CS70: Lecture 11. Outline.

1. RSA system (continued)
   1.1 Correctness: Fermat's Theorem.
   1.2 Construction.
2. Signature Schemes.
3. Warnings.

Bijections

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Consider $m = 5$, $n = 9$, then if $(a,b) = (3,7)$ then $x = 43 \pmod{45}$.

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correspond to actions in $\pmod{45}$!

Public key crypography.

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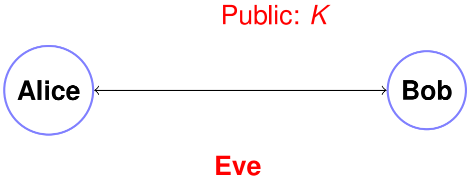

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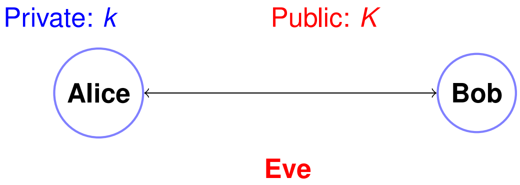

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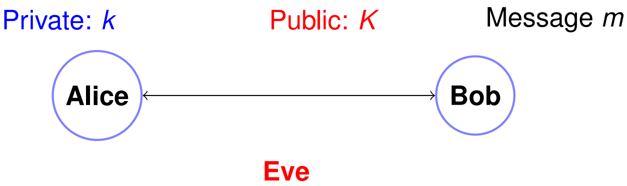

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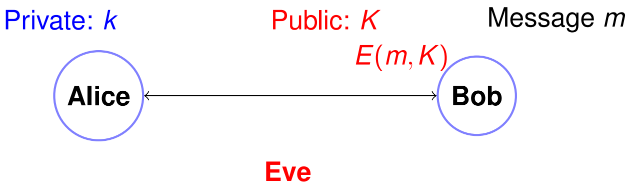

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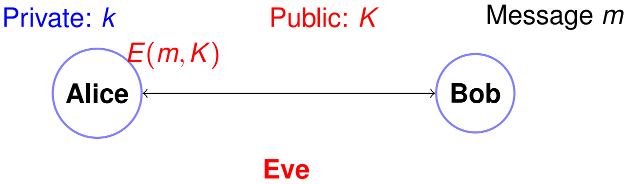

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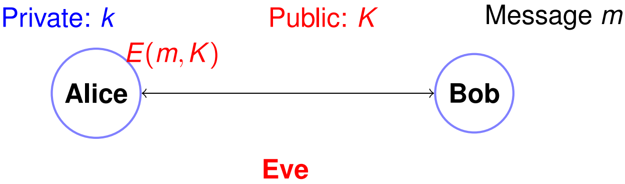
$m = D(E(m,K),k)$



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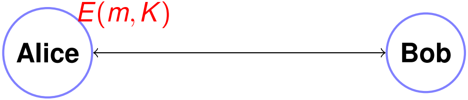
Private: k Public: K Message m

$E(m,K)$



Eve

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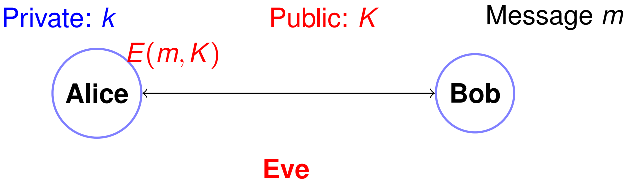

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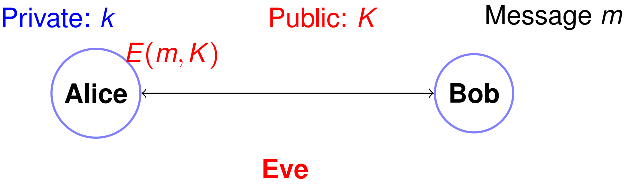

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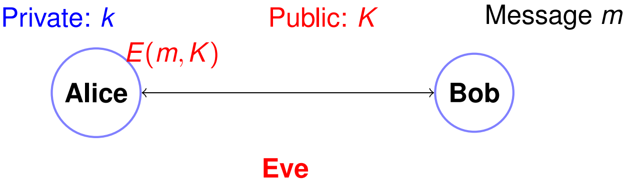

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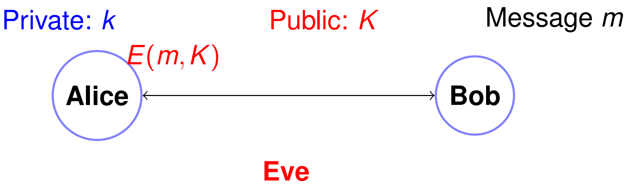

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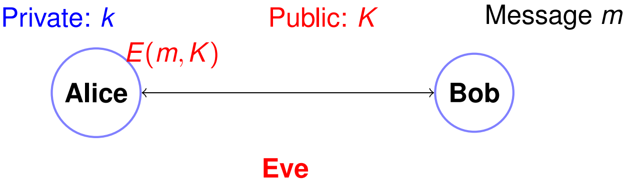

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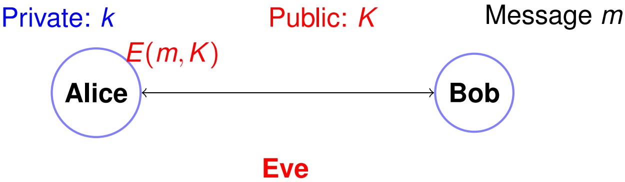

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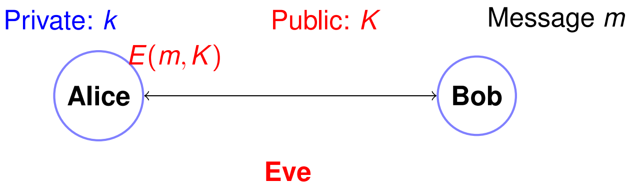

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Similar, not same, but useful.

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All steps are polynomial in $O(\log N)$, the number of bits.

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Breaking in general sense $\implies$ factoring algorithm.

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CS161...

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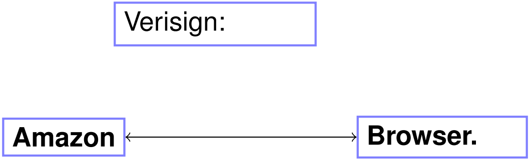

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Amazon $\longleftrightarrow$ **Browser.**

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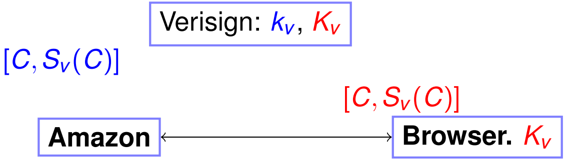

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Security: Eve can't forge unless she "breaks" RSA scheme.

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