70: Discrete Math and Probability Theory

70: Discrete Math and Probability Theory

Programming + Microprocessors

70: Discrete Math and Probability Theory

Programming + Microprocessors $\equiv$ Superpower!

70: Discrete Math and Probability Theory

Programming + Microprocessors $\equiv$ Superpower!

What are your super powerful programs/processors doing?

70: Discrete Math and Probability Theory

Programming + Microprocessors $\equiv$ Superpower!

What are your super powerful programs/processors doing?
  Logic and Proofs!

70: Discrete Math and Probability Theory

Programming + Microprocessors $\equiv$ Superpower!

What are your super powerful programs/processors doing?
- Logic and Proofs!
- Induction $\equiv$ Recursion.

70: Discrete Math and Probability Theory

Programming + Microprocessors $\equiv$ Superpower!

What are your super powerful programs/processors doing?
  Logic and Proofs!
  Induction $\equiv$ Recursion.

What can computers do?

70: Discrete Math and Probability Theory

Programming + Microprocessors $\equiv$ Superpower!

What are your super powerful programs/processors doing?
  Logic and Proofs!
  Induction $\equiv$ Recursion.

What can computers do?
  Work with discrete objects.

70: Discrete Math and Probability Theory

Programming + Microprocessors $\equiv$ Superpower!

What are your super powerful programs/processors doing?
  Logic and Proofs!
  Induction $\equiv$ Recursion.

What can computers do?
  Work with discrete objects.
  Discrete Math

70: Discrete Math and Probability Theory

Programming + Microprocessors $\equiv$ Superpower!

What are your super powerful programs/processors doing?
 Logic and Proofs!
 Induction $\equiv$ Recursion.

What can computers do?
 Work with discrete objects.
 Discrete Math $\implies$ immense application.

70: Discrete Math and Probability Theory

Programming + Microprocessors $\equiv$ Superpower!

What are your super powerful programs/processors doing?
  Logic and Proofs!
  Induction $\equiv$ Recursion.

What can computers do?
  Work with discrete objects.
  Discrete Math $\implies$ immense application.

Computers learn and interact with the world?

70: Discrete Math and Probability Theory

Programming + Microprocessors $\equiv$ Superpower!

What are your super powerful programs/processors doing?
  Logic and Proofs!
  Induction $\equiv$ Recursion.

What can computers do?
  Work with discrete objects.
  Discrete Math $\implies$ immense application.

Computers learn and interact with the world?
  E.g. machine learning, data analysis, robotics, ...

70: Discrete Math and Probability Theory

Programming + Microprocessors $\equiv$ Superpower!

What are your super powerful programs/processors doing?
Logic and Proofs!
Induction $\equiv$ Recursion.

What can computers do?
Work with discrete objects.
Discrete Math $\implies$ immense application.

Computers learn and interact with the world?
E.g. machine learning, data analysis, robotics, ...
Probability!

70: Discrete Math and Probability Theory

Programming + Microprocessors $\equiv$ Superpower!

What are your super powerful programs/processors doing?
Logic and Proofs!
Induction $\equiv$ Recursion.

What can computers do?
Work with discrete objects.
Discrete Math $\implies$ immense application.

Computers learn and interact with the world?
E.g. machine learning, data analysis, robotics, ...
Probability!

See Professor Sahai's note under Resources, for more discussion.

The Truth about CS70

...according to me.

The Truth about CS70

...according to me.

It's transformative.

The Truth about CS70

...according to me.

It's transformative.

Programming is a superpower.

The Truth about CS70

...according to me.

It's transformative.

Programming is a superpower.

Reasoning is perhaps even better.

The Truth about CS70

...according to me.

It's transformative.

Programming is a superpower.

Reasoning is perhaps even better.

Each Topic:

The Truth about CS70

...according to me.

It's transformative.

Programming is a superpower.

Reasoning is perhaps even better.

Each Topic:

The Truth about CS70

...according to me.

It's transformative.

Programming is a superpower.

Reasoning is perhaps even better.

Each Topic:

Define,

The Truth about CS70

...according to me.

It's transformative.

Programming is a superpower.

Reasoning is perhaps even better.

Each Topic:

Define, understand,

The Truth about CS70

...according to me.

It's transformative.

Programming is a superpower.

Reasoning is perhaps even better.

Each Topic:

  Define, understand, and build.

The Truth about CS70

...according to me.

It's transformative.

Programming is a superpower.

Reasoning is perhaps even better.

Each Topic:

Define, understand, and build.

Clearly and correctly.

The Truth about CS70

...according to me.

It's transformative.

Programming is a superpower.

Reasoning is perhaps even better.

Each Topic:

Define, understand, and build.

Clearly and correctly.

Rate of Change

The Truth about CS70

...according to me.

It's transformative.

Programming is a superpower.

Reasoning is perhaps even better.

Each Topic:

Define, understand, and build.

Clearly and correctly.

Rate of Change + Newton

The Truth about CS70

...according to me.

It's transformative.

Programming is a superpower.

Reasoning is perhaps even better.

Each Topic:

Define, understand, and build.

Clearly and correctly.

Rate of Change + Newton –¿ Calculus.

Satish Rao

21st year at Berkeley.

Satish Rao

21st year at Berkeley.
PhD: Long time ago,

Satish Rao

21st year at Berkeley.
PhD: Long time ago, far

Satish Rao

21st year at Berkeley.
PhD: Long time ago, far far away.

Satish Rao

21st year at Berkeley.
PhD: Long time ago, far far away.
Research: Theory

Satish Rao

21st year at Berkeley.
PhD: Long time ago, far far away.
Research: Theory (Algorithms)

Satish Rao

21st year at Berkeley.
PhD: Long time ago, far far away.
Research: Theory (Algorithms)
Taught in CS: 70, 170, 174, 188, 270, 273, 294,

Satish Rao

21st year at Berkeley.
PhD: Long time ago, far far away.
Research: Theory (Algorithms)
Taught in CS: 70, 170, 174, 188, 270, 273, 294, 375, ...

Satish Rao

21st year at Berkeley.
PhD: Long time ago, far far away.
Research: Theory (Algorithms)
Taught in CS: 70, 170, 174, 188, 270, 273, 294, 375, ...

Other: Three adult kids. One Cal Grad!

Satish Rao

21st year at Berkeley.
PhD: Long time ago, far far away.
Research: Theory (Algorithms)
Taught in CS: 70, 170, 174, 188, 270, 273, 294, 375, ...

Other: Three adult kids. One Cal Grad!

Lecturing Style:

Satish Rao

21st year at Berkeley.
PhD: Long time ago, far far away.
Research: Theory (Algorithms)
Taught in CS: 70, 170, 174, 188, 270, 273, 294, 375, ...

Other: Three adult kids. One Cal Grad!

Lecturing Style: I have used slides.

Why I use Slides and some Advice.

Why I use Slides and some Advice.

Lots of arguments are demonstrated well by examples or verbal explanations,

Why I use Slides and some Advice.

Lots of arguments are demonstrated well by examples or verbal explanations, but sometimes painful to write down,

Why I use Slides and some Advice.

Lots of arguments are demonstrated well by examples or verbal explanations, but sometimes painful to write down, which works for me with slides.

Why I use Slides and some Advice.

Lots of arguments are demonstrated well by examples or verbal explanations, but sometimes painful to write down, which works for me with slides.

(1) Is there value for you to watch me write on screen or paper?

Why I use Slides and some Advice.

Lots of arguments are demonstrated well by examples or verbal explanations, but sometimes painful to write down, which works for me with slides.

(1) Is there value for you to watch me write on screen or paper?

(2) You have them!

Why I use Slides and some Advice.

Lots of arguments are demonstrated well by examples or verbal explanations, but sometimes painful to write down, which works for me with slides.

(1) Is there value for you to watch me write on screen or paper?

(2) You have them!

Use the slides to guide you.

Why I use Slides and some Advice.

Lots of arguments are demonstrated well by examples or verbal explanations, but sometimes painful to write down, which works for me with slides.

(1) Is there value for you to watch me write on screen or paper?

(2) You have them!

Use the slides to guide you.
Handout Version: for getting landscape.

Why I use Slides and some Advice.

Lots of arguments are demonstrated well by examples or verbal explanations, but sometimes painful to write down, which works for me with slides.

(1) Is there value for you to watch me write on screen or paper?

(2) You have them!

Use the slides to guide you.
Handout Version: for getting landscape.
Slide Version: for detailed understanding.

Why I use Slides and some Advice.

Lots of arguments are demonstrated well by examples or verbal explanations, but sometimes painful to write down, which works for me with slides.

(1) Is there value for you to watch me write on screen or paper?

(2) You have them!

Use the slides to guide you.
Handout Version: for getting landscape.
Slide Version: for detailed understanding.

It is easier to present more.

Why I use Slides and some Advice.

Lots of arguments are demonstrated well by examples or verbal explanations, but sometimes painful to write down, which works for me with slides.

(1) Is there value for you to watch me write on screen or paper?

(2) You have them!

Use the slides to guide you.
Handout Version: for getting landscape.
Slide Version: for detailed understanding.

It is easier to present more.

Why I use Slides and some Advice.

Lots of arguments are demonstrated well by examples or verbal explanations, but sometimes painful to write down, which works for me with slides.

(1) Is there value for you to watch me write on screen or paper?

(2) You have them!

Use the slides to guide you.
Handout Version: for getting landscape.
Slide Version: for detailed understanding.

It is easier to present more.

“More” is overview, connections, insights, some jokes (breaks), the details.

Why I use Slides and some Advice.

Lots of arguments are demonstrated well by examples or verbal explanations, but sometimes painful to write down, which works for me with slides.

(1) Is there value for you to watch me write on screen or paper?

(2) You have them!

Use the slides to guide you.
Handout Version: for getting landscape.
Slide Version: for detailed understanding.

It is easier to present more.

"More" is overview, connections, insights, some jokes (breaks), the details.

Risk: Students get frustrated at not understanding everything.

Why I use Slides and some Advice.

Lots of arguments are demonstrated well by examples or verbal explanations, but sometimes painful to write down, which works for me with slides.

(1) Is there value for you to watch me write on screen or paper?

(2) You have them!

Use the slides to guide you.
Handout Version: for getting landscape.
Slide Version: for detailed understanding.

It is easier to present more.

"More" is overview, connections, insights, some jokes (breaks), the details.

Risk: Students get frustrated at not understanding everything.

The truth: Students don't understand everything.

Why I use Slides and some Advice.

Lots of arguments are demonstrated well by examples or verbal explanations, but sometimes painful to write down, which works for me with slides.

(1) Is there value for you to watch me write on screen or paper?

(2) You have them!

Use the slides to guide you.
Handout Version: for getting landscape.
Slide Version: for detailed understanding.

It is easier to present more.

"More" is overview, connections, insights, some jokes (breaks), the details.

Risk: Students get frustrated at not understanding everything.

The truth: Students don't understand everything.
I certainly don't in real time

Why I use Slides and some Advice.

Lots of arguments are demonstrated well by examples or verbal explanations, but sometimes painful to write down, which works for me with slides.

(1) Is there value for you to watch me write on screen or paper?

(2) You have them!

Use the slides to guide you.
Handout Version: for getting landscape.
Slide Version: for detailed understanding.

It is easier to present more.

"More" is overview, connections, insights, some jokes (breaks), the details.

Risk: Students get frustrated at not understanding everything.

The truth: Students don't understand everything.
I certainly don't in real time or sometimes ever.

Why I use Slides and some Advice.

Lots of arguments are demonstrated well by examples or verbal explanations, but sometimes painful to write down, which works for me with slides.

(1) Is there value for you to watch me write on screen or paper?

(2) You have them!

Use the slides to guide you.
Handout Version: for getting landscape.
Slide Version: for detailed understanding.

It is easier to present more.

“More” is overview, connections, insights, some jokes (breaks), the details.

Risk: Students get frustrated at not understanding everything.

The truth: Students don’t understand everything.
I certainly don’t in real time or sometimes ever.

It is ok:

Why I use Slides and some Advice.

Lots of arguments are demonstrated well by examples or verbal explanations, but sometimes painful to write down, which works for me with slides.

(1) Is there value for you to watch me write on screen or paper?

(2) You have them!

Use the slides to guide you.
Handout Version: for getting landscape.
Slide Version: for detailed understanding.

It is easier to present more.

"More" is overview, connections, insights, some jokes (breaks), the details.

Risk: Students get frustrated at not understanding everything.

The truth: Students don't understand everything.
I certainly don't in real time or sometimes ever.

It is ok: many levels to grok.

Why I use Slides and some Advice.

Lots of arguments are demonstrated well by examples or verbal explanations, but sometimes painful to write down, which works for me with slides.

(1) Is there value for you to watch me write on screen or paper?

(2) You have them!

Use the slides to guide you.
Handout Version: for getting landscape.
Slide Version: for detailed understanding.

It is easier to present more.

"More" is overview, connections, insights, some jokes (breaks), the details.

Risk: Students get frustrated at not understanding everything.

The truth: Students don't understand everything.
I certainly don't in real time or sometimes ever.

It is ok: many levels to grok. Lecture is one pass.

Why I use Slides and some Advice.

Lots of arguments are demonstrated well by examples or verbal explanations, but sometimes painful to write down, which works for me with slides.

(1) Is there value for you to watch me write on screen or paper?

(2) You have them!

Use the slides to guide you.
Handout Version: for getting landscape.
Slide Version: for detailed understanding.

It is easier to present more.

"More" is overview, connections, insights, some jokes (breaks), the details.

Risk: Students get frustrated at not understanding everything.

The truth: Students don't understand everything.
I certainly don't in real time or sometimes ever.

It is ok: many levels to grok. Lecture is one pass.

Notes are sufficient.

Admin

Course Webpage: **http://www.eecs70.org/**

Admin

Course Webpage: **http://www.eecs70.org/**

Explains policies, has office hours, homework, midterm dates, etc.

Admin

Course Webpage: **http://www.eecs70.org/**

Explains policies, has office hours, homework, midterm dates, etc.

One midterm, final.

Admin

Course Webpage: **http://www.eecs70.org/**

Explains policies, has office hours, homework, midterm dates, etc.

One midterm, final.
midterm.

Admin

Course Webpage: **http://www.eecs70.org/**

Explains policies, has office hours, homework, midterm dates, etc.

One midterm, final.
midterm.

Admin

Course Webpage: **http://www.eecs70.org/**

Explains policies, has office hours, homework, midterm dates, etc.

One midterm, final.
midterm.

Questions

Admin

Course Webpage: **http://www.eecs70.org/**

Explains policies, has office hours, homework, midterm dates, etc.

One midterm, final.
midterm.

Questions $\implies$ piazza:

Admin

Course Webpage: **http://www.eecs70.org/**

Explains policies, has office hours, homework, midterm dates, etc.

One midterm, final.
midterm.

Questions $\Longrightarrow$ piazza:

Logistics, etc.

Content Support: other students!

Plus Piazza hours.

Admin

Course Webpage: **http://www.eecs70.org/**

Explains policies, has office hours, homework, midterm dates, etc.

One midterm, final.
midterm.

Questions $\Longrightarrow$ piazza:
Logistics, etc.
Content Support: other students!
Plus Piazza hours.

Weekly Post.

Admin

Course Webpage: **http://www.eecs70.org/**

Explains policies, has office hours, homework, midterm dates, etc.

One midterm, final.
midterm.

Questions $\Longrightarrow$ piazza:

Logistics, etc.

Content Support: other students!

Plus Piazza hours.

Weekly Post.

It's weekly.

Admin

Course Webpage: **http://www.eecs70.org/**

Explains policies, has office hours, homework, midterm dates, etc.

One midterm, final.
midterm.

Questions $\implies$ piazza:
Logistics, etc.
Content Support: other students!
Plus Piazza hours.

Weekly Post.

It's weekly.
Read it!!!!

Admin

Course Webpage: **http://www.eecs70.org/**

Explains policies, has office hours, homework, midterm dates, etc.

