus.

P_2 - Person 2 rides the bus.

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But we can't have either of the following happen; That either person 1 or person 2 ride the bus and person 3 or 4 ride the bus. Or that person 2 or person 3 ride the bus and that either person 4 rides the bus or person 5 doesn't.

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We need a way to keep track!

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DeMorgan's Law's for Negation: distribute and flip!

$$\neg(P \wedge Q)$$

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DeMorgan's Law's for Negation: distribute and flip!

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Distributive?

$$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)?$$

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$$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)?$$

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

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$$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)?$$

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

$$\text{LHS: } T \wedge (Q \vee R)$$

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P is False .

$$\text{LHS: } F \wedge (Q \vee R)$$

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Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

$$\text{LHS: } T \wedge (Q \vee R) \equiv (Q \vee R).$$

$$\text{RHS: } (T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R).$$

P is False .

$$\text{LHS: } F \wedge (Q \vee R) \equiv F.$$

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Simplify: $T \vee Q \equiv T$, $F \vee Q \equiv Q$.

Distributive?

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$$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)?$$

Simplify: $T \vee Q \equiv T$, $F \vee Q \equiv Q$.

Foil 1:

Distributive?

$$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)?$$

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

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P is True .

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$$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)?$$

Simplify: $T \vee Q \equiv T$, $F \vee Q \equiv Q$.

Foil 1:

$$(A \vee B) \wedge (C \vee D) \equiv (A \wedge C) \vee (A \wedge D) \vee (B \wedge C) \vee (B \wedge D)?$$

Distributive?

$$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)?$$

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

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Simplify: $T \vee Q \equiv T$, $F \vee Q \equiv Q$.

Foil 1:

$$(A \vee B) \wedge (C \vee D) \equiv (A \wedge C) \vee (A \wedge D) \vee (B \wedge C) \vee (B \wedge D)?$$

Foil 2:

Distributive?

$$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)?$$

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

$$\text{LHS: } T \wedge (Q \vee R) \equiv (Q \vee R).$$

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Simplify: $T \vee Q \equiv T$, $F \vee Q \equiv Q$.

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Implication.

$P \implies Q$ interpreted as

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Examples:

Statement: If you stand in the rain, then you'll get wet.

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P = "you stand in the rain"

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Statement: If you stand in the rain, then you'll get wet.

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Examples:

Statement: If you stand in the rain, then you'll get wet.

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Statement: If a right triangle has sidelengths $a \leq b \leq c$, then $a^2 + b^2 = c^2$.

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Non-Consequences/consequences of Implication

The statement " $P \implies Q$ "

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only is **False** if P is **True** and Q is **False** .

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Some Fun: use propositional formulas to describe implication?

$((P \implies Q) \wedge P) \implies Q$.

Implication and English.

$$P \implies Q$$

- ▶ If P , then Q .

Implication and English.

$$P \implies Q$$

- ▶ If P , then Q .
- ▶ Q if P .
Just reversing the order.

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Implication and English.

$$P \implies Q$$

- ▶ If P , then Q .
- ▶ Q if P .
Just reversing the order.
- ▶ P only if Q .
Remember if P is true then Q must be true.
this suggests that P can only be true if Q is true.
since if Q is false P must have been false.

Implication and English.

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This means that proving P allows you
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this suggests that P can only be true if Q is true.
since if Q is false P must have been false.
- ▶ P is sufficient for Q .
This means that proving P allows you
to conclude that Q is true.
- ▶ Q is necessary for P .
For P to be true it is necessary that Q is true.
Or if Q is false then we know that P is false.

Truth Table: implication.

| P | Q | $P \implies Q$ |
|-----|-----|----------------|
| T | T | T |
| T | F | |
| F | T | |
| F | F | |

Truth Table: implication.

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| P | Q | $P \implies Q$ |
|-----|-----|----------------|
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

| P | Q | $\neg P \vee Q$ |
|-----|-----|-----------------|
| T | T | |
| T | F | |
| F | T | |
| F | F | |

Truth Table: implication.

| P | Q | $P \implies Q$ |
|-----|-----|----------------|
| T | T | T |
| T | F | F |
| F | T | T |
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| T | F | F |
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$$\neg P \vee Q \equiv P \implies Q.$$

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| T | F | F |
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| F | F | T |

$$\neg P \vee Q \equiv P \implies Q.$$

These two propositional forms are logically equivalent!

