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Computation as a lens

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Today: Quantum computing, evolution models, models of the brain, complexity of Nash equilibria, ...

Your future (in this course).

What's to come?

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What's to come? Probability.

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A bag contains:

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A bag contains:

What is the chance that a ball taken from the bag is blue?

Your future (in this course).

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A bag contains:

What is the chance that a ball taken from the bag is blue?

Count blue.

Your future (in this course).

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A bag contains:

What is the chance that a ball taken from the bag is blue?

Count blue. Count total.

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Count blue. Count total. Divide.

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Babak $\equiv$ "Bob" Back.

Outline: basics

1. Counting.
2. Tree
3. Rules of Counting
4. Sample with/without replacement where order does/doesn't matter.

Probability is soon..but first let’s count.

Count?

How many outcomes possible for k coin tosses?
How many poker hands?
How many handshakes for n people?
How many diagonals in a convex polygon?
How many 10 digit numbers?
How many 10 digit numbers without repetition?

Using a tree..

How many 3-bit strings?

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How many different sequences of three bits from $\{0, 1\}$?

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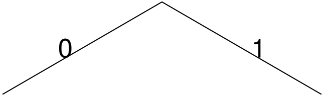

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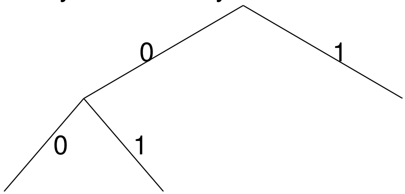

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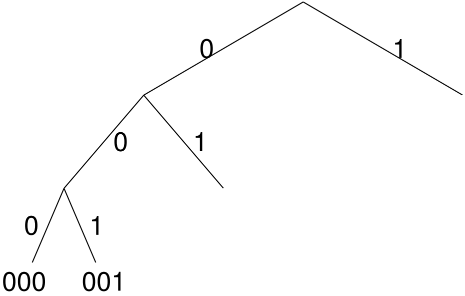

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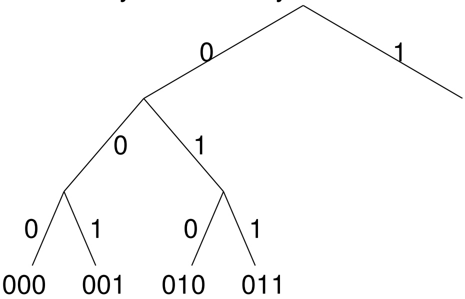

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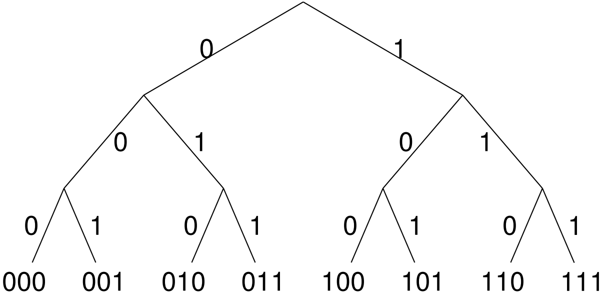

8 leaves which is $2 \times 2 \times 2$.

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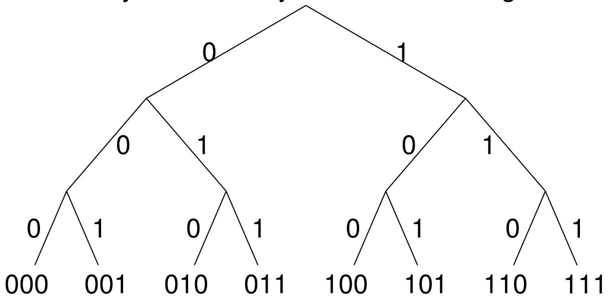

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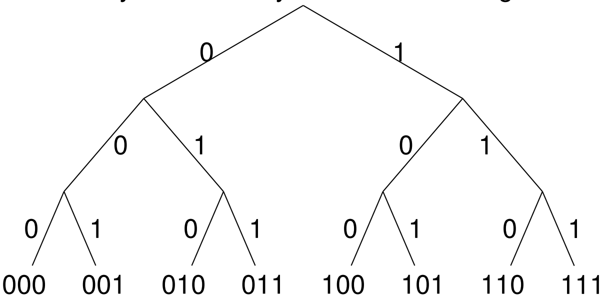