One midterm, final.
midterm.

Questions $\Longrightarrow$ piazza:
Logistics, etc.
Content Support: other students!
Plus Piazza hours.

Weekly Post.

It's weekly.
Read it!!!!
Announcements, logistics, critical advice.

Wason's experiment:1

Suppose we have four cards on a table:

- 1st about Alice, 2nd about Bob, 3rd Charlie, 4th Donna.

Wason's experiment:1

Suppose we have four cards on a table:

- 1st about Alice, 2nd about Bob, 3rd Charlie, 4th Donna.
- Card contains person's destination on one side,
  and mode of travel.

Wason's experiment:1

Suppose we have four cards on a table:

- 1st about Alice, 2nd about Bob, 3rd Charlie, 4th Donna.
- Card contains person's destination on one side,
  and mode of travel.
- Consider the theory:

Wason's experiment:1

Suppose we have four cards on a table:

- 1st about Alice, 2nd about Bob, 3rd Charlie, 4th Donna.
- Card contains person's destination on one side,
  and mode of travel.
- Consider the theory:
  "If a person travels to Chicago, he/she flies."

Wason's experiment:1

Suppose we have four cards on a table:

- 1st about Alice, 2nd about Bob, 3rd Charlie, 4th Donna.
- Card contains person's destination on one side,
  and mode of travel.
- Consider the theory:
  "If a person travels to Chicago, he/she flies."

Wason's experiment:1

Suppose we have four cards on a table:

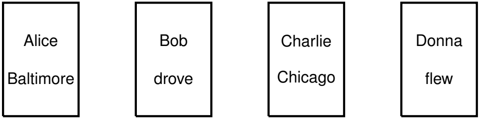
- 1st about Alice, 2nd about Bob, 3rd Charlie, 4th Donna.
- Card contains person's destination on one side, and mode of travel.
- Consider the theory: "If a person travels to Chicago, he/she flies."
- Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.

Alice	Bob	Charlie	Donna
Baltimore	drove	Chicago	flew

Wason's experiment:1

Suppose we have four cards on a table:

- 1st about Alice, 2nd about Bob, 3rd Charlie, 4th Donna.
- Card contains person's destination on one side, and mode of travel.
- Consider the theory: "If a person travels to Chicago, he/she flies."
- Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.

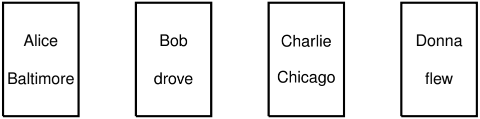


- Which cards must you flip to test the theory?

Wason's experiment:1

Suppose we have four cards on a table:

- 1st about Alice, 2nd about Bob, 3rd Charlie, 4th Donna.
- Card contains person's destination on one side, and mode of travel.
- Consider the theory: "If a person travels to Chicago, he/she flies."
- Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.



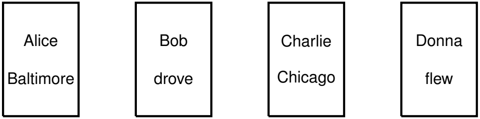
- Which cards must you flip to test the theory?

Answer:

Wason's experiment:1

Suppose we have four cards on a table:

- 1st about Alice, 2nd about Bob, 3rd Charlie, 4th Donna.
- Card contains person's destination on one side,
  and mode of travel.
- Consider the theory:
  "If a person travels to Chicago, he/she flies."
- Suppose you see that Alice went to Baltimore, Bob drove,
  Charlie went to Chicago, and Donna flew.



- Which cards must you flip to test the theory?

Answer: Later.

CS70: Lecture 1. Outline.

Today: Note 1.

CS70: Lecture 1. Outline.

Today: Note 1. Note 0 is background.

CS70: Lecture 1. Outline.

Today: Note 1. Note 0 is background. Do read it.

CS70: Lecture 1. Outline.

Today: Note 1.   Note 0 is background. Do read it.

The language of proofs!

CS70: Lecture 1. Outline.

Today: Note 1. Note 0 is background. Do read it.

The language of proofs!

1. Propositions.
2. Propositional Forms.
3. Implication.
4. Truth Tables
5. Quantifiers
6. More De Morgan's Laws

Propositions: Statements that are true or false.

$\sqrt{2}$ is irrational
2+2 = 4
2+2 = 3
826th digit of pi is 4
Johnny Depp is a good actor
Any even > 2 is sum of 2 primes
$4 + 5$
$x + x$
Alice travelled to Chicago

Propositions: Statements that are true or false.

	Proposition
$\sqrt{2}$ is irrational	
2+2 = 4	
2+2 = 3	
826th digit of pi is 4	
Johnny Depp is a good actor	
Any even >2 is sum of 2 primes	
$4+5$	
$x+x$	
Alice travelled to Chicago	

Propositions: Statements that are true or false.

$\sqrt{2}$ is irrational	**Proposition**	True
2+2 = 4		
2+2 = 3		
826th digit of pi is 4		
Johnny Depp is a good actor		
Any even > 2 is sum of 2 primes		
$4 + 5$		
$x + x$		
Alice travelled to Chicago		

Propositions: Statements that are true or false.

$\sqrt{2}$ is irrational	**Proposition**	True
2+2 = 4	**Proposition**	
2+2 = 3		
826th digit of pi is 4		
Johnny Depp is a good actor		
Any even >2 is sum of 2 primes		
$4+5$		
$x+x$		
Alice travelled to Chicago		

Propositions: Statements that are true or false.

$\sqrt{2}$ is irrational	**Proposition**	True
2+2 = 4	**Proposition**	True
2+2 = 3		
826th digit of pi is 4		
Johnny Depp is a good actor		
Any even > 2 is sum of 2 primes		
$4 + 5$		
$x + x$		
Alice travelled to Chicago		

Propositions: Statements that are true or false.

$\sqrt{2}$ is irrational	**Proposition**	True
2+2 = 4	**Proposition**	True
2+2 = 3	**Proposition**	
826th digit of pi is 4		
Johnny Depp is a good actor		
Any even >2 is sum of 2 primes		
$4+5$		
$x+x$		
Alice travelled to Chicago		

Propositions: Statements that are true or false.

$\sqrt{2}$ is irrational	**Proposition**	True
2+2 = 4	**Proposition**	True
2+2 = 3	**Proposition**	False
826th digit of pi is 4		
Johnny Depp is a good actor		
Any even > 2 is sum of 2 primes		
$4 + 5$		
$x + x$		
Alice travelled to Chicago		

Propositions: Statements that are true or false.

$\sqrt{2}$ is irrational	**Proposition**	True
2+2 = 4	**Proposition**	True
2+2 = 3	**Proposition**	False
826th digit of pi is 4	**Proposition**	
Johnny Depp is a good actor		
Any even >2 is sum of 2 primes		
$4+5$		
$x+x$		
Alice travelled to Chicago		

Propositions: Statements that are true or false.

$\sqrt{2}$ is irrational	**Proposition**	True
2+2 = 4	**Proposition**	True
2+2 = 3	**Proposition**	False
826th digit of pi is 4	**Proposition**	False
Johnny Depp is a good actor		
Any even >2 is sum of 2 primes		
$4+5$		
$x+x$		
Alice travelled to Chicago		

Propositions: Statements that are true or false.

$\sqrt{2}$ is irrational	**Proposition**	True
2+2 = 4	**Proposition**	True
2+2 = 3	**Proposition**	False
826th digit of pi is 4	**Proposition**	False
Johnny Depp is a good actor	**Not Proposition**	
Any even > 2 is sum of 2 primes		
$4 + 5$		
$x + x$		
Alice travelled to Chicago		

Propositions: Statements that are true or false.

$\sqrt{2}$ is irrational	**Proposition**	True
2+2 = 4	**Proposition**	True
2+2 = 3	**Proposition**	False
826th digit of pi is 4	**Proposition**	False
Johnny Depp is a good actor	**Not Proposition**	
Any even > 2 is sum of 2 primes	**Proposition**	
$4 + 5$		
$x + x$		
Alice travelled to Chicago		

Propositions: Statements that are true or false.

$\sqrt{2}$ is irrational	**Proposition**	True
2+2 = 4	**Proposition**	True
2+2 = 3	**Proposition**	False
826th digit of pi is 4	**Proposition**	False
Johnny Depp is a good actor	**Not Proposition**	
Any even > 2 is sum of 2 primes	**Proposition**	False
$4 + 5$		
$x + x$		
Alice travelled to Chicago		

Propositions: Statements that are true or false.

$\sqrt{2}$ is irrational	**Proposition**	True
2+2 = 4	**Proposition**	True
2+2 = 3	**Proposition**	False
826th digit of pi is 4	**Proposition**	False
Johnny Depp is a good actor	**Not Proposition**	
Any even > 2 is sum of 2 primes	**Proposition**	False
$4 + 5$	**Not Proposition.**	
$x + x$		
Alice travelled to Chicago		

Propositions: Statements that are true or false.

$\sqrt{2}$ is irrational	**Proposition**	True
2+2 = 4	**Proposition**	True
2+2 = 3	**Proposition**	False
826th digit of pi is 4	**Proposition**	False
Johnny Depp is a good actor	**Not Proposition**	
Any even > 2 is sum of 2 primes	**Proposition**	False
$4 + 5$	**Not Proposition.**	
$x + x$	**Not a Proposition.**	
Alice travelled to Chicago		

Propositions: Statements that are true or false.

$\sqrt{2}$ is irrational	**Proposition**	True
2+2 = 4	**Proposition**	True
2+2 = 3	**Proposition**	False
826th digit of pi is 4	**Proposition**	False
Johnny Depp is a good actor	**Not Proposition**	
Any even > 2 is sum of 2 primes	**Proposition**	False
$4 + 5$	**Not Proposition.**	
$x + x$	**Not a Proposition.**	
Alice travelled to Chicago	**Proposition.**	

Propositions: Statements that are true or false.

$\sqrt{2}$ is irrational	**Proposition**	True
2+2 = 4	**Proposition**	True
2+2 = 3	**Proposition**	False
826th digit of pi is 4	**Proposition**	False
Johnny Depp is a good actor	**Not Proposition**	
Any even > 2 is sum of 2 primes	**Proposition**	False
$4 + 5$	**Not Proposition.**	
$x + x$	**Not a Proposition.**	
Alice travelled to Chicago	**Proposition.**	False

Propositions: Statements that are true or false.

$\sqrt{2}$ is irrational	**Proposition**	True
2+2 = 4	**Proposition**	True
2+2 = 3	**Proposition**	False
826th digit of pi is 4	**Proposition**	False
Johnny Depp is a good actor	**Not Proposition**	
Any even > 2 is sum of 2 primes	**Proposition**	False
$4 + 5$	**Not Proposition.**	
$x + x$	**Not a Proposition.**	
Alice travelled to Chicago	**Proposition.**	False
I love you.		

Propositions: Statements that are true or false.

$\sqrt{2}$ is irrational	**Proposition**	True
2+2 = 4	**Proposition**	True
2+2 = 3	**Proposition**	False
826th digit of pi is 4	**Proposition**	False
Johnny Depp is a good actor	**Not Proposition**	
Any even > 2 is sum of 2 primes	**Proposition**	False
$4 + 5$	**Not Proposition.**	
$x + x$	**Not a Proposition.**	
Alice travelled to Chicago	**Proposition.**	False
I love you.	**Hmmm.**	

Propositions: Statements that are true or false.

$\sqrt{2}$ is irrational	**Proposition**	True
2+2 = 4	**Proposition**	True
2+2 = 3	**Proposition**	False
826th digit of pi is 4	**Proposition**	False
Johnny Depp is a good actor	**Not Proposition**	
Any even > 2 is sum of 2 primes	**Proposition**	False
$4+5$	**Not Proposition.**	
$x+x$	**Not a Proposition.**	
Alice travelled to Chicago	**Proposition.**	False
I love you.	**Hmmm.**	

Again: “value” of a proposition is ...

Propositions: Statements that are true or false.

$\sqrt{2}$ is irrational	**Proposition**	True
2+2 = 4	**Proposition**	True
2+2 = 3	**Proposition**	False
826th digit of pi is 4	**Proposition**	False
Johnny Depp is a good actor	**Not Proposition**	
Any even > 2 is sum of 2 primes	**Proposition**	False
$4+5$	**Not Proposition.**	
$x+x$	**Not a Proposition.**	
Alice travelled to Chicago	**Proposition.**	False
I love you.	**Hmmm.**	

Again: "value" of a proposition is ... True or False

Propositions: Statements that are true or false.

$\sqrt{2}$ is irrational	**Proposition**	True
2+2 = 4	**Proposition**	True
2+2 = 3	**Proposition**	False
826th digit of pi is 4	**Proposition**	False
Johnny Depp is a good actor	**Not Proposition**	
Any even > 2 is sum of 2 primes	**Proposition**	False
$4 + 5$	**Not Proposition.**	
$x + x$	**Not a Proposition.**	
Alice travelled to Chicago	**Proposition.**	False
I love you.	**Hmmm.**	Its complicated.

Again: “value” of a proposition is ... True or False

Propositional Forms.

Put propositions together to make another...

Propositional Forms.

Put propositions together to make another...

Conjunction ("and"): $P \wedge Q$

Propositional Forms.

Put propositions together to make another...

Conjunction ("and"): $P \wedge Q$

"$P \wedge Q$" is True when both P and Q are True .

Propositional Forms.

Put propositions together to make another...

Conjunction ("and"): $P \wedge Q$

"$P \wedge Q$" is True when both P and Q are True . Else False .

Propositional Forms.

Put propositions together to make another...

Conjunction ("and"): $P \wedge Q$

"$P \wedge Q$" is True when both P and Q are True . Else False .

Disjunction ("or"): $P \vee Q$

Propositional Forms.

Put propositions together to make another...

Conjunction ("and"): $P \wedge Q$

"$P \wedge Q$" is True when both P and Q are True . Else False .

Disjunction ("or"): $P \vee Q$

"$P \vee Q$" is True when at least one P or Q is True . Else False .

Propositional Forms.

Put propositions together to make another...

Conjunction ("and"): $P \wedge Q$

"$P \wedge Q$" is True when both P and Q are True . Else False .

Disjunction ("or"): $P \vee Q$

"$P \vee Q$" is True when at least one P or Q is True . Else False .

Negation ("not"): $\neg P$

Propositional Forms.

Put propositions together to make another...

Conjunction ("and"): $P \wedge Q$

"$P \wedge Q$" is True when both P and Q are True . Else False .

Disjunction ("or"): $P \vee Q$

"$P \vee Q$" is True when at least one P or Q is True . Else False .

Negation ("not"): $\neg P$

"$\neg P$" is True when P is False .

Propositional Forms.

Put propositions together to make another...

Conjunction ("and"): $P \wedge Q$

"$P \wedge Q$" is True when both P and Q are True . Else False .

Disjunction ("or"): $P \vee Q$

"$P \vee Q$" is True when at least one P or Q is True . Else False .

Negation ("not"): $\neg P$

"$\neg P$" is True when P is False . Else False .

Propositional Forms.

Put propositions together to make another...

Conjunction ("and"): $P \wedge Q$

"$P \wedge Q$" is True when both P and Q are True . Else False .

Disjunction ("or"): $P \vee Q$

"$P \vee Q$" is True when at least one P or Q is True . Else False .

Negation ("not"): $\neg P$

"$\neg P$" is True when P is False . Else False .

Examples:

Propositional Forms.

Put propositions together to make another...

Conjunction (“and”): $P \wedge Q$

“$P \wedge Q$” is True when both P and Q are True . Else False .

Disjunction (“or”): $P \vee Q$

“$P \vee Q$” is True when at least one P or Q is True . Else False .

Negation (“not”): $\neg P$

“$\neg P$” is True when P is False . Else False .

Examples:

$\neg$ “$(2+2=4)$” – a proposition that is ...

Propositional Forms.

Put propositions together to make another...

Conjunction ("and"): $P \wedge Q$

"$P \wedge Q$" is True when both P and Q are True . Else False .