Contrapositive, Converse

- ▶ Contrapositive of $P \implies Q$ is $\neg Q \implies \neg P$.

Contrapositive, Converse

- ▶ Contrapositive of $P \implies Q$ is $\neg Q \implies \neg P$.
 - ▶ If the plant pollutes, fish die.

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- ▶ Contrapositive of $P \implies Q$ is $\neg Q \implies \neg P$.
 - ▶ If the plant pollutes, fish die.
 - ▶ If the fish don't die, the plant does not pollute.

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- ▶ Contrapositive of $P \implies Q$ is $\neg Q \implies \neg P$.
 - ▶ If the plant pollutes, fish die.
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(contrapositive)

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- ▶ Contrapositive of $P \implies Q$ is $\neg Q \implies \neg P$.
 - ▶ If the plant pollutes, fish die.
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 - ▶ If you stand in the rain, you get wet.

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- ▶ Contrapositive of $P \implies Q$ is $\neg Q \implies \neg P$.
 - ▶ If the plant pollutes, fish die.
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(contrapositive)
 - ▶ If you stand in the rain, you get wet.
 - ▶ If you did not stand in the rain, you did not get wet.

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- ▶ Contrapositive of $P \implies Q$ is $\neg Q \implies \neg P$.
 - ▶ If the plant pollutes, fish die.
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- ▶ **Definition:** If $P \implies Q$ and $Q \implies P$ is P if and only if Q or $P \iff Q$.
(Logically Equivalent: \iff .)

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- ▶ Any set: $S = \{Alice, Bob, Charlie, Donna\}$.

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- ▶ $\mathbb{N} = \{0, 1, \dots\}$ (natural numbers).
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- ▶ \mathbb{Z}^+ (positive integers)
- ▶ \mathbb{R} (real numbers)
- ▶ Any set: $S = \{Alice, Bob, Charlie, Donna\}$.
- ▶ See note 0 for more!

Back to: Wason's experiment:1

Theory:

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Theory: "If a person travels to Chicago, he/she flies."

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Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

$P(x)$ = "Person x went to Chicago." $Q(x)$ = "Person x flew"

Statement/theory: $\forall x \in \{A, B, C, D\}, P(x)$

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

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Theory: "If a person travels to Chicago, he/she flies."

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Statement/theory: $\forall x \in \{A, B, C, D\}, P(x) \implies Q(x)$

$P(A)$ = **False** .

Back to: Wason's experiment:1

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Statement/theory: $\forall x \in \{A, B, C, D\}, P(x) \implies Q(x)$

$P(A)$ = **False** . Do we care about $Q(A)$?

Back to: Wason's experiment:1

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$P(A)$ = **False** . Do we care about $Q(A)$?

No.

Back to: Wason's experiment:1

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Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

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Statement/theory: $\forall x \in \{A, B, C, D\}, P(x) \implies Q(x)$

$P(A)$ = **False** . Do we care about $Q(A)$?

No. $P(A) \implies Q(A)$, when $P(A)$ is **False** , $Q(A)$ can be anything.

Back to: Wason's experiment:1

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$Q(B)$ = **False** .

Back to: Wason's experiment:1

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No. $P(A) \implies Q(A)$, when $P(A)$ is **False** , $Q(A)$ can be anything.

$Q(B)$ = **False** . Do we care about $P(B)$?

Yes.

Back to: Wason's experiment:1

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Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

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$Q(B)$ = **False** . Do we care about $P(B)$?

Yes. $P(B) \implies Q(B)$

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

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$P(A)$ = **False** . Do we care about $Q(A)$?

No. $P(A) \implies Q(A)$, when $P(A)$ is **False** , $Q(A)$ can be anything.

$Q(B)$ = **False** . Do we care about $P(B)$?

Yes. $P(B) \implies Q(B) \equiv \neg Q(B) \implies \neg P(B)$.

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.

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$Q(B)$ = **False** . Do we care about $P(B)$?

Yes. $P(B) \implies Q(B) \equiv \neg Q(B) \implies \neg P(B)$.

So $P(\text{Bob})$ must be **False** .

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

$P(x)$ = "Person x went to Chicago." $Q(x)$ = "Person x flew"

Statement/theory: $\forall x \in \{A, B, C, D\}, P(x) \implies Q(x)$

$P(A)$ = **False** . Do we care about $Q(A)$?

No. $P(A) \implies Q(A)$, when $P(A)$ is **False** , $Q(A)$ can be anything.