Disjunction ("or"): $P \vee Q$

"$P \vee Q$" is True when at least one P or Q is True . Else False .

Negation ("not"): $\neg P$

"$\neg P$" is True when P is False . Else False .

Examples:

$\neg$ "$(2+2=4)$" – a proposition that is ... False

Propositional Forms.

Put propositions together to make another...

Conjunction ("and"): $P \wedge Q$

"$P \wedge Q$" is True when both P and Q are True . Else False .

Disjunction ("or"): $P \vee Q$

"$P \vee Q$" is True when at least one P or Q is True . Else False .

Negation ("not"): $\neg P$

"$\neg P$" is True when P is False . Else False .

Examples:

$\neg$ "$(2+2=4)$" – a proposition that is ... False

"$2+2=3$" $\bigwedge$ "$2+2=4$" – a proposition that is ...

Propositional Forms.

Put propositions together to make another...

Conjunction ("and"): $P \wedge Q$

"$P \wedge Q$" is True when both P and Q are True . Else False .

Disjunction ("or"): $P \vee Q$

"$P \vee Q$" is True when at least one P or Q is True . Else False .

Negation ("not"): $\neg P$

"$\neg P$" is True when P is False . Else False .

Examples:

$\neg$ "$(2+2=4)$" – a proposition that is ... False

"$2+2=3$" $\bigwedge$ "$2+2=4$" – a proposition that is ... False

Propositional Forms.

Put propositions together to make another...

Conjunction (“and”): $P \wedge Q$

“$P \wedge Q$” is True when both P and Q are True . Else False .

Disjunction (“or”): $P \vee Q$

“$P \vee Q$” is True when at least one P or Q is True . Else False .

Negation (“not”): $\neg P$

“$\neg P$” is True when P is False . Else False .

Examples:

$\neg$ “$(2+2=4)$” – a proposition that is ... False

“$2+2=3$” $\bigwedge$ “$2+2=4$” – a proposition that is ... False

“$2+2=3$” $\bigvee$ “$2+2=4$” – a proposition that is ...

Propositional Forms.

Put propositions together to make another...

Conjunction ("and"): $P \wedge Q$

"$P \wedge Q$" is True when both P and Q are True . Else False .

Disjunction ("or"): $P \vee Q$

"$P \vee Q$" is True when at least one P or Q is True . Else False .

Negation ("not"): $\neg P$

"$\neg P$" is True when P is False . Else False .

Examples:

$\neg$ "$(2+2=4)$" – a proposition that is ... False

"$2+2=3$" $\bigwedge$ "$2+2=4$" – a proposition that is ... False

"$2+2=3$" $\bigvee$ "$2+2=4$" – a proposition that is ... True

Propositional Forms.

Put propositions together to make another...

Conjunction ("and"): $P \wedge Q$

"$P \wedge Q$" is True when both P and Q are True . Else False .

Disjunction ("or"): $P \vee Q$

"$P \vee Q$" is True when at least one P or Q is True . Else False .

Negation ("not"): $\neg P$

"$\neg P$" is True when P is False . Else False .

Examples:

$\neg$ "$(2+2=4)$" – a proposition that is ... False

"$2+2=3$" $\bigwedge$ "$2+2=4$" – a proposition that is ... False

"$2+2=3$" $\bigvee$ "$2+2=4$" – a proposition that is ... True

Propositional Forms: quick check!

$P =$ "$\sqrt{2}$ is rational"

Propositional Forms: quick check!

$P =$ "$\sqrt{2}$ is rational"
$Q =$ "826th digit of pi is 2"

Propositional Forms: quick check!

$P =$ "$\sqrt{2}$ is rational"
$Q =$ "826th digit of pi is 2"

Propositional Forms: quick check!

$P =$ "$\sqrt{2}$ is rational"
$Q =$ "826th digit of pi is 2"

P is ...

Propositional Forms: quick check!

$P =$ "$\sqrt{2}$ is rational"
$Q =$ "826th digit of pi is 2"

P is ...False .

Propositional Forms: quick check!

$P =$ "$\sqrt{2}$ is rational"
$Q =$ "826th digit of pi is 2"

P is ...False .
Q is ...

Propositional Forms: quick check!

$P =$ "$\sqrt{2}$ is rational"
$Q =$ "826th digit of pi is 2"

P is ...False .
Q is ...True .

Propositional Forms: quick check!

$P =$ "$\sqrt{2}$ is rational"
$Q =$ "826th digit of pi is 2"

P is ...False .
Q is ...True .

$P \wedge Q$ ...

Propositional Forms: quick check!

$P =$ "$\sqrt{2}$ is rational"
$Q =$ "826th digit of pi is 2"

P is ...False .
Q is ...True .

$P \wedge Q$ ... *False* $\wedge$ *True* $\rightarrow$ False

Propositional Forms: quick check!

$P =$ "$\sqrt{2}$ is rational"
$Q =$ "826th digit of pi is 2"

P is ...False .
Q is ...True .

$P \wedge Q$ ... *False* $\wedge$ *True* $\rightarrow$ False

$P \vee Q$ ...

Propositional Forms: quick check!

$P =$ "$\sqrt{2}$ is rational"
$Q =$ "826th digit of pi is 2"

P is ...False .
Q is ...True .

$P \wedge Q$ ... *False* $\wedge$ *True* $\rightarrow$ False

$P \vee Q$ ... *False* $\vee$ *True* $\rightarrow$ True

Propositional Forms: quick check!

$P =$ "$\sqrt{2}$ is rational"
$Q =$ "826th digit of pi is 2"

P is ...False .
Q is ...True .

$P \wedge Q$ ... *False* $\wedge$ *True* $\rightarrow$ False

$P \vee Q$ ... *False* $\vee$ *True* $\rightarrow$ True

$\neg P$ ...

Propositional Forms: quick check!

$P =$ "$\sqrt{2}$ is rational"
$Q =$ "826th digit of pi is 2"

P is ...False .
Q is ...True .

$P \wedge Q$ ... *False* $\wedge$ *True* $\rightarrow$ False

$P \vee Q$ ... *False* $\vee$ *True* $\rightarrow$ True

$\neg P$ ... $\neg$*False* $\rightarrow$ True

Put them together..

Propositions:

P_1 - Person 1 rides the bus.

Put them together..

Propositions:

P_1 - Person 1 rides the bus.

P_2 - Person 2 rides the bus.

Put them together..

Propositions:

P_1 - Person 1 rides the bus.

P_2 - Person 2 rides the bus.

....

Put them together..

Propositions:

P_1 - Person 1 rides the bus.
P_2 - Person 2 rides the bus.
....

But we can't have either of the following happen; That either person 1 or person 2 ride the bus and person 3 or 4 ride the bus. Or that person 2 or person 3 ride the bus and that either person 4 rides the bus or person 5 doesn't.

Put them together..

Propositions:

P_1 - Person 1 rides the bus.
P_2 - Person 2 rides the bus.
....

But we can’t have either of the following happen; That either person 1 or person 2 ride the bus and person 3 or 4 ride the bus. Or that person 2 or person 3 ride the bus and that either person 4 rides the bus or person 5 doesn’t.

Propositional Form:

$\neg(((P_1 \vee P_2) \wedge (P_3 \vee P_4)) \vee ((P_2 \vee P_3) \wedge (P_4 \vee \neg P_5)))$

Put them together..

Propositions:

P_1 - Person 1 rides the bus.
P_2 - Person 2 rides the bus.
....

But we can't have either of the following happen; That either person 1 or person 2 ride the bus and person 3 or 4 ride the bus. Or that person 2 or person 3 ride the bus and that either person 4 rides the bus or person 5 doesn't.

Propositional Form:

$\neg(((P_1 \vee P_2) \wedge (P_3 \vee P_4)) \vee ((P_2 \vee P_3) \wedge (P_4 \vee \neg P_5)))$

Can person 3 ride the bus?

Put them together..

Propositions:
P_1 - Person 1 rides the bus.
P_2 - Person 2 rides the bus.
....

But we can't have either of the following happen; That either person 1 or person 2 ride the bus and person 3 or 4 ride the bus. Or that person 2 or person 3 ride the bus and that either person 4 rides the bus or person 5 doesn't.

Propositional Form:
$\neg(((P_1 \vee P_2) \wedge (P_3 \vee P_4)) \vee ((P_2 \vee P_3) \wedge (P_4 \vee \neg P_5)))$

Can person 3 ride the bus?
Can person 3 and person 4 ride the bus together?

Put them together..

Propositions:

P_1 - Person 1 rides the bus.

P_2 - Person 2 rides the bus.

....

But we can't have either of the following happen; That either person 1 or person 2 ride the bus and person 3 or 4 ride the bus. Or that person 2 or person 3 ride the bus and that either person 4 rides the bus or person 5 doesn't.

Propositional Form:

$\neg(((P_1 \vee P_2) \wedge (P_3 \vee P_4)) \vee ((P_2 \vee P_3) \wedge (P_4 \vee \neg P_5)))$

Can person 3 ride the bus?

Can person 3 and person 4 ride the bus together?

Put them together..

Propositions:

P_1 - Person 1 rides the bus.
P_2 - Person 2 rides the bus.
....

But we can’t have either of the following happen; That either person 1 or person 2 ride the bus and person 3 or 4 ride the bus. Or that person 2 or person 3 ride the bus and that either person 4 rides the bus or person 5 doesn’t.

Propositional Form:

$\neg(((P_1 \vee P_2) \wedge (P_3 \vee P_4)) \vee ((P_2 \vee P_3) \wedge (P_4 \vee \neg P_5)))$

Can person 3 ride the bus?
Can person 3 and person 4 ride the bus together?

This seems ...

Put them together..

Propositions:

P_1 - Person 1 rides the bus.

P_2 - Person 2 rides the bus.

....

But we can't have either of the following happen; That either person 1 or person 2 ride the bus and person 3 or 4 ride the bus. Or that person 2 or person 3 ride the bus and that either person 4 rides the bus or person 5 doesn't.

Propositional Form:

$\neg(((P_1 \vee P_2) \wedge (P_3 \vee P_4)) \vee ((P_2 \vee P_3) \wedge (P_4 \vee \neg P_5)))$

Can person 3 ride the bus?

Can person 3 and person 4 ride the bus together?

This seems ...complicated.

Put them together..

Propositions:
P_1 - Person 1 rides the bus.
P_2 - Person 2 rides the bus.
....

But we can’t have either of the following happen; That either person 1 or person 2 ride the bus and person 3 or 4 ride the bus. Or that person 2 or person 3 ride the bus and that either person 4 rides the bus or person 5 doesn’t.

Propositional Form:

$$\neg(((P_1 \vee P_2) \wedge (P_3 \vee P_4)) \vee ((P_2 \vee P_3) \wedge (P_4 \vee \neg P_5)))$$

Can person 3 ride the bus?
Can person 3 and person 4 ride the bus together?

This seems ...complicated.

We can program!!!!

Put them together..

Propositions:
P_1 - Person 1 rides the bus.
P_2 - Person 2 rides the bus.
....

But we can't have either of the following happen; That either person 1 or person 2 ride the bus and person 3 or 4 ride the bus. Or that person 2 or person 3 ride the bus and that either person 4 rides the bus or person 5 doesn't.

Propositional Form:
$\neg(((P_1 \vee P_2) \wedge (P_3 \vee P_4)) \vee ((P_2 \vee P_3) \wedge (P_4 \vee \neg P_5)))$

Can person 3 ride the bus?
Can person 3 and person 4 ride the bus together?

This seems ...complicated.

We can program!!!!

We need a way to keep track!

Truth Tables for Propositional Forms.

"$P \wedge Q$" is True when
both P and Q are True .

P	Q	$P \wedge Q$
T	T	T
T	F	
F	T	
F	F	

Truth Tables for Propositional Forms.

"$P \wedge Q$" is True when
both P and Q are True .

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	
F	F	

Truth Tables for Propositional Forms.

"$P \wedge Q$" is True when
both P and Q are True .

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	

Truth Tables for Propositional Forms.

"$P \wedge Q$" is True when both P and Q are True .

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

Truth Tables for Propositional Forms.

"$P \wedge Q$" is True when both P and Q are True .

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

"$P \vee Q$" is True when $\geq$ one of P or Q is True .

P	Q	$P \vee Q$
T	T	
T	F	
F	T	
F	F	

Truth Tables for Propositional Forms.

"$P \wedge Q$" is True when
both P and Q are True .

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

"$P \vee Q$" is True when
$\geq$ one of P or Q is True .

P	Q	$P \vee Q$
T	T	T
T	F	
F	T	
F	F	

Truth Tables for Propositional Forms.

"$P \wedge Q$" is True when both P and Q are True .

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

"$P \vee Q$" is True when $\geq$ one of P or Q is True .

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	
F	F	

Truth Tables for Propositional Forms.

"$P \wedge Q$" is True when both P and Q are True .

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

"$P \vee Q$" is True when $\geq$ one of P or Q is True .

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Truth Tables for Propositional Forms.

"$P \wedge Q$" is True when both P and Q are True .

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

"$P \vee Q$" is True when $\geq$ one of P or Q is True .

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Check: $\wedge$ and $\vee$ are commutative.

Truth Tables for Propositional Forms.

"$P \wedge Q$" is True when both P and Q are True .

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

"$P \vee Q$" is True when $\geq$ one of P or Q is True .

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Check: $\wedge$ and $\vee$ are commutative.

One use for truth tables: Logical Equivalence of propositional forms!

Truth Tables for Propositional Forms.

"$P \wedge Q$" is True when both P and Q are True .

"$P \vee Q$" is True when $\geq$ one of P or Q is True .

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Check: $\wedge$ and $\vee$ are commutative.

One use for truth tables: Logical Equivalence of propositional forms!

Example: $\neg(P \wedge Q)$ logically equivalent to $\neg P \vee \neg Q$.

Truth Tables for Propositional Forms.

"$P \wedge Q$" is True when both P and Q are True .

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

"$P \vee Q$" is True when $\geq$ one of P or Q is True .

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Check: $\wedge$ and $\vee$ are commutative.

One use for truth tables: Logical Equivalence of propositional forms!

Example: $\neg(P \wedge Q)$ logically equivalent to $\neg P \vee \neg Q$. Same

Truth Tables for Propositional Forms.

"$P \wedge Q$" is True when both P and Q are True .

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

"$P \vee Q$" is True when $\geq$ one of P or Q is True .

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Check: $\wedge$ and $\vee$ are commutative.

One use for truth tables: Logical Equivalence of propositional forms!

Example: $\neg(P \wedge Q)$ logically equivalent to $\neg P \vee \neg Q$. Same Truth Table!

Truth Tables for Propositional Forms.

"$P \wedge Q$" is True when both P and Q are True .

"$P \vee Q$" is True when $\geq$ one of P or Q is True .

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Check: $\wedge$ and $\vee$ are commutative.

One use for truth tables: Logical Equivalence of propositional forms!

Example: $\neg(P \wedge Q)$ logically equivalent to $\neg P \vee \neg Q$. Same Truth Table!

P	Q	$\neg(P \vee Q)$	$\neg P \wedge \neg Q$
T	T	F	
T	F		
F	T		
F	F		

Truth Tables for Propositional Forms.

"$P \wedge Q$" is True when both P and Q are True .

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

"$P \vee Q$" is True when $\geq$ one of P or Q is True .

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Check: $\wedge$ and $\vee$ are commutative.

One use for truth tables: Logical Equivalence of propositional forms!

Example: $\neg(P \wedge Q)$ logically equivalent to $\neg P \vee \neg Q$. Same Truth Table!

P	Q	$\neg(P \vee Q)$	$\neg P \wedge \neg Q$
T	T	F	F
T	F		
F	T		
F	F		

Truth Tables for Propositional Forms.

"$P \wedge Q$" is True when both P and Q are True .

"$P \vee Q$" is True when $\geq$ one of P or Q is True .

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Check: $\wedge$ and $\vee$ are commutative.

One use for truth tables: Logical Equivalence of propositional forms!

Example: $\neg(P \wedge Q)$ logically equivalent to $\neg P \vee \neg Q$. Same Truth Table!

P	Q	$\neg(P \vee Q)$	$\neg P \wedge \neg Q$
T	T	F	F
T	F	F	
F	T		
F	F		

Truth Tables for Propositional Forms.

"$P \wedge Q$" is True when both P and Q are True .

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

"$P \vee Q$" is True when $\geq$ one of P or Q is True .

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Check: $\wedge$ and $\vee$ are commutative.

One use for truth tables: Logical Equivalence of propositional forms!

Example: $\neg(P \wedge Q)$ logically equivalent to $\neg P \vee \neg Q$. Same Truth Table!

P	Q	$\neg(P \vee Q)$	$\neg P \wedge \neg Q$
T	T	F	F
T	F	F	F
F	T		
F	F		

Truth Tables for Propositional Forms.

"$P \wedge Q$" is True when both P and Q are True .

"$P \vee Q$" is True when $\geq$ one of P or Q is True .

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Check: $\wedge$ and $\vee$ are commutative.

One use for truth tables: Logical Equivalence of propositional forms!

Example: $\neg(P \wedge Q)$ logically equivalent to $\neg P \vee \neg Q$. Same Truth Table!

P	Q	$\neg(P \vee Q)$	$\neg P \wedge \neg Q$
T	T	F	F
T	F	F	F
F	T	F	
F	F		

Truth Tables for Propositional Forms.

"$P \wedge Q$" is True when both P and Q are True .

"$P \vee Q$" is True when $\geq$ one of P or Q is True .

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Check: $\wedge$ and $\vee$ are commutative.

One use for truth tables: Logical Equivalence of propositional forms!

Example: $\neg(P \wedge Q)$ logically equivalent to $\neg P \vee \neg Q$. Same Truth Table!

P	Q	$\neg(P \vee Q)$	$\neg P \wedge \neg Q$
T	T	F	F
T	F	F	F
F	T	F	F
F	F		

Truth Tables for Propositional Forms.

"$P \wedge Q$" is True when both P and Q are True .

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

"$P \vee Q$" is True when $\geq$ one of P or Q is True .

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Check: $\wedge$ and $\vee$ are commutative.

One use for truth tables: Logical Equivalence of propositional forms!

Example: $\neg(P \wedge Q)$ logically equivalent to $\neg P \vee \neg Q$. Same Truth Table!

P	Q	$\neg(P \vee Q)$	$\neg P \wedge \neg Q$
T	T	F	F
T	F	F	F
F	T	F	F
F	F	T	

Truth Tables for Propositional Forms.

"$P \wedge Q$" is True when both P and Q are True .

"$P \vee Q$" is True when $\geq$ one of P or Q is True .

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Check: $\wedge$ and $\vee$ are commutative.

One use for truth tables: Logical Equivalence of propositional forms!

Example: $\neg(P \wedge Q)$ logically equivalent to $\neg P \vee \neg Q$. Same Truth Table!

P	Q	$\neg(P \vee Q)$	$\neg P \wedge \neg Q$
T	T	F	F
T	F	F	F
F	T	F	F
F	F	T	T

Truth Tables for Propositional Forms.

"$P \wedge Q$" is True when both P and Q are True .

"$P \vee Q$" is True when $\geq$ one of P or Q is True .

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Check: $\wedge$ and $\vee$ are commutative.

One use for truth tables: Logical Equivalence of propositional forms!

Example: $\neg(P \wedge Q)$ logically equivalent to $\neg P \vee \neg Q$. Same Truth Table!

P	Q	$\neg(P \vee Q)$	$\neg P \wedge \neg Q$
T	T	F	F
T	F	F	F
F	T	F	F
F	F	T	T

DeMorgan's Law's for Negation: distribute and flip!

$\neg(P \wedge Q)$

Truth Tables for Propositional Forms.

"$P \wedge Q$" is True when both P and Q are True .

"$P \vee Q$" is True when $\geq$ one of P or Q is True .

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Check: $\wedge$ and $\vee$ are commutative.

One use for truth tables: Logical Equivalence of propositional forms!

Example: $\neg(P \wedge Q)$ logically equivalent to $\neg P \vee \neg Q$. Same Truth Table!

P	Q	$\neg(P \vee Q)$	$\neg P \wedge \neg Q$
T	T	F	F
T	F	F	F
F	T	F	F
F	F	T	T

DeMorgan's Law's for Negation: distribute and flip!

$$\neg(P \wedge Q) \quad \equiv \quad \neg P \vee \neg Q$$

Truth Tables for Propositional Forms.

"$P \wedge Q$" is True when both P and Q are True .

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

"$P \vee Q$" is True when $\geq$ one of P or Q is True .

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Check: $\wedge$ and $\vee$ are commutative.

One use for truth tables: Logical Equivalence of propositional forms!

Example: $\neg(P \wedge Q)$ logically equivalent to $\neg P \vee \neg Q$. Same Truth Table!

P	Q	$\neg(P \vee Q)$	$\neg P \wedge \neg Q$
T	T	F	F
T	F	F	F
F	T	F	F
F	F	T	T

DeMorgan's Law's for Negation: distribute and flip!

$\neg(P \wedge Q) \quad \equiv \quad \neg P \vee \neg Q \qquad\qquad \neg(P \vee Q)$

Truth Tables for Propositional Forms.

"$P \wedge Q$" is True when both P and Q are True .

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

"$P \vee Q$" is True when $\geq$ one of P or Q is True .

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Check: $\wedge$ and $\vee$ are commutative.

One use for truth tables: Logical Equivalence of propositional forms!

Example: $\neg(P \wedge Q)$ logically equivalent to $\neg P \vee \neg Q$. Same Truth Table!

P	Q	$\neg(P \vee Q)$	$\neg P \wedge \neg Q$
T	T	F	F
T	F	F	F
F	T	F	F
F	F	T	T

DeMorgan's Law's for Negation: distribute and flip!

$$\neg(P \wedge Q) \equiv \neg P \vee \neg Q \qquad \neg(P \vee Q) \equiv \neg P \wedge \neg Q$$

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

Is $(T \wedge Q) \equiv Q$?

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

Is $(T \wedge Q) \equiv Q$? Yes?

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Is $(T \wedge Q) \equiv Q$? Yes? No?

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Is $(T \wedge Q) \equiv Q$? Yes? No?

Yes!

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Is $(T \wedge Q) \equiv Q$? Yes? No?

Yes! Look at rows in truth table for $P = T$.

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Is $(T \wedge Q) \equiv Q$? Yes? No?

Yes! Look at rows in truth table for $P = T$.

What is $(F \wedge Q)$?

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Is $(T \wedge Q) \equiv Q$? Yes? No?

Yes! Look at rows in truth table for $P = T$.

What is $(F \wedge Q)$? F or False.

Quick Questions

P	Q	$P \land Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \lor Q$
T	T	T
T	F	T
F	T	T
F	F	F

Is $(T \land Q) \equiv Q$? Yes? No?

Yes! Look at rows in truth table for $P = T$.

What is $(F \land Q)$? F or False.

What is $(T \lor Q)$?

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Is $(T \wedge Q) \equiv Q$? Yes? No?

Yes! Look at rows in truth table for $P = T$.

What is $(F \wedge Q)$? F or False.

What is $(T \vee Q)$? T

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Is $(T \wedge Q) \equiv Q$? Yes? No?

Yes! Look at rows in truth table for $P = T$.

What is $(F \wedge Q)$? F or False.

What is $(T \vee Q)$? T

What is $(F \vee Q)$?

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Is $(T \wedge Q) \equiv Q$? Yes? No?

Yes! Look at rows in truth table for $P = T$.

What is $(F \wedge Q)$? F or False.

What is $(T \vee Q)$? T

What is $(F \vee Q)$? Q

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$,

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:
P is True .
LHS: $T \wedge (Q \vee R)$

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R)$

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R)$

Distributive?

$P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)$?

Simplify: $(T \land Q) \equiv Q$, $(F \land Q) \equiv F$.

Cases:

P is True .

LHS: $T \land (Q \lor R) \equiv (Q \lor R)$.

RHS: $(T \land Q) \lor (T \land R) \equiv (Q \lor R)$.

P is False .

LHS: $F \land (Q \lor R) \equiv F$.

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

RHS: $(F \wedge Q) \vee (F \wedge R)$

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F)$

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Simplify: $T \vee Q \equiv T$,

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Simplify: $T \vee Q \equiv T$, $F \vee Q \equiv Q$. ...

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Simplify: $T \vee Q \equiv T$, $F \vee Q \equiv Q$. ...

Foil 1:

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Simplify: $T \vee Q \equiv T$, $F \vee Q \equiv Q$. ...

Foil 1:

$(A \vee B) \wedge (C \vee D) \equiv (A \wedge C) \vee (A \wedge D) \vee (B \wedge C) \vee (B \wedge D)$?

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Simplify: $T \vee Q \equiv T$, $F \vee Q \equiv Q$. ...

Foil 1:

$(A \vee B) \wedge (C \vee D) \equiv (A \wedge C) \vee (A \wedge D) \vee (B \wedge C) \vee (B \wedge D)$?

Foil 2:

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:
P is True .
LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.
RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.
P is False .
LHS: $F \wedge (Q \vee R) \equiv F$.
RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Simplify: $T \vee Q \equiv T$, $F \vee Q \equiv Q$. ...

Foil 1:
$(A \vee B) \wedge (C \vee D) \equiv (A \wedge C) \vee (A \wedge D) \vee (B \wedge C) \vee (B \wedge D)$?

Foil 2:
$(A \wedge B) \vee (C \wedge D) \equiv (A \vee C) \wedge (A \vee D) \wedge (B \vee C) \wedge (B \vee D)$?

Implication.

$P \implies Q$ interpreted as

Implication.

$P \implies Q$ interpreted as

If P, then Q.

Implication.

$P \implies Q$ interpreted as

If P, then Q.

Implication.

$P \implies Q$ interpreted as

If P, then Q.

True Statements: P, $P \implies Q$.

Implication.

$P \implies Q$ interpreted as

If P, then Q.

True Statements: P, $P \implies Q$.
Conclude: Q is true.

Implication.

$P \implies Q$ interpreted as

If P, then Q.

True Statements: P, $P \implies Q$.
Conclude: Q is true.

Examples:

Implication.

$P \implies Q$ interpreted as

If P, then Q.

True Statements: P, $P \implies Q$.
Conclude: Q is true.

Examples:

Statement: If you stand in the rain, then you'll get wet.

Implication.

$P \implies Q$ interpreted as

If P, then Q.

True Statements: P, $P \implies Q$.
Conclude: Q is true.

Examples:

Statement: If you stand in the rain, then you'll get wet.
P = "you stand in the rain"

Implication.

$P \implies Q$ interpreted as

If P, then Q.

True Statements: P, $P \implies Q$.
Conclude: Q is true.

Examples:

Statement: If you stand in the rain, then you'll get wet.
P = "you stand in the rain"
Q = "you will get wet"

Implication.

$P \implies Q$ interpreted as

If P, then Q.

True Statements: P, $P \implies Q$.
Conclude: Q is true.

Examples:

Statement: If you stand in the rain, then you'll get wet.
P = "you stand in the rain"
Q = "you will get wet"
Statement: "Stand in the rain"

Implication.

$P \implies Q$ interpreted as

If P, then Q.

True Statements: P, $P \implies Q$.
Conclude: Q is true.

Examples:

Statement: If you stand in the rain, then you'll get wet.
P = "you stand in the rain"
Q = "you will get wet"
Statement: "Stand in the rain"
Can conclude: "you'll get wet."

Implication.

$P \implies Q$ interpreted as

If P, then Q.

True Statements: P, $P \implies Q$.
Conclude: Q is true.

Examples:

Statement: If you stand in the rain, then you'll get wet.
P = "you stand in the rain"
Q = "you will get wet"
Statement: "Stand in the rain"
Can conclude: "you'll get wet."

Statement:
If a right triangle has sidelengths $a \leq b \leq c$, then $a^2 + b^2 = c^2$.

Implication.

$P \implies Q$ interpreted as

If P, then Q.

True Statements: P, $P \implies Q$.
Conclude: Q is true.

Examples:

Statement: If you stand in the rain, then you'll get wet.
P = "you stand in the rain"
Q = "you will get wet"
Statement: "Stand in the rain"
Can conclude: "you'll get wet."

Statement:
If a right triangle has sidelengths $a \leq b \leq c$, then $a^2 + b^2 = c^2$.

P = "a right triangle has sidelengths $a \leq b \leq c$",

Implication.

$P \implies Q$ interpreted as

If P, then Q.

True Statements: P, $P \implies Q$.
Conclude: Q is true.

Examples:

Statement: If you stand in the rain, then you'll get wet.
P = "you stand in the rain"
Q = "you will get wet"
Statement: "Stand in the rain"
Can conclude: "you'll get wet."

Statement:
If a right triangle has sidelengths $a \leq b \leq c$, then $a^2 + b^2 = c^2$.

P = "a right triangle has sidelengths $a \leq b \leq c$",
Q = "$a^2 + b^2 = c^2$".

Non-Consequences/consequences of Implication

The statement "$P \implies Q$"

Non-Consequences/consequences of Implication

The statement "$P \implies Q$"

only is False if P is True and Q is False .

Non-Consequences/consequences of Implication

The statement “$P \implies Q$”

only is False if P is True and Q is False .

False implies nothing

Non-Consequences/consequences of Implication

The statement "$P \implies Q$"

only is False if P is True and Q is False .

False implies nothing

P False means

Non-Consequences/consequences of Implication

The statement "$P \implies Q$"

only is False if P is True and Q is False .

False implies nothing
P False means Q can be True

Non-Consequences/consequences of Implication

The statement "$P \implies Q$"

only is False if P is True and Q is False .

False implies nothing
P False means Q can be True or False

Non-Consequences/consequences of Implication

The statement "$P \implies Q$"

only is False if P is True and Q is False .

False implies nothing
P False means Q can be True or False
Anything implies true.

Non-Consequences/consequences of Implication

The statement "$P \implies Q$"

only is False if P is True and Q is False .

False implies nothing
P False means Q can be True or False
Anything implies true.
P can be True or False when

Non-Consequences/consequences of Implication

The statement "$P \implies Q$"

only is False if P is True and Q is False .

False implies nothing
P False means Q can be True or False
Anything implies true.
P can be True or False when Q is True

Non-Consequences/consequences of Implication

The statement "$P \implies Q$"

only is False if P is True and Q is False .

False implies nothing
P False means Q can be True or False
Anything implies true.
P can be True or False when Q is True

If chemical plant pollutes river, fish die.

Non-Consequences/consequences of Implication

The statement "$P \implies Q$"

only is False if P is True and Q is False .

False implies nothing
P False means Q can be True or False
Anything implies true.
P can be True or False when Q is True

If chemical plant pollutes river, fish die.
If fish die, did chemical plant pollute river?

Non-Consequences/consequences of Implication

The statement “$P \implies Q$”

only is False if P is True and Q is False .

False implies nothing
P False means Q can be True or False
Anything implies true.
P can be True or False when Q is True

If chemical plant pollutes river, fish die.
If fish die, did chemical plant pollute river?

Not necessarily.

Non-Consequences/consequences of Implication

The statement "$P \implies Q$"

only is False if P is True and Q is False .

False implies nothing
P False means Q can be True or False
Anything implies true.
P can be True or False when Q is True

If chemical plant pollutes river, fish die.
If fish die, did chemical plant pollute river?

Not necessarily.

$P \implies Q$ and Q are True does not mean P is True

Non-Consequences/consequences of Implication

The statement "$P \implies Q$"

only is False if P is True and Q is False .

False implies nothing
P False means Q can be True or False
Anything implies true.
P can be True or False when Q is True

If chemical plant pollutes river, fish die.
If fish die, did chemical plant pollute river?