$Q(B)$ = **False** . Do we care about $P(B)$?

Yes. $P(B) \implies Q(B) \equiv \neg Q(B) \implies \neg P(B)$.

So $P(\text{Bob})$ must be **False** .

$P(C)$ = **True** .

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

$P(x)$ = "Person x went to Chicago." $Q(x)$ = "Person x flew"

Statement/theory: $\forall x \in \{A, B, C, D\}, P(x) \implies Q(x)$

$P(A)$ = **False** . Do we care about $Q(A)$?

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$Q(B)$ = **False** . Do we care about $P(B)$?

Yes. $P(B) \implies Q(B) \equiv \neg Q(B) \implies \neg P(B)$.

So $P(\text{Bob})$ must be **False** .

$P(C)$ = **True** . Do we care about $Q(C)$?

Back to: Wason's experiment:1

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Which cards do you need to flip to test the theory?

$P(x)$ = "Person x went to Chicago." $Q(x)$ = "Person x flew"

Statement/theory: $\forall x \in \{A, B, C, D\}, P(x) \implies Q(x)$

$P(A)$ = **False** . Do we care about $Q(A)$?

No. $P(A) \implies Q(A)$, when $P(A)$ is **False** , $Q(A)$ can be anything.

$Q(B)$ = **False** . Do we care about $P(B)$?

Yes. $P(B) \implies Q(B) \equiv \neg Q(B) \implies \neg P(B)$.

So $P(\text{Bob})$ must be **False** .

$P(C)$ = **True** . Do we care about $Q(C)$?

Yes.

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

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So $P(\text{Bob})$ must be **False** .

$P(C)$ = **True** . Do we care about $Q(C)$?

Yes. $P(C) \implies Q(C)$ means $Q(C)$ must be true.

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

$P(x)$ = "Person x went to Chicago." $Q(x)$ = "Person x flew"

Statement/theory: $\forall x \in \{A, B, C, D\}, P(x) \implies Q(x)$

$P(A)$ = **False** . Do we care about $Q(A)$?

No. $P(A) \implies Q(A)$, when $P(A)$ is **False** , $Q(A)$ can be anything.

$Q(B)$ = **False** . Do we care about $P(B)$?

Yes. $P(B) \implies Q(B) \equiv \neg Q(B) \implies \neg P(B)$.

So $P(\text{Bob})$ must be **False** .

$P(C)$ = **True** . Do we care about $Q(C)$?

Yes. $P(C) \implies Q(C)$ means $Q(C)$ must be true.

$Q(D)$ = **True** .

Back to: Wason's experiment:1

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Which cards do you need to flip to test the theory?

$P(x)$ = "Person x went to Chicago." $Q(x)$ = "Person x flew"

Statement/theory: $\forall x \in \{A, B, C, D\}, P(x) \implies Q(x)$

$P(A)$ = **False** . Do we care about $Q(A)$?

No. $P(A) \implies Q(A)$, when $P(A)$ is **False** , $Q(A)$ can be anything.

$Q(B)$ = **False** . Do we care about $P(B)$?

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$P(A)$ = **False** . Do we care about $Q(A)$?

No. $P(A) \implies Q(A)$, when $P(A)$ is **False** , $Q(A)$ can be anything.

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$P(C)$ = **True** . Do we care about $Q(C)$?

Yes. $P(C) \implies Q(C)$ means $Q(C)$ must be true.

$Q(D)$ = **True** . Do we care about $P(D)$?

No.

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

$P(x)$ = "Person x went to Chicago." $Q(x)$ = "Person x flew"

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$P(A)$ = **False** . Do we care about $Q(A)$?

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So $P(\text{Bob})$ must be **False** .

$P(C)$ = **True** . Do we care about $Q(C)$?

Yes. $P(C) \implies Q(C)$ means $Q(C)$ must be true.

$Q(D)$ = **True** . Do we care about $P(D)$?

No. $P(D) \implies Q(D)$ holds whatever $P(D)$ is when $Q(D)$ is true.

Back to: Wason's experiment:1

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$P(C)$ = **True** . Do we care about $Q(C)$?

Yes. $P(C) \implies Q(C)$ means $Q(C)$ must be true.

$Q(D)$ = **True** . Do we care about $P(D)$?

No. $P(D) \implies Q(D)$ holds whatever $P(D)$ is when $Q(D)$ is true.

Only have to turn over cards for Bob and Charlie.