Not necessarily.

$P \implies Q$ and Q are True does not mean P is True

Be careful!

Non-Consequences/consequences of Implication

The statement "$P \implies Q$"

only is False if P is True and Q is False .

False implies nothing
P False means Q can be True or False
Anything implies true.
P can be True or False when Q is True

If chemical plant pollutes river, fish die.
If fish die, did chemical plant pollute river?

Not necessarily.

$P \implies Q$ and Q are True does not mean P is True

Be careful!

Instead we have:

Non-Consequences/consequences of Implication

The statement "$P \implies Q$"

only is False if P is True and Q is False .

False implies nothing
P False means Q can be True or False
Anything implies true.
P can be True or False when Q is True

If chemical plant pollutes river, fish die.
If fish die, did chemical plant pollute river?

Not necessarily.

$P \implies Q$ and Q are True does not mean P is True

Be careful!

Instead we have:
$P \implies Q$ and P are True *does* mean Q is True .

Non-Consequences/consequences of Implication

The statement "$P \implies Q$"

only is False if P is True and Q is False .

False implies nothing
P False means Q can be True or False
Anything implies true.
P can be True or False when Q is True

If chemical plant pollutes river, fish die.
If fish die, did chemical plant pollute river?

Not necessarily.

$P \implies Q$ and Q are True does not mean P is True

Be careful!

Instead we have:
$P \implies Q$ and P are True *does* mean Q is True .

The chemical plant pollutes river.

Non-Consequences/consequences of Implication

The statement "$P \implies Q$"

only is False if P is True and Q is False .

False implies nothing
P False means Q can be True or False
Anything implies true.
P can be True or False when Q is True

If chemical plant pollutes river, fish die.
If fish die, did chemical plant pollute river?

Not necessarily.

$P \implies Q$ and Q are True does not mean P is True

Be careful!

Instead we have:
$P \implies Q$ and P are True *does* mean Q is True .

The chemical plant pollutes river. Can we conclude fish die?

Non-Consequences/consequences of Implication

The statement "$P \implies Q$"

only is False if P is True and Q is False .

False implies nothing
P False means Q can be True or False
Anything implies true.
P can be True or False when Q is True

If chemical plant pollutes river, fish die.
If fish die, did chemical plant pollute river?

Not necessarily.

$P \implies Q$ and Q are True does not mean P is True

Be careful!

Instead we have:
$P \implies Q$ and P are True *does* mean Q is True .

The chemical plant pollutes river. Can we conclude fish die?

Non-Consequences/consequences of Implication

The statement "$P \implies Q$"

only is False if P is True and Q is False .

False implies nothing
P False means Q can be True or False
Anything implies true.
P can be True or False when Q is True

If chemical plant pollutes river, fish die.
If fish die, did chemical plant pollute river?

Not necessarily.

$P \implies Q$ and Q are True does not mean P is True

Be careful!

Instead we have:
$P \implies Q$ and P are True *does* mean Q is True .

The chemical plant pollutes river. Can we conclude fish die?

Some Fun: use propositional formulas to describe implication?

Non-Consequences/consequences of Implication

The statement "$P \implies Q$"

only is False if P is True and Q is False .

False implies nothing
P False means Q can be True or False
Anything implies true.
P can be True or False when Q is True

If chemical plant pollutes river, fish die.
If fish die, did chemical plant pollute river?

Not necessarily.

$P \implies Q$ and Q are True does not mean P is True

Be careful!

Instead we have:
$P \implies Q$ and P are True *does* mean Q is True .

The chemical plant pollutes river. Can we conclude fish die?

Some Fun: use propositional formulas to describe implication?
$((P \implies Q) \land P) \implies Q$.

Implication and English.

$P \implies Q$

- If P, then Q.

Implication and English.

$P \implies Q$

- If P, then Q.
- Q if P.
  Just reversing the order.

Implication and English.

$P \implies Q$

- If P, then Q.
- Q if P.
  Just reversing the order.

Implication and English.

$P \implies Q$

- If P, then Q.
- Q if P.
  Just reversing the order.
- P only if Q.

Implication and English.

$P \implies Q$

- If P, then Q.
- Q if P.
  Just reversing the order.
- P only if Q.
  Remember if P is true then Q must be true.

Implication and English.

$P \implies Q$

- If P, then Q.
- Q if P.
  Just reversing the order.
- P only if Q.
  Remember if P is true then Q must be true.
  this suggests that P can only be true if Q is true.

Implication and English.

$P \implies Q$

- If P, then Q.
- Q if P.
  Just reversing the order.
- P only if Q.
  Remember if P is true then Q must be true.
  this suggests that P can only be true if Q is true.
  since if Q is false P must have been false.

Implication and English.

$P \implies Q$

- If P, then Q.
- Q if P.
  Just reversing the order.
- P only if Q.
  Remember if P is true then Q must be true.
  this suggests that P can only be true if Q is true.
  since if Q is false P must have been false.
- P is sufficient for Q.

Implication and English.

$P \implies Q$

- If P, then Q.
- Q if P.
  Just reversing the order.
- P only if Q.
  Remember if P is true then Q must be true.
  this suggests that P can only be true if Q is true.
  since if Q is false P must have been false.
- P is sufficient for Q.
  This means that proving P allows you

Implication and English.

$P \implies Q$

- If P, then Q.
- Q if P.
  Just reversing the order.
- P only if Q.
  Remember if P is true then Q must be true.
  this suggests that P can only be true if Q is true.
  since if Q is false P must have been false.
- P is sufficient for Q.
  This means that proving P allows you
  to conclude that Q is true.

Implication and English.

$P \implies Q$

- If P, then Q.
- Q if P.
  Just reversing the order.
- P only if Q.
  Remember if P is true then Q must be true.
  this suggests that P can only be true if Q is true.
  since if Q is false P must have been false.
- P is sufficient for Q.
  This means that proving P allows you
  to conclude that Q is true.
  Example: Showing $n > 4$ is sufficient for showing $n > 3$.

Implication and English.

$P \implies Q$

- If P, then Q.
- Q if P.
  Just reversing the order.
- P only if Q.
  Remember if P is true then Q must be true.
  this suggests that P can only be true if Q is true.
  since if Q is false P must have been false.
- P is sufficient for Q.
  This means that proving P allows you
  to conclude that Q is true.
  Example: Showing $n > 4$ is sufficient for showing $n > 3$.
- Q is necessary for P.
  For P to be true it is necessary that Q is true.
  Or if Q is false then we know that P is false.

Implication and English.

$P \implies Q$

- If P, then Q.
- Q if P.
  Just reversing the order.
- P only if Q.
  Remember if P is true then Q must be true.
  this suggests that P can only be true if Q is true.
  since if Q is false P must have been false.
- P is sufficient for Q.
  This means that proving P allows you
  to conclude that Q is true.
  Example: Showing $n > 4$ is sufficient for showing $n > 3$.
- Q is necessary for P.
  For P to be true it is necessary that Q is true.
  Or if Q is false then we know that P is false.
  Example: It is necessary that $n > 3$ for $n > 4$.

Truth Table: implication.

P	Q	$P \implies Q$
T	T	T
T	F	
F	T	
F	F	

Truth Table: implication.

P	Q	$P \implies Q$
T	T	T
T	F	F
F	T	
F	F	

Truth Table: implication.

P	Q	$P \implies Q$
T	T	T
T	F	F
F	T	T
F	F	

Truth Table: implication.

P	Q	$P \implies Q$
T	T	T
T	F	F
F	T	T
F	F	T

Truth Table: implication.

P	Q	$P \implies Q$
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	$\neg P \vee Q$
T	T	
T	F	
F	T	
F	F	

Truth Table: implication.

P	Q	$P \implies Q$
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	$\neg P \lor Q$
T	T	T
T	F	
F	T	
F	F	

Truth Table: implication.

P	Q	$P \implies Q$
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	$\neg P \vee Q$
T	T	T
T	F	F
F	T	
F	F	

Truth Table: implication.

P	Q	$P \implies Q$
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	$\neg P \vee Q$
T	T	T
T	F	F
F	T	T
F	F	

Truth Table: implication.

P	Q	$P \implies Q$
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	$\neg P \vee Q$
T	T	T
T	F	F
F	T	T
F	F	T

Truth Table: implication.

P	Q	$P \implies Q$
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	$\neg P \vee Q$
T	T	T
T	F	F
F	T	T
F	F	T

$\neg P \vee Q \quad \equiv P \implies Q.$

Truth Table: implication.

P	Q	$P \implies Q$
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	$\neg P \vee Q$
T	T	T
T	F	F
F	T	T
F	F	T

$\neg P \vee Q \quad \equiv P \implies Q.$

These two propositional forms are logically equivalent!

Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.

Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.

Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.
  - If the fish don't die, the plant does not pollute.

Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.
  - If the fish don't die, the plant does not pollute. (contrapositive)

Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.
  - If the fish don't die, the plant does not pollute. (contrapositive)
  - If you stand in the rain, you get wet.

Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.
  - If the fish don't die, the plant does not pollute. (contrapositive)
  - If you stand in the rain, you get wet.
  - If you did not stand in the rain, you did not get wet.

Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.
  - If the fish don't die, the plant does not pollute. (contrapositive)
  - If you stand in the rain, you get wet.
  - If you did not stand in the rain, you did not get wet. (not contrapositive!)

Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.
  - If the fish don't die, the plant does not pollute. (contrapositive)
  - If you stand in the rain, you get wet.
  - If you did not stand in the rain, you did not get wet. (not contrapositive!)
  - If you did not get wet, you did not stand in the rain.

Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.
  - If the fish don't die, the plant does not pollute. (contrapositive)
  - If you stand in the rain, you get wet.
  - If you did not stand in the rain, you did not get wet. (not contrapositive!)
  - If you did not get wet, you did not stand in the rain. (contrapositive.)

Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.
  - If the fish don't die, the plant does not pollute. (contrapositive)

  - If you stand in the rain, you get wet.
  - If you did not stand in the rain, you did not get wet. (not contrapositive!)
  - If you did not get wet, you did not stand in the rain. (contrapositive.)

  Logically equivalent! Notation: $\equiv$.

Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.
  - If the fish don't die, the plant does not pollute. (contrapositive)
  - If you stand in the rain, you get wet.
  - If you did not stand in the rain, you did not get wet. (not contrapositive!)
  - If you did not get wet, you did not stand in the rain. (contrapositive.)

  Logically equivalent! Notation: $\equiv$.
  $P \implies Q$

Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.
  - If the fish don't die, the plant does not pollute. (contrapositive)

  - If you stand in the rain, you get wet.
  - If you did not stand in the rain, you did not get wet. (not contrapositive!)
  - If you did not get wet, you did not stand in the rain. (contrapositive.)

Logically equivalent! Notation: $\equiv$.
$P \implies Q \equiv \neg P \vee Q$

Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.
  - If the fish don't die, the plant does not pollute. (contrapositive)

  - If you stand in the rain, you get wet.
  - If you did not stand in the rain, you did not get wet. (not contrapositive!)
  - If you did not get wet, you did not stand in the rain. (contrapositive.)

  Logically equivalent! Notation: $\equiv$.

  $P \implies Q \equiv \neg P \vee Q \equiv \neg(\neg Q) \vee \neg P$

Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.
  - If the fish don't die, the plant does not pollute. (contrapositive)
  - If you stand in the rain, you get wet.
  - If you did not stand in the rain, you did not get wet. (not contrapositive!)
  - If you did not get wet, you did not stand in the rain. (contrapositive.)

Logically equivalent! Notation: $\equiv$.
$P \implies Q \equiv \neg P \vee Q \equiv \neg(\neg Q) \vee \neg P \equiv \neg Q \implies \neg P$.

Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.
  - If the fish don't die, the plant does not pollute. (contrapositive)
  - If you stand in the rain, you get wet.
  - If you did not stand in the rain, you did not get wet. (not contrapositive!)
  - If you did not get wet, you did not stand in the rain. (contrapositive.)

  Logically equivalent! Notation: $\equiv$.
  $P \implies Q \equiv \neg P \vee Q \equiv \neg(\neg Q) \vee \neg P \equiv \neg Q \implies \neg P$.
- **Converse** of $P \implies Q$ is $Q \implies P$.

Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.
  - If the fish don't die, the plant does not pollute.
    (contrapositive)
  - If you stand in the rain, you get wet.
  - If you did not stand in the rain, you did not get wet.
    (not contrapositive!)
  - If you did not get wet, you did not stand in the rain.
    (contrapositive.)

  Logically equivalent! Notation: $\equiv$.
  $P \implies Q \equiv \neg P \vee Q \equiv \neg(\neg Q) \vee \neg P \equiv \neg Q \implies \neg P$.
- **Converse** of $P \implies Q$ is $Q \implies P$.

  If fish die the plant pollutes.

Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.
  - If the fish don't die, the plant does not pollute. (contrapositive)
  - If you stand in the rain, you get wet.
  - If you did not stand in the rain, you did not get wet. (not contrapositive!) converse!
  - If you did not get wet, you did not stand in the rain. (contrapositive.)

  Logically equivalent! Notation: $\equiv$.
  $P \implies Q \equiv \neg P \vee Q \equiv \neg(\neg Q) \vee \neg P \equiv \neg Q \implies \neg P$.
- **Converse** of $P \implies Q$ is $Q \implies P$.

  If fish die the plant pollutes.

Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.
  - If the fish don't die, the plant does not pollute. (contrapositive)

  - If you stand in the rain, you get wet.
  - If you did not stand in the rain, you did not get wet. (not contrapositive!) converse!
  - If you did not get wet, you did not stand in the rain. (contrapositive.)

  Logically equivalent! Notation: $\equiv$.
  $P \implies Q \equiv \neg P \vee Q \equiv \neg(\neg Q) \vee \neg P \equiv \neg Q \implies \neg P$.
- **Converse** of $P \implies Q$ is $Q \implies P$.

  If fish die the plant pollutes.

  Not logically equivalent!

Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.
  - If the fish don't die, the plant does not pollute. (contrapositive)
  - If you stand in the rain, you get wet.
  - If you did not stand in the rain, you did not get wet. (not contrapositive!) converse!
  - If you did not get wet, you did not stand in the rain. (contrapositive.)

  Logically equivalent! Notation: $\equiv$.
  $P \implies Q \equiv \neg P \vee Q \equiv \neg(\neg Q) \vee \neg P \equiv \neg Q \implies \neg P$.
- **Converse** of $P \implies Q$ is $Q \implies P$.

  If fish die the plant pollutes.

  Not logically equivalent!
- **Definition:** If $P \implies Q$ and $Q \implies P$ is P if and only if Q or $P \iff Q$.
  (Logically Equivalent: $\iff$.)

Variables.

Propositions?

- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.

Variables.

Propositions?

- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.
- $x > 2$

Variables.

Propositions?

- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.
- $x > 2$
- n is even and the sum of two primes

Variables.

Propositions?

- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.
- $x > 2$
- n is even and the sum of two primes

No.

Variables.

Propositions?

- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.
- $x > 2$
- n is even and the sum of two primes

No. They have a free variable.

Variables.

Propositions?

- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.
- $x > 2$
- n is even and the sum of two primes

No. They have a free variable.

We call them **predicates**, e.g., $Q(x) =$ "x is even"

Variables.

Propositions?

- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.
- $x > 2$
- n is even and the sum of two primes

No. They have a free variable.

We call them **predicates**, e.g., $Q(x) =$ "x is even"
Same as boolean valued functions from 61A!

Variables.

Propositions?

- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.
- $x > 2$
- n is even and the sum of two primes

No. They have a free variable.

We call them **predicates**, e.g., $Q(x) =$ "x is even"
Same as boolean valued functions from 61A!

- $P(n) =$ "$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$."

Variables.

Propositions?

- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.
- $x > 2$
- n is even and the sum of two primes

No. They have a free variable.

We call them **predicates**, e.g., $Q(x) =$ "x is even"
Same as boolean valued functions from 61A!

- $P(n) =$ "$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$."
- $R(x) =$ "$x > 2$"

Variables.

Propositions?

- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.
- $x > 2$
- n is even and the sum of two primes

No. They have a free variable.

We call them **predicates**, e.g., $Q(x) =$ "x is even"
Same as boolean valued functions from 61A!

- $P(n) =$ "$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$."
- $R(x) =$ "$x > 2$"
- $G(n) =$ "n is even and the sum of two primes"

Variables.

Propositions?

- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.
- $x > 2$
- n is even and the sum of two primes

No. They have a free variable.

We call them **predicates**, e.g., $Q(x) =$ "x is even"
Same as boolean valued functions from 61A!

- $P(n) =$ "$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$."
- $R(x) =$ "$x > 2$"
- $G(n) =$ "n is even and the sum of two primes"
- Remember Wason's experiment!
  $F(x) =$ "Person x flew."

Variables.

Propositions?

- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.
- $x > 2$
- n is even and the sum of two primes

No. They have a free variable.

We call them **predicates**, e.g., $Q(x) =$ "x is even"
Same as boolean valued functions from 61A!

- $P(n) =$ "$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$."
- $R(x) =$ "$x > 2$"
- $G(n) =$ "n is even and the sum of two primes"
- Remember Wason's experiment!
  $F(x) =$ "Person x flew."
  $C(x) =$ "Person x went to Chicago

Variables.

Propositions?

- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.
- $x > 2$
- n is even and the sum of two primes

No. They have a free variable.

We call them **predicates**, e.g., $Q(x)$ = "x is even"
Same as boolean valued functions from 61A!

- $P(n)$ = "$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$."
- $R(x)$ = "$x > 2$"
- $G(n)$ = "n is even and the sum of two primes"
- Remember Wason's experiment!
  $F(x)$ = "Person x flew."
  $C(x)$ = "Person x went to Chicago
- $C(x) \implies F(x)$.

Variables.

Propositions?

- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.
- $x > 2$
- n is even and the sum of two primes

No. They have a free variable.

We call them **predicates**, e.g., $Q(x) =$ "x is even"
Same as boolean valued functions from 61A!

- $P(n) =$ "$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$."
- $R(x) =$ "$x > 2$"
- $G(n) =$ "n is even and the sum of two primes"
- Remember Wason's experiment!
  $F(x) =$ "Person x flew."
  $C(x) =$ "Person x went to Chicago
- $C(x) \implies F(x)$. Theory from Wason's.

Variables.

Propositions?

- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.
- $x > 2$
- n is even and the sum of two primes

No. They have a free variable.

We call them **predicates**, e.g., $Q(x) =$ "x is even"
Same as boolean valued functions from 61A!

- $P(n) =$ "$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$."
- $R(x) =$ "$x > 2$"
- $G(n) =$ "n is even and the sum of two primes"
- Remember Wason's experiment!
  $F(x) =$ "Person x flew."
  $C(x) =$ "Person x went to Chicago
- $C(x) \implies F(x)$. Theory from Wason's.
  If person x goes to Chicago then person x flew.

Variables.

Propositions?

- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.
- $x > 2$
- n is even and the sum of two primes

No. They have a free variable.

We call them **predicates**, e.g., $Q(x) =$ "x is even"
Same as boolean valued functions from 61A!

- $P(n) =$ "$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$."
- $R(x) =$ "$x > 2$"
- $G(n) =$ "n is even and the sum of two primes"
- Remember Wason's experiment!
  $F(x) =$ "Person x flew."
  $C(x) =$ "Person x went to Chicago
- $C(x) \implies F(x)$. Theory from Wason's.
  If person x goes to Chicago then person x flew.

Next:

Variables.

Propositions?

- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.
- $x > 2$
- n is even and the sum of two primes

No. They have a free variable.

We call them **predicates**, e.g., $Q(x) =$ "x is even"
Same as boolean valued functions from 61A!

- $P(n) =$ "$\sum_{i=1}^{n} \ i = \frac{n(n+1)}{2}$."
- $R(x) =$ "$x > 2$"
- $G(n) =$ "n is even and the sum of two primes"
- Remember Wason's experiment!
  $F(x) =$ "Person x flew."
  $C(x) =$ "Person x went to Chicago
- $C(x) \implies F(x)$. Theory from Wason's.
  If person x goes to Chicago then person x flew.

Next: Statements about boolean valued functions!!

Quantifiers..

There exists quantifier:

Quantifiers..

There exists quantifier:

$(\exists x \in S)(P(x))$ means "There exists an x in S where $P(x)$ is true."

Quantifiers..

There exists quantifier:

$(\exists x \in S)(P(x))$ means "There exists an x in S where $P(x)$ is true."

For example:

$$(\exists x \in \mathbb{N})(x = x^2)$$

Quantifiers..

There exists quantifier:

$(\exists x \in S)(P(x))$ means "There exists an x in S where $P(x)$ is true."

For example:

$$(\exists x \in \mathbb{N})(x = x^2)$$

Equivalent to "$(0 = 0)$

Quantifiers..

There exists quantifier:

$(\exists x \in S)(P(x))$ means "There exists an x in S where $P(x)$ is true."

For example:

$$(\exists x \in \mathbb{N})(x = x^2)$$

Equivalent to "$(0 = 0) \vee (1 = 1)$

Quantifiers..

There exists quantifier:

$(\exists x \in S)(P(x))$ means "There exists an x in S where $P(x)$ is true."

For example:

$$(\exists x \in \mathbb{N})(x = x^2)$$

Equivalent to "$(0 = 0) \vee (1 = 1) \vee (2 = 4)$

Quantifiers..

There exists quantifier:

$(\exists x \in S)(P(x))$ means "There exists an x in S where $P(x)$ is true."

For example:

$$(\exists x \in \mathbb{N})(x = x^2)$$

Equivalent to "$(0 = 0) \vee (1 = 1) \vee (2 = 4) \vee \ldots$"

Quantifiers..

There exists quantifier:

$(\exists x \in S)(P(x))$ means "There exists an x in S where $P(x)$ is true."

For example:

$$(\exists x \in \mathbb{N})(x = x^2)$$

Equivalent to "$(0 = 0) \vee (1 = 1) \vee (2 = 4) \vee \ldots$"

Much shorter to use a quantifier!

Quantifiers..

There exists quantifier:

$(\exists x \in S)(P(x))$ means "There exists an x in S where $P(x)$ is true."

For example:

$$(\exists x \in \mathbb{N})(x = x^2)$$

Equivalent to "$(0 = 0) \vee (1 = 1) \vee (2 = 4) \vee \ldots$"

Much shorter to use a quantifier!

For all quantifier;
$(\forall x \in S)\ (P(x))$. means "For all x in S, $P(x)$ is True ."

Quantifiers..

There exists quantifier:

$(\exists x \in S)(P(x))$ means "There exists an x in S where $P(x)$ is true."

For example:

$$(\exists x \in \mathbb{N})(x = x^2)$$

Equivalent to "$(0 = 0) \vee (1 = 1) \vee (2 = 4) \vee \ldots$"

Much shorter to use a quantifier!

For all quantifier;
$(\forall x \in S)\ (P(x))$. means "For all x in S, $P(x)$ is True ."

Examples:

Quantifiers..

There exists quantifier:

$(\exists x \in S)(P(x))$ means "There exists an x in S where $P(x)$ is true."

For example:

$$(\exists x \in \mathbb{N})(x = x^2)$$

Equivalent to "$(0 = 0) \vee (1 = 1) \vee (2 = 4) \vee \ldots$"

Much shorter to use a quantifier!

For all quantifier;
$(\forall x \in S)\ (P(x))$. means "For all x in S, $P(x)$ is True ."

Examples:

"Adding 1 makes a bigger number."

Quantifiers..

There exists quantifier:

$(\exists x \in S)(P(x))$ means "There exists an x in S where $P(x)$ is true."

For example:

$$(\exists x \in \mathbb{N})(x = x^2)$$

Equivalent to "$(0 = 0) \vee (1 = 1) \vee (2 = 4) \vee \ldots$"

Much shorter to use a quantifier!

For all quantifier;
$(\forall x \in S)\ (P(x))$. means "For all x in S, $P(x)$ is True ."

Examples:

"Adding 1 makes a bigger number."

$$(\forall x \in \mathbb{N})\ (x + 1 > x)$$

Quantifiers..

There exists quantifier:

$(\exists x \in S)(P(x))$ means "There exists an x in S where $P(x)$ is true."

For example:

$$(\exists x \in \mathbb{N})(x = x^2)$$

Equivalent to "$(0 = 0) \vee (1 = 1) \vee (2 = 4) \vee \ldots$"

Much shorter to use a quantifier!

For all quantifier;
$(\forall x \in S)\ (P(x))$. means "For all x in S, $P(x)$ is True ."

Examples:

"Adding 1 makes a bigger number."

$(\forall x \in \mathbb{N})\ (x + 1 > x)$

"the square of a number is always non-negative"

Quantifiers..

There exists quantifier:

$(\exists x \in S)(P(x))$ means "There exists an x in S where $P(x)$ is true."

For example:

$$(\exists x \in \mathbb{N})(x = x^2)$$

Equivalent to "$(0 = 0) \vee (1 = 1) \vee (2 = 4) \vee \ldots$"

Much shorter to use a quantifier!

For all quantifier;
$(\forall x \in S)\ (P(x))$. means "For all x in S, $P(x)$ is True ."

Examples:

"Adding 1 makes a bigger number."

$$(\forall x \in \mathbb{N})\ (x + 1 > x)$$

"the square of a number is always non-negative"

$$(\forall x \in \mathbb{N})(x^2 >= 0)$$

Quantifiers..

There exists quantifier:

$(\exists x \in S)(P(x))$ means "There exists an x in S where $P(x)$ is true."

For example:

$$(\exists x \in \mathbb{N})(x = x^2)$$

Equivalent to "$(0 = 0) \vee (1 = 1) \vee (2 = 4) \vee \ldots$"

Much shorter to use a quantifier!

For all quantifier;
$(\forall x \in S)\ (P(x))$. means "For all x in S, $P(x)$ is True ."

Examples:

"Adding 1 makes a bigger number."

$$(\forall x \in \mathbb{N})\ (x + 1 > x)$$

"the square of a number is always non-negative"

$$(\forall x \in \mathbb{N})(x^2 >= 0)$$

Wait!

Quantifiers..

There exists quantifier:

$(\exists x \in S)(P(x))$ means "There exists an x in S where $P(x)$ is true."

For example:

$$(\exists x \in \mathbb{N})(x = x^2)$$

Equivalent to "$(0 = 0) \vee (1 = 1) \vee (2 = 4) \vee \ldots$"

Much shorter to use a quantifier!

For all quantifier;
$(\forall x \in S)\ (P(x))$. means "For all x in S, $P(x)$ is True ."

Examples:

"Adding 1 makes a bigger number."

$$(\forall x \in \mathbb{N})\ (x + 1 > x)$$

"the square of a number is always non-negative"

$$(\forall x \in \mathbb{N})(x^2 >= 0)$$

Wait! What is $\mathbb{N}$?

Quantifiers: universes.

Proposition: "For all natural numbers n**,** $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$**."**

Proposition has **universe**:

Quantifiers: universes.

Proposition: "For all natural numbers n**,** $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$**."**

Proposition has **universe**: "the natural numbers".

Quantifiers: universes.

Proposition: "For all natural numbers n**,** $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$**."**

Proposition has **universe**: "the natural numbers".

Universe examples include..

Quantifiers: universes.

Proposition: "For all natural numbers n, $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$."

Proposition has **universe**: "the natural numbers".

Universe examples include..

- $\mathbb{N} = \{0, 1, \ldots\}$ (natural numbers).

Quantifiers: universes.

Proposition: "For all natural numbers n**,** $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$**."**

Proposition has **universe**: "the natural numbers".

Universe examples include..

- $\mathbb{N} = \{0, 1, \ldots\}$ (natural numbers).
- $\mathbb{Z} = \{\ldots, -1, 0, \ldots\}$ (integers)

Quantifiers: universes.

Proposition: "For all natural numbers n**,** $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$**."**

Proposition has **universe**: "the natural numbers".

Universe examples include..

- $\mathbb{N} = \{0, 1, \ldots\}$ (natural numbers).
- $\mathbb{Z} = \{\ldots, -1, 0, \ldots\}$ (integers)
- $\mathbb{Z}^+$ (positive integers)

Quantifiers: universes.

Proposition: "For all natural numbers n**,** $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$**."**

Proposition has **universe**: "the natural numbers".

Universe examples include..

- $\mathbb{N} = \{0, 1, \ldots\}$ (natural numbers).
- $\mathbb{Z} = \{\ldots, -1, 0, \ldots\}$ (integers)
- $\mathbb{Z}^+$ (positive integers)
- $\mathbb{R}$ (real numbers)

Quantifiers: universes.

Proposition: "For all natural numbers n**,** $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$**."**

Proposition has **universe**: "the natural numbers".

Universe examples include..

- $\mathbb{N} = \{0, 1, \ldots\}$ (natural numbers).
- $\mathbb{Z} = \{\ldots, -1, 0, \ldots\}$ (integers)
- $\mathbb{Z}^{+}$ (positive integers)
- $\mathbb{R}$ (real numbers)
- Any set: $S = \{Alice, Bob, Charlie, Donna\}$.

Quantifiers: universes.

Proposition: "For all natural numbers n**,** $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$**."**

Proposition has **universe**: "the natural numbers".

Universe examples include..

- $\mathbb{N} = \{0, 1, \ldots\}$ (natural numbers).
- $\mathbb{Z} = \{\ldots, -1, 0, \ldots\}$ (integers)
- $\mathbb{Z}^+$ (positive integers)
- $\mathbb{R}$ (real numbers)
- Any set: $S = \{Alice, Bob, Charlie, Donna\}$.
- See note 0 for more!

Back to: Wason's experiment:1

Theory:

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

$Chicago(x)$ = "x went to Chicago."

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

$Chicago(x)$ = "x went to Chicago." $Flew(x)$ = "x flew"

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

$Chicago(x)$ = "x went to Chicago." $Flew(x)$ = "x flew"

Statement/theory: $\forall x \in \{A, B, C, D\}, Chicago(x)$

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

$\mathit{Chicago}(x)$ = "x went to Chicago." $\mathit{Flew}(x)$ = "x flew"

Statement/theory: $\forall x \in \{A, B, C, D\}, \mathit{Chicago}(x) \implies \mathit{Flew}(x)$

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

$Chicago(x)$ = "x went to Chicago." $Flew(x)$ = "x flew"

Statement/theory: $\forall x \in \{A, B, C, D\}, Chicago(x) \implies Flew(x)$

$Chicago(A)$ = False .

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

$Chicago(x)$ = "x went to Chicago." $Flew(x)$ = "x flew"

Statement/theory: $\forall x \in \{A, B, C, D\}, Chicago(x) \implies Flew(x)$

$Chicago(A)$ = False . Do we care about $Flew(A)$?

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

$Chicago(x)$ = "x went to Chicago." $Flew(x)$ = "x flew"

Statement/theory: $\forall x \in \{A, B, C, D\}, Chicago(x) \implies Flew(x)$

$Chicago(A)$ = False . Do we care about $Flew(A)$?

No.

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

$Chicago(x)$ = "x went to Chicago." $Flew(x)$ = "x flew"

Statement/theory: $\forall x \in \{A, B, C, D\}, Chicago(x) \implies Flew(x)$

$Chicago(A)$ = False . Do we care about $Flew(A)$?
No. $Chicago(A) \implies Flew(A)$, when $Chicago(A)$ is False , $Flew(A)$ can be anything.

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

$Chicago(x)$ = "x went to Chicago." $Flew(x)$ = "x flew"

Statement/theory: $\forall x \in \{A, B, C, D\}, Chicago(x) \implies Flew(x)$

$Chicago(A)$ = False . Do we care about $Flew(A)$?
No. $Chicago(A) \implies Flew(A)$, when $Chicago(A)$ is False , $Flew(A)$ can be anything.