More for all quantifiers examples.

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- ▶ “doubling a number always makes it larger”

More for all quantifiers examples.

- ▶ “doubling a number always makes it larger”

$$(\forall x \in \mathcal{N}) (2x > x)$$

More for all quantifiers examples.

- ▶ “doubling a number always makes it larger”

$$(\forall x \in \mathcal{N}) (2x > x) \quad \text{False}$$

More for all quantifiers examples.

- ▶ “doubling a number always makes it larger”

$(\forall x \in \mathbb{N}) (2x > x)$ **False** **Consider** $x = 0$

More for all quantifiers examples.

- ▶ “doubling a number always makes it larger”

$(\forall x \in \mathbb{N}) (2x > x)$ **False** **Consider** $x = 0$

Can fix statement...

More for all quantifiers examples.

- ▶ “doubling a number always makes it larger”

$$(\forall x \in \mathbb{N}) (2x > x) \quad \text{False} \quad \text{Consider } x = 0$$

Can fix statement...

$$(\forall x \in \mathbb{N}) (2x \geq x)$$

More for all quantifiers examples.

- ▶ “doubling a number always makes it larger”

$$(\forall x \in \mathbb{N}) (2x > x) \quad \text{False} \quad \text{Consider } x = 0$$

Can fix statement...

$$(\forall x \in \mathbb{N}) (2x \geq x) \quad \text{True}$$

More for all quantifiers examples.

- ▶ “doubling a number always makes it larger”

$$(\forall x \in \mathbb{N}) (2x > x) \quad \text{False} \quad \text{Consider } x = 0$$

Can fix statement...

$$(\forall x \in \mathbb{N}) (2x \geq x) \quad \text{True}$$

- ▶ “Square of any natural number greater than 5 is greater than 25.”

More for all quantifiers examples.

- ▶ “doubling a number always makes it larger”

$$(\forall x \in \mathbb{N}) (2x > x) \quad \text{False} \quad \text{Consider } x = 0$$

Can fix statement...

$$(\forall x \in \mathbb{N}) (2x \geq x) \quad \text{True}$$

- ▶ “Square of any natural number greater than 5 is greater than 25.”

$$(\forall x \in \mathbb{N})$$

More for all quantifiers examples.

- ▶ “doubling a number always makes it larger”

$$(\forall x \in \mathbb{N}) (2x > x) \quad \text{False} \quad \text{Consider } x = 0$$

Can fix statement...

$$(\forall x \in \mathbb{N}) (2x \geq x) \quad \text{True}$$

- ▶ “Square of any natural number greater than 5 is greater than 25.”

$$(\forall x \in \mathbb{N})(x > 5)$$

More for all quantifiers examples.

- ▶ “doubling a number always makes it larger”

$$(\forall x \in \mathbb{N}) (2x > x) \quad \text{False} \quad \text{Consider } x = 0$$

Can fix statement...

$$(\forall x \in \mathbb{N}) (2x \geq x) \quad \text{True}$$

- ▶ “Square of any natural number greater than 5 is greater than 25.”

$$(\forall x \in \mathbb{N})(x > 5 \implies$$

More for all quantifiers examples.

- ▶ “doubling a number always makes it larger”

$$(\forall x \in \mathbb{N}) (2x > x) \quad \text{False} \quad \text{Consider } x = 0$$

Can fix statement...

$$(\forall x \in \mathbb{N}) (2x \geq x) \quad \text{True}$$

- ▶ “Square of any natural number greater than 5 is greater than 25.”

$$(\forall x \in \mathbb{N})(x > 5 \implies x^2 > 25).$$

More for all quantifiers examples.

- ▶ “doubling a number always makes it larger”

$$(\forall x \in \mathbb{N}) (2x > x) \quad \text{False} \quad \text{Consider } x = 0$$

Can fix statement...

$$(\forall x \in \mathbb{N}) (2x \geq x) \quad \text{True}$$

- ▶ “Square of any natural number greater than 5 is greater than 25.”

$$(\forall x \in \mathbb{N})(x > 5 \implies x^2 > 25).$$

Idea alert:

More for all quantifiers examples.

- ▶ “doubling a number always makes it larger”

$$(\forall x \in \mathbb{N}) (2x > x) \quad \text{False} \quad \text{Consider } x = 0$$

Can fix statement...

$$(\forall x \in \mathbb{N}) (2x \geq x) \quad \text{True}$$

- ▶ “Square of any natural number greater than 5 is greater than 25.”

$$(\forall x \in \mathbb{N})(x > 5 \implies x^2 > 25).$$

Idea alert: Restrict domain using implication.