$Flew(B)$ = False .

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

$Chicago(x)$ = "x went to Chicago." $Flew(x)$ = "x flew"

Statement/theory: $\forall x \in \{A, B, C, D\}, Chicago(x) \implies Flew(x)$

$Chicago(A)$ = False . Do we care about $Flew(A)$?

No. $Chicago(A) \implies Flew(A)$, when $Chicago(A)$ is False , $Flew(A)$ can be anything.

$Flew(B)$ = False . Do we care about $Chicago(B)$?

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

$Chicago(x)$ = "x went to Chicago." $Flew(x)$ = "x flew"

Statement/theory: $\forall x \in \{A, B, C, D\}, Chicago(x) \implies Flew(x)$

$Chicago(A)$ = False . Do we care about $Flew(A)$?
No. $Chicago(A) \implies Flew(A)$, when $Chicago(A)$ is False , $Flew(A)$ can be anything.

$Flew(B)$ = False . Do we care about $Chicago(B)$?
Yes.

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

$Chicago(x)$ = "x went to Chicago." $Flew(x)$ = "x flew"

Statement/theory: $\forall x \in \{A, B, C, D\}, Chicago(x) \implies Flew(x)$

$Chicago(A)$ = False . Do we care about $Flew(A)$?
No. $Chicago(A) \implies Flew(A)$, when $Chicago(A)$ is False , $Flew(A)$ can be anything.

$Flew(B)$ = False . Do we care about $Chicago(B)$?
Yes. $Chicago(B) \implies Flew(B)$

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

$Chicago(x)$ = "x went to Chicago." $Flew(x)$ = "x flew"

Statement/theory: $\forall x \in \{A, B, C, D\}, Chicago(x) \implies Flew(x)$

$Chicago(A)$ = False . Do we care about $Flew(A)$?
No. $Chicago(A) \implies Flew(A)$, when $Chicago(A)$ is False , $Flew(A)$ can be anything.

$Flew(B)$ = False . Do we care about $Chicago(B)$?
Yes. $Chicago(B) \implies Flew(B) \equiv \neg Flew(B) \implies \neg Chicago(B)$.

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

$Chicago(x)$ = "x went to Chicago." $Flew(x)$ = "x flew"

Statement/theory: $\forall x \in \{A, B, C, D\}, Chicago(x) \implies Flew(x)$

$Chicago(A)$ = False . Do we care about $Flew(A)$?
No. $Chicago(A) \implies Flew(A)$, when $Chicago(A)$ is False , $Flew(A)$ can be anything.

$Flew(B)$ = False . Do we care about $Chicago(B)$?
Yes. $Chicago(B) \implies Flew(B) \equiv \neg Flew(B) \implies \neg Chicago(B)$.
So $Chicago(Bob)$ must be False .

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

$Chicago(x)$ = "x went to Chicago." $Flew(x)$ = "x flew"

Statement/theory: $\forall x \in \{A, B, C, D\}, Chicago(x) \implies Flew(x)$

$Chicago(A)$ = False . Do we care about $Flew(A)$?
No. $Chicago(A) \implies Flew(A)$, when $Chicago(A)$ is False , $Flew(A)$ can be anything.

$Flew(B)$ = False . Do we care about $Chicago(B)$?
Yes. $Chicago(B) \implies Flew(B) \equiv \neg Flew(B) \implies \neg Chicago(B)$.
So $Chicago(Bob)$ must be False .

$Chicago(C)$ = True .

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

$Chicago(x)$ = "x went to Chicago." $Flew(x)$ = "x flew"

Statement/theory: $\forall x \in \{A, B, C, D\}, Chicago(x) \implies Flew(x)$

$Chicago(A)$ = False . Do we care about $Flew(A)$?
No. $Chicago(A) \implies Flew(A)$, when $Chicago(A)$ is False , $Flew(A)$ can be anything.

$Flew(B)$ = False . Do we care about $Chicago(B)$?
Yes. $Chicago(B) \implies Flew(B) \equiv \neg Flew(B) \implies \neg Chicago(B)$.
So $Chicago(Bob)$ must be False .

$Chicago(C)$ = True . Do we care about $Flew(C)$?

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

$Chicago(x)$ = "x went to Chicago." $Flew(x)$ = "x flew"

Statement/theory: $\forall x \in \{A, B, C, D\}, Chicago(x) \implies Flew(x)$

$Chicago(A)$ = False . Do we care about $Flew(A)$?
No. $Chicago(A) \implies Flew(A)$, when $Chicago(A)$ is False , $Flew(A)$ can be anything.

$Flew(B)$ = False . Do we care about $Chicago(B)$?
Yes. $Chicago(B) \implies Flew(B) \equiv \neg Flew(B) \implies \neg Chicago(B)$.
So $Chicago(Bob)$ must be False .

$Chicago(C)$ = True . Do we care about $Flew(C)$?
Yes.

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

$Chicago(x)$ = "x went to Chicago." $Flew(x)$ = "x flew"

Statement/theory: $\forall x \in \{A, B, C, D\}, Chicago(x) \implies Flew(x)$

$Chicago(A)$ = False . Do we care about $Flew(A)$?
No. $Chicago(A) \implies Flew(A)$, when $Chicago(A)$ is False , $Flew(A)$ can be anything.

$Flew(B)$ = False . Do we care about $Chicago(B)$?
Yes. $Chicago(B) \implies Flew(B) \equiv \neg Flew(B) \implies \neg Chicago(B)$.
So $Chicago(Bob)$ must be False .

$Chicago(C)$ = True . Do we care about $Flew(C)$?
Yes. $Chicago(C) \implies Flew(C)$ means $Flew(C)$ must be true.

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

$Chicago(x)$ = "x went to Chicago." $Flew(x)$ = "x flew"

Statement/theory: $\forall x \in \{A, B, C, D\}, Chicago(x) \implies Flew(x)$

$Chicago(A)$ = False . Do we care about $Flew(A)$?
No. $Chicago(A) \implies Flew(A)$, when $Chicago(A)$ is False , $Flew(A)$ can be anything.

$Flew(B)$ = False . Do we care about $Chicago(B)$?
Yes. $Chicago(B) \implies Flew(B) \equiv \neg Flew(B) \implies \neg Chicago(B)$.
So $Chicago(Bob)$ must be False .

$Chicago(C)$ = True . Do we care about $Flew(C)$?
Yes. $Chicago(C) \implies Flew(C)$ means $Flew(C)$ must be true.

$Flew(D)$ = True .

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

$Chicago(x)$ = "x went to Chicago." $Flew(x)$ = "x flew"

Statement/theory: $\forall x \in \{A, B, C, D\}, Chicago(x) \implies Flew(x)$

$Chicago(A) =$ False . Do we care about $Flew(A)$?
No. $Chicago(A) \implies Flew(A)$, when $Chicago(A)$ is False , $Flew(A)$ can be anything.

$Flew(B) =$ False . Do we care about $Chicago(B)$?
Yes. $Chicago(B) \implies Flew(B) \equiv \neg Flew(B) \implies \neg Chicago(B)$.
So $Chicago(Bob)$ must be False .

$Chicago(C) =$ True . Do we care about $Flew(C)$?
Yes. $Chicago(C) \implies Flew(C)$ means $Flew(C)$ must be true.

$Flew(D) =$ True . Do we care about $Chicago(D)$?

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

$Chicago(x)$ = "x went to Chicago." $Flew(x)$ = "x flew"

Statement/theory: $\forall x \in \{A, B, C, D\}, Chicago(x) \implies Flew(x)$

$Chicago(A)$ = False . Do we care about $Flew(A)$?
No. $Chicago(A) \implies Flew(A)$, when $Chicago(A)$ is False , $Flew(A)$ can be anything.

$Flew(B)$ = False . Do we care about $Chicago(B)$?
Yes. $Chicago(B) \implies Flew(B) \equiv \neg Flew(B) \implies \neg Chicago(B)$.
So $Chicago(Bob)$ must be False .

$Chicago(C)$ = True . Do we care about $Flew(C)$?
Yes. $Chicago(C) \implies Flew(C)$ means $Flew(C)$ must be true.

$Flew(D)$ = True . Do we care about $Chicago(D)$?
No.

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

$Chicago(x)$ = "x went to Chicago." $Flew(x)$ = "x flew"

Statement/theory: $\forall x \in \{A, B, C, D\}, Chicago(x) \implies Flew(x)$

$Chicago(A)$ = False . Do we care about $Flew(A)$?
No. $Chicago(A) \implies Flew(A)$, when $Chicago(A)$ is False , $Flew(A)$ can be anything.

$Flew(B)$ = False . Do we care about $Chicago(B)$?
Yes. $Chicago(B) \implies Flew(B) \equiv \neg Flew(B) \implies \neg Chicago(B)$.
So $Chicago(Bob)$ must be False .

$Chicago(C)$ = True . Do we care about $Flew(C)$?
Yes. $Chicago(C) \implies Flew(C)$ means $Flew(C)$ must be true.

$Flew(D)$ = True . Do we care about $Chicago(D)$?
No. $Chicago(D) \implies Flew(D)$ holds whatever $Chicago(D)$ is when $Flew(D)$ is true.

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Alice to Baltimore, Bob drove, Charlie to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

$Chicago(x)$ = "x went to Chicago." $Flew(x)$ = "x flew"

Statement/theory: $\forall x \in \{A, B, C, D\}, Chicago(x) \implies Flew(x)$

$Chicago(A)$ = False . Do we care about $Flew(A)$?
No. $Chicago(A) \implies Flew(A)$, when $Chicago(A)$ is False , $Flew(A)$ can be anything.

$Flew(B)$ = False . Do we care about $Chicago(B)$?
Yes. $Chicago(B) \implies Flew(B) \equiv \neg Flew(B) \implies \neg Chicago(B)$.
So $Chicago(Bob)$ must be False .

$Chicago(C)$ = True . Do we care about $Flew(C)$?
Yes. $Chicago(C) \implies Flew(C)$ means $Flew(C)$ must be true.

$Flew(D)$ = True . Do we care about $Chicago(D)$?
No. $Chicago(D) \implies Flew(D)$ holds whatever $Chicago(D)$ is when $Flew(D)$ is true.

Only have to turn over cards for Bob and Charlie.

More for all quantifiers examples.

More for all quantifiers examples.

- “doubling a number always makes it larger”

More for all quantifiers examples.

- "doubling a number always makes it larger"

$$(\forall x \in N)\ (2x > x)$$

More for all quantifiers examples.

- "doubling a number always makes it larger"

$$(\forall x \in N)\ (2x > x) \qquad \textbf{False}$$

More for all quantifiers examples.

- "doubling a number always makes it larger"

$$(\forall x \in N)\,(2x > x) \quad \textbf{False} \quad \textbf{Consider } x = 0$$

More for all quantifiers examples.

- "doubling a number always makes it larger"

  $(\forall x \in N)\ (2x > x)$ **False** **Consider** $x = 0$

  Can fix statement...

More for all quantifiers examples.

- "doubling a number always makes it larger"

  $(\forall x \in N)\ (2x > x)$ **False** **Consider** $x = 0$

  Can fix statement...

  $(\forall x \in N)\ (2x \geq x)$

More for all quantifiers examples.

- “doubling a number always makes it larger”

  $(\forall x \in N)\ (2x > x)$ **False** **Consider** $x = 0$

  Can fix statement...

  $(\forall x \in N)\ (2x \geq x)$ **True**

More for all quantifiers examples.

- "doubling a number always makes it larger"

  $(\forall x \in N)\ (2x > x)$ **False** **Consider** $x = 0$

  Can fix statement...

  $(\forall x \in N)\ (2x \geq x)$ **True**

- "Square of any natural number greater than 5 is greater than 25."

More for all quantifiers examples.

- "doubling a number always makes it larger"

  $(\forall x \in N)\ (2x > x)$ **False** **Consider $x = 0$**

  Can fix statement...

  $(\forall x \in N)\ (2x \geq x)$ **True**

- "Square of any natural number greater than 5 is greater than 25."

  $(\forall x \in N)$

More for all quantifiers examples.

- "doubling a number always makes it larger"

  $$(\forall x \in N)\ (2x > x) \quad \textbf{False} \quad \textbf{Consider } x = 0$$

  Can fix statement...

  $$(\forall x \in N)\ (2x \geq x) \quad \textbf{True}$$

- "Square of any natural number greater than 5 is greater than 25."

  $$(\forall x \in N)(x > 5$$

More for all quantifiers examples.

- "doubling a number always makes it larger"

$$(\forall x \in N)\ (2x > x)$$ **False** **Consider** $x = 0$

Can fix statement...

$$(\forall x \in N)\ (2x \geq x)$$ **True**

- "Square of any natural number greater than 5 is greater than 25."

$$(\forall x \in N)(x > 5 \implies$$

More for all quantifiers examples.

- "doubling a number always makes it larger"

  $(\forall x \in N)\ (2x > x)$ **False** **Consider** $x = 0$

  Can fix statement...

  $(\forall x \in N)\ (2x \geq x)$ **True**

- "Square of any natural number greater than 5 is greater than 25."

  $$(\forall x \in N)(x > 5 \implies x^2 > 25).$$

More for all quantifiers examples.

- "doubling a number always makes it larger"

  $(\forall x \in N)\ (2x > x)$ **False** **Consider** $x = 0$

  Can fix statement...

  $(\forall x \in N)\ (2x \geq x)$ **True**

- "Square of any natural number greater than 5 is greater than 25."

  $$(\forall x \in N)(x > 5 \implies x^2 > 25).$$

  Idea alert:

More for all quantifiers examples.

- "doubling a number always makes it larger"

  $$(\forall x \in N)\ (2x > x)$$ **False** **Consider** $x = 0$

  Can fix statement...

  $$(\forall x \in N)\ (2x \geq x)$$ **True**

- "Square of any natural number greater than 5 is greater than 25."

  $$(\forall x \in N)(x > 5 \implies x^2 > 25).$$

  Idea alert: Restrict domain using implication.

More for all quantifiers examples.

- "doubling a number always makes it larger"

  $(\forall x \in N)\ (2x > x)$ **False** **Consider** $x = 0$

  Can fix statement...

  $(\forall x \in N)\ (2x \geq x)$ **True**

- "Square of any natural number greater than 5 is greater than 25."

  $$(\forall x \in N)(x > 5 \implies x^2 > 25).$$

  Idea alert: Restrict domain using implication.

Later we may omit universe if clear from context.

Quantifiers..not commutative.

- In English: “there is a natural number that is the square of every natural number”.

Quantifiers..not commutative.

- In English: “there is a natural number that is the square of every natural number”.

$$(\exists y \in N)$$

Quantifiers..not commutative.

- In English: "there is a natural number that is the square of every natural number".

$$(\exists y \in N)\ (\forall x \in N)$$

Quantifiers..not commutative.

- In English: "there is a natural number that is the square of every natural number".

$$(\exists y \in N)\ (\forall x \in N)\ (y = x^2)$$

Quantifiers..not commutative.

- In English: "there is a natural number that is the square of every natural number".

$$(\exists y \in N)\ (\forall x \in N)\ (y = x^2) \qquad \text{False}$$

Quantifiers..not commutative.

- In English: "there is a natural number that is the square of every natural number".

$$(\exists y \in N)\ (\forall x \in N)\ (y = x^2) \qquad \text{False}$$

- In English: "the square of every natural number is a natural number."

Quantifiers..not commutative.

- In English: "there is a natural number that is the square of every natural number".

$$(\exists y \in N)\ (\forall x \in N)\ (y = x^2) \quad \text{False}$$

- In English: "the square of every natural number is a natural number."

$$(\forall x \in N)$$

Quantifiers..not commutative.

- In English: “there is a natural number that is the square of every natural number”.

$$(\exists y \in N)\ (\forall x \in N)\ (y = x^2) \quad \text{False}$$

- In English: “the square of every natural number is a natural number.”

$$(\forall x \in N)(\exists y \in N)$$

Quantifiers..not commutative.