More for all quantifiers examples.

- ▶ “doubling a number always makes it larger”

$$(\forall x \in \mathbf{N}) (2x > x) \quad \text{False} \quad \text{Consider } x = 0$$

Can fix statement...

$$(\forall x \in \mathbf{N}) (2x \geq x) \quad \text{True}$$

- ▶ “Square of any natural number greater than 5 is greater than 25.”

$$(\forall x \in \mathbf{N})(x > 5 \implies x^2 > 25).$$

Idea alert: Restrict domain using implication.

Note that we may omit universe if clear from context.

Quantifiers..not commutative.

- ▶ In English: “there is a natural number that is the square of every natural number”.

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- ▶ In English: “there is a natural number that is the square of every natural number”.

$$(\exists y \in \mathcal{N})$$

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- ▶ In English: “there is a natural number that is the square of every natural number”.

$$(\exists y \in \mathcal{N}) (\forall x \in \mathcal{N})$$

Quantifiers..not commutative.

- ▶ In English: “there is a natural number that is the square of every natural number”.

$$(\exists y \in \mathcal{N}) (\forall x \in \mathcal{N}) (y = x^2)$$

Quantifiers..not commutative.

- ▶ In English: “there is a natural number that is the square of every natural number”.

$$(\exists y \in \mathcal{N}) (\forall x \in \mathcal{N}) (y = x^2) \quad \text{False}$$

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- ▶ In English: “there is a natural number that is the square of every natural number”.

$$(\exists y \in \mathcal{N}) (\forall x \in \mathcal{N}) (y = x^2) \quad \text{False}$$

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- ▶ In English: “there is a natural number that is the square of every natural number”.

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- ▶ In English: “the square of every natural number is a natural number.”

$$(\forall x \in \mathcal{N})$$

Quantifiers..not commutative.

- ▶ In English: “there is a natural number that is the square of every natural number”.

$$(\exists y \in \mathcal{N}) (\forall x \in \mathcal{N}) (y = x^2) \quad \text{False}$$

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$$(\forall x \in \mathcal{N})(\exists y \in \mathcal{N})$$

Quantifiers..not commutative.

- ▶ In English: “there is a natural number that is the square of every natural number”.

$$(\exists y \in \mathcal{N}) (\forall x \in \mathcal{N}) (y = x^2) \quad \text{False}$$

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Quantifiers..not commutative.

- ▶ In English: “there is a natural number that is the square of every natural number”.

$$(\exists y \in \mathcal{N})(\forall x \in \mathcal{N})(y = x^2) \quad \text{False}$$

- ▶ In English: “the square of every natural number is a natural number.”

$$(\forall x \in \mathcal{N})(\exists y \in \mathcal{N})(y = x^2) \quad \text{True}$$

Quantifiers..not commutative.

- ▶ In English: “there is a natural number that is the square of every natural number”.

$$(\exists y \in \mathcal{N}) (\forall x \in \mathcal{N}) (y = x^2) \quad \text{False}$$

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$$(\forall x \in \mathcal{N})(\exists y \in \mathcal{N}) (y = x^2) \quad \text{True}$$

Quantifiers....negation...DeMorgan again.

Consider

$$\neg(\forall x \in S)(P(x)),$$

Quantifiers....negation...DeMorgan again.

Consider

$$\neg(\forall x \in S)(P(x)),$$

English: there is an x in S where $P(x)$ does not hold.

Quantifiers....negation...DeMorgan again.

Consider

$$\neg(\forall x \in S)(P(x)),$$

English: there is an x in S where $P(x)$ does not hold.

That is,

Quantifiers....negation...DeMorgan again.

Consider

$$\neg(\forall x \in S)(P(x)),$$

English: there is an x in S where $P(x)$ does not hold.

That is,

$$\neg(\forall x \in S)(P(x)) \iff \exists(x \in S)(\neg P(x)).$$

Quantifiers....negation...DeMorgan again.

Consider

$$\neg(\forall x \in S)(P(x)),$$

English: there is an x in S where $P(x)$ does not hold.

That is,

$$\neg(\forall x \in S)(P(x)) \iff \exists(x \in S)(\neg P(x)).$$

What we do in this course! We consider claims.

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Next Time: proofs!