- In English: "there is a natural number that is the square of every natural number".

$$(\exists y \in N)\ (\forall x \in N)\ (y = x^2) \quad \text{False}$$

- In English: "the square of every natural number is a natural number."

$$(\forall x \in N)(\exists y \in N)\ (y = x^2)$$

Quantifiers..not commutative.

- In English: “there is a natural number that is the square of every natural number”.

$$(\exists y \in N)\ (\forall x \in N)\ (y = x^2) \quad \text{False}$$

- In English: “the square of every natural number is a natural number.”

$$(\forall x \in N)(\exists y \in N)\ (y = x^2) \quad \text{True}$$

Quantifiers..not commutative.

- In English: "there is a natural number that is the square of every natural number".

$$(\exists y \in N)\ (\forall x \in N)\ (y = x^2) \qquad \text{False}$$

- In English: "the square of every natural number is a natural number."

$$(\forall x \in N)(\exists y \in N)\ (y = x^2) \quad \text{True}$$

Quantifiers....negation...DeMorgan again.

Consider

$$\neg(\forall x \in S)(P(x)),$$

Quantifiers....negation...DeMorgan again.

Consider

$$\neg(\forall x \in S)(P(x)),$$

English: there is an x in S where $P(x)$ does not hold.

Quantifiers....negation...DeMorgan again.

Consider

$$\neg(\forall x \in S)(P(x)),$$

English: there is an x in S where $P(x)$ does not hold.

That is,

Quantifiers....negation...DeMorgan again.

Consider

$$\neg(\forall x \in S)(P(x)),$$

English: there is an x in S where $P(x)$ does not hold.

That is,

$$\neg(\forall x \in S)(P(x)) \iff \exists(x \in S)(\neg P(x)).$$

Quantifiers....negation...DeMorgan again.

Consider

$$\neg(\forall x \in S)(P(x)),$$

English: there is an x in S where $P(x)$ does not hold.

That is,

$$\neg(\forall x \in S)(P(x)) \iff \exists(x \in S)(\neg P(x)).$$

What we do in this course! We consider claims.

Quantifiers....negation...DeMorgan again.

Consider

$$\neg(\forall x \in S)(P(x)),$$

English: there is an x in S where $P(x)$ does not hold.

That is,

$$\neg(\forall x \in S)(P(x)) \iff \exists(x \in S)(\neg P(x)).$$

What we do in this course! We consider claims.

Claim: $(\forall x)\ P(x)$

Quantifiers....negation...DeMorgan again.

Consider

$$\neg(\forall x \in S)(P(x)),$$

English: there is an x in S where $P(x)$ does not hold.

That is,

$$\neg(\forall x \in S)(P(x)) \iff \exists(x \in S)(\neg P(x)).$$

What we do in this course! We consider claims.

Claim: $(\forall x)\ P(x)$ "For all inputs x the program works."

Quantifiers....negation...DeMorgan again.

Consider

$$\neg(\forall x \in S)(P(x)),$$

English: there is an x in S where $P(x)$ does not hold.

That is,

$$\neg(\forall x \in S)(P(x)) \iff \exists(x \in S)(\neg P(x)).$$

What we do in this course! We consider claims.

Claim: $(\forall x)\ P(x)$ "For all inputs x the program works."
For False , find x, where $\neg P(x)$.

Quantifiers....negation...DeMorgan again.

Consider

$$\neg(\forall x \in S)(P(x)),$$

English: there is an x in S where $P(x)$ does not hold.

That is,

$$\neg(\forall x \in S)(P(x)) \iff \exists(x \in S)(\neg P(x)).$$

What we do in this course! We consider claims.

Claim: $(\forall x)\ P(x)$ "For all inputs x the program works."
For False , find x, where $\neg P(x)$.
Counterexample.

Quantifiers....negation...DeMorgan again.

Consider

$$\neg(\forall x \in S)(P(x)),$$

English: there is an x in S where $P(x)$ does not hold.

That is,

$$\neg(\forall x \in S)(P(x)) \iff \exists(x \in S)(\neg P(x)).$$

What we do in this course! We consider claims.

Claim: $(\forall x)\ P(x)$ "For all inputs x the program works."
For False , find x, where $\neg P(x)$.
  Counterexample.
  Bad input.

Quantifiers....negation...DeMorgan again.

Consider

$$\neg(\forall x \in S)(P(x)),$$

English: there is an x in S where $P(x)$ does not hold.

That is,

$$\neg(\forall x \in S)(P(x)) \iff \exists(x \in S)(\neg P(x)).$$

What we do in this course! We consider claims.

Claim: $(\forall x)\ P(x)$ "For all inputs x the program works."
For False , find x, where $\neg P(x)$.
Counterexample.
Bad input.
Case that illustrates bug.

Quantifiers....negation...DeMorgan again.

Consider

$$\neg(\forall x \in S)(P(x)),$$

English: there is an x in S where $P(x)$ does not hold.

That is,

$$\neg(\forall x \in S)(P(x)) \iff \exists(x \in S)(\neg P(x)).$$

What we do in this course! We consider claims.

Claim: $(\forall x)\ P(x)$ "For all inputs x the program works."
For False , find x, where $\neg P(x)$.
Counterexample.
Bad input.
Case that illustrates bug.
For True : prove claim.

Quantifiers....negation...DeMorgan again.

Consider

$$\neg(\forall x \in S)(P(x)),$$

English: there is an x in S where $P(x)$ does not hold.

That is,

$$\neg(\forall x \in S)(P(x)) \iff \exists(x \in S)(\neg P(x)).$$

What we do in this course! We consider claims.

Claim: $(\forall x)\ P(x)$ "For all inputs x the program works."
For False , find x, where $\neg P(x)$.
  Counterexample.
  Bad input.
  Case that illustrates bug.
For True : prove claim. Next lectures...

Negation of exists.

Consider

Negation of exists.

Consider

$$\neg(\exists x \in S)(P(x))$$

Negation of exists.

Consider

$$\neg(\exists x \in S)(P(x))$$

English: means that there is no $x \in S$ where $P(x)$ is true.

Negation of exists.

Consider

$$\neg(\exists x \in S)(P(x))$$

English: means that there is no $x \in S$ where $P(x)$ is true. English: means that for all $x \in S$, $P(x)$ does not hold.

Negation of exists.

Consider

$$\neg(\exists x \in S)(P(x))$$

English: means that there is no $x \in S$ where $P(x)$ is true. English: means that for all $x \in S$, $P(x)$ does not hold.

That is,

$$\neg(\exists x \in S)(P(x)) \iff \forall(x \in S)\neg P(x).$$

Which Theorem?

Theorem: $(\forall n \in N) \neg(\exists a, b, c \in N)\ (n \geq 3 \implies a^n + b^n = c^n)$

Which Theorem?

Theorem: $(\forall n \in N)\, \neg(\exists a, b, c \in N)\, (n \geq 3 \implies a^n + b^n = c^n)$

Which Theorem?

Which Theorem?

Theorem: $(\forall n \in N)\ \neg(\exists a, b, c \in N)\ (n \geq 3 \implies a^n + b^n = c^n)$

Which Theorem?

Fermat's Last Theorem!

Which Theorem?

Theorem: $(\forall n \in N)\, \neg(\exists a, b, c \in N)\, (n \geq 3 \implies a^n + b^n = c^n)$

Which Theorem?

Fermat's Last Theorem!

Remember Special Triangles: for $n = 2$, we have 3,4,5 and 5,7, 12 and ...

Which Theorem?

Theorem: $(\forall n \in N)\, \neg(\exists a, b, c \in N)\, (n \geq 3 \implies a^n + b^n = c^n)$

Which Theorem?

Fermat's Last Theorem!

Remember Special Triangles: for $n = 2$, we have 3,4,5 and 5,7, 12 and ...

1637: Proof doesn't fit in the margins.

Which Theorem?

Theorem: $(\forall n \in N)\, \neg(\exists a, b, c \in N)\, (n \geq 3 \implies a^n + b^n = c^n)$

Which Theorem?

Fermat's Last Theorem!

Remember Special Triangles: for $n = 2$, we have 3,4,5 and 5,7, 12 and ...

1637: Proof doesn't fit in the margins.

1993: Wiles ...(based in part on Ribet's Theorem)

Which Theorem?

Theorem: $(\forall n \in N)\, \neg(\exists a, b, c \in N)\, (n \geq 3 \implies a^n + b^n = c^n)$

Which Theorem?

Fermat's Last Theorem!

Remember Special Triangles: for $n = 2$, we have 3,4,5 and 5,7, 12 and ...

1637: Proof doesn't fit in the margins.

1993: Wiles ...(based in part on Ribet's Theorem)

DeMorgan Restatement:

Which Theorem?

Theorem: $(\forall n \in N)\ \neg(\exists a, b, c \in N)\ (n \geq 3 \implies a^n + b^n = c^n)$

Which Theorem?

Fermat's Last Theorem!

Remember Special Triangles: for $n = 2$, we have 3,4,5 and 5,7, 12 and ...

1637: Proof doesn't fit in the margins.

1993: Wiles ...(based in part on Ribet's Theorem)

DeMorgan Restatement:
Theorem: $\neg(\exists n \in N)\ (\exists a, b, c \in N)\ (n \geq 3 \implies a^n + b^n = c^n)$

Summary.

Propositions are statements that are true or false.

Summary.

Propositions are statements that are true or false.

Proprositional forms use $\wedge, \vee, \neg$.

Summary.

Propositions are statements that are true or false.

Proprositional forms use $\wedge, \vee, \neg$.

Propositional forms correspond to truth tables.

Summary.

Propositions are statements that are true or false.

Proprositional forms use $\wedge, \vee, \neg$.

Propositional forms correspond to truth tables.

Logical equivalence of forms means same truth tables.

Summary.

Propositions are statements that are true or false.

Proprositional forms use $\wedge, \vee, \neg$.

Propositional forms correspond to truth tables.

Logical equivalence of forms means same truth tables.

Implication: $P \implies Q$

Summary.

Propositions are statements that are true or false.

Proprositional forms use $\wedge, \vee, \neg$.

Propositional forms correspond to truth tables.

Logical equivalence of forms means same truth tables.

Implication: $P \implies Q \iff \neg P \vee Q$.

Summary.

Propositions are statements that are true or false.

Proprositional forms use $\wedge, \vee, \neg$.

Propositional forms correspond to truth tables.

Logical equivalence of forms means same truth tables.

Implication: $P \implies Q \iff \neg P \vee Q$.

Contrapositive: $\neg Q \implies \neg P$

Summary.

Propositions are statements that are true or false.

Proprositional forms use $\wedge, \vee, \neg$.

Propositional forms correspond to truth tables.

Logical equivalence of forms means same truth tables.

Implication: $P \implies Q \iff \neg P \vee Q$.

Contrapositive: $\neg Q \implies \neg P$
Converse: $Q \implies P$

Summary.

Propositions are statements that are true or false.

Proprositional forms use $\wedge, \vee, \neg$.

Propositional forms correspond to truth tables.

Logical equivalence of forms means same truth tables.

Implication: $P \implies Q \iff \neg P \vee Q$.

Contrapositive: $\neg Q \implies \neg P$
Converse: $Q \implies P$

Predicates: Statements with “free” variables.

Summary.

Propositions are statements that are true or false.

Proprositional forms use $\wedge, \vee, \neg$.

Propositional forms correspond to truth tables.

Logical equivalence of forms means same truth tables.

Implication: $P \implies Q \iff \neg P \vee Q$.

Contrapositive: $\neg Q \implies \neg P$
Converse: $Q \implies P$

Predicates: Statements with “free” variables.

Quantifiers: $\forall x\ P(x)$, $\exists y\ Q(y)$

Summary.

Propositions are statements that are true or false.

Proprositional forms use $\wedge, \vee, \neg$.

Propositional forms correspond to truth tables.

Logical equivalence of forms means same truth tables.

Implication: $P \implies Q \iff \neg P \vee Q$.

Contrapositive: $\neg Q \implies \neg P$
Converse: $Q \implies P$

Predicates: Statements with “free” variables.

Quantifiers: $\forall x\ P(x)$, $\exists y\ Q(y)$

Now can state theorems!

Summary.

Propositions are statements that are true or false.

Proprositional forms use $\wedge, \vee, \neg$.

Propositional forms correspond to truth tables.

Logical equivalence of forms means same truth tables.

Implication: $P \implies Q \iff \neg P \vee Q$.

Contrapositive: $\neg Q \implies \neg P$
Converse: $Q \implies P$

Predicates: Statements with “free” variables.

Quantifiers: $\forall x\ P(x)$, $\exists y\ Q(y)$

Now can state theorems! And disprove false ones!

Summary.

Propositions are statements that are true or false.

Proprositional forms use $\wedge, \vee, \neg$.

Propositional forms correspond to truth tables.

Logical equivalence of forms means same truth tables.

Implication: $P \implies Q \iff \neg P \vee Q$.

Contrapositive: $\neg Q \implies \neg P$
Converse: $Q \implies P$

Predicates: Statements with “free” variables.

Quantifiers: $\forall x\ P(x)$, $\exists y\ Q(y)$

Now can state theorems! And disprove false ones!

DeMorgans Laws: “Flip and Distribute negation”

Summary.

Propositions are statements that are true or false.

Proprositional forms use $\wedge, \vee, \neg$.

Propositional forms correspond to truth tables.

Logical equivalence of forms means same truth tables.

Implication: $P \implies Q \iff \neg P \vee Q$.

Contrapositive: $\neg Q \implies \neg P$
Converse: $Q \implies P$

Predicates: Statements with “free” variables.

Quantifiers: $\forall x\ P(x)$, $\exists y\ Q(y)$

Now can state theorems! And disprove false ones!

DeMorgans Laws: “Flip and Distribute negation”
$\neg(P \vee Q) \iff$

Summary.

Propositions are statements that are true or false.

Proprositional forms use $\wedge, \vee, \neg$.

Propositional forms correspond to truth tables.

Logical equivalence of forms means same truth tables.

Implication: $P \implies Q \iff \neg P \vee Q$.

Contrapositive: $\neg Q \implies \neg P$
Converse: $Q \implies P$

Predicates: Statements with “free” variables.

Quantifiers: $\forall x\ P(x)$, $\exists y\ Q(y)$

Now can state theorems! And disprove false ones!

DeMorgans Laws: “Flip and Distribute negation”
$\neg(P \vee Q) \iff (\neg P \wedge \neg Q)$
$\neg \forall x\ P(x) \iff$

Summary.

Propositions are statements that are true or false.

Proprositional forms use $\wedge, \vee, \neg$.

Propositional forms correspond to truth tables.

Logical equivalence of forms means same truth tables.

Implication: $P \implies Q \iff \neg P \vee Q$.

Contrapositive: $\neg Q \implies \neg P$
Converse: $Q \implies P$

Predicates: Statements with “free” variables.

Quantifiers: $\forall x\ P(x)$, $\exists y\ Q(y)$

Now can state theorems! And disprove false ones!

DeMorgans Laws: “Flip and Distribute negation”

$$\neg(P \vee Q) \iff (\neg P \wedge \neg Q)$$
$$\neg \forall x\ P(x) \iff \exists x\ \neg P(x).$$

Summary.

Propositions are statements that are true or false.

Proprositional forms use $\wedge, \vee, \neg$.

Propositional forms correspond to truth tables.

Logical equivalence of forms means same truth tables.

Implication: $P \implies Q \iff \neg P \vee Q$.

Contrapositive: $\neg Q \implies \neg P$
Converse: $Q \implies P$

Predicates: Statements with "free" variables.

Quantifiers: $\forall x\ P(x)$, $\exists y\ Q(y)$

Now can state theorems! And disprove false ones!

DeMorgans Laws: "Flip and Distribute negation"
$\neg(P \vee Q) \iff (\neg P \wedge \neg Q)$
$\neg\forall x\ P(x) \iff \quad \exists x\ \neg P(x)$.

Next Time: proofs!