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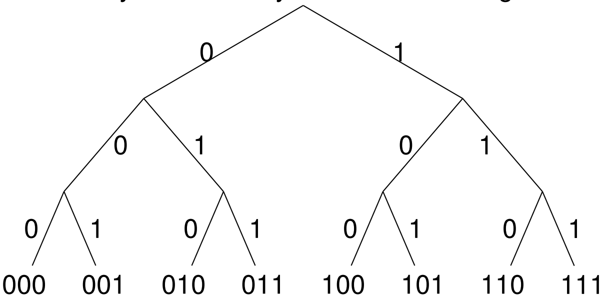

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Objects made by choosing from n_1, then n_2, ..., then n_k
the number of objects is $n_1 \times n_2 \cdots \times n_k$.

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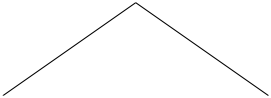

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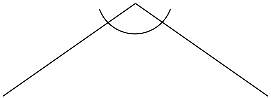

n_1

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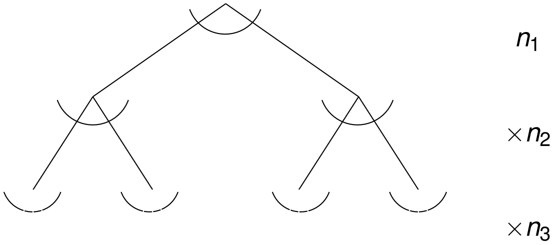

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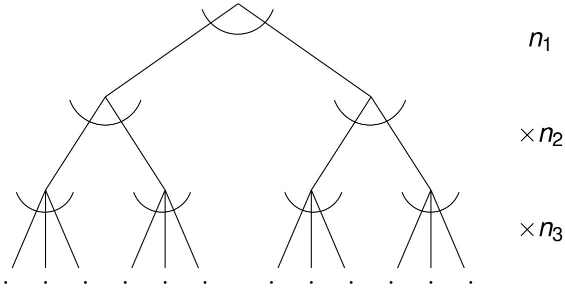

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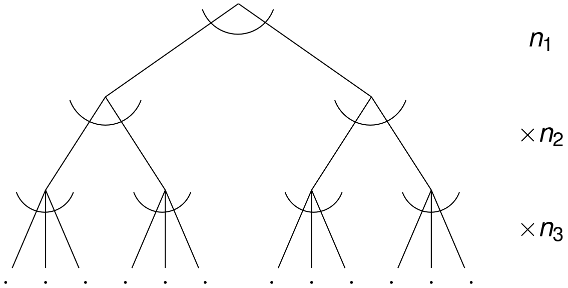

In picture, $2 \times 2 \times 3 = 12$!

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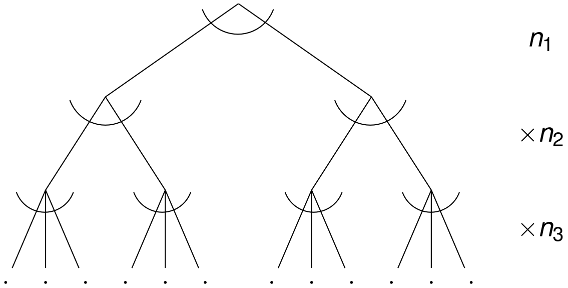

In picture, $2 \times 2 \times 3 = 12$!

Using the first rule..

How many outcomes possible for k coin tosses?

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2 ways for first choice,

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...
2

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...
2×2

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...
$2 \times 2 \cdots$

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$2 \times 2 \cdots \times 2 = 2^k$

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If no. Then $(m-1)m^{n-1}$.

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How many functions f mapping S to T?

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Questions?

Permutations.

[1]By definition: $0! = 1$.

Permutations.

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Permutations.

How many 10 digit numbers **without repeating a digit**?

10 ways for first, 9 ways for second,

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Permutations.

How many 10 digit numbers **without repeating a digit**?

10 ways for first, 9 ways for second, 8 ways for third,

---

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n ways for first choice, $n-1$ ways for second,
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[1] By definition: 0! = 1.

Permutations.

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---

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One-to-One Functions.

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How many one-to-one functions from $|S|$ to $|S|$.

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So total number is $|S| \times |S|-1 \cdots 1 = |S|!$
A one-to-one function is a permutation!

Counting sets..when order doesn't matter.

How many poker hands?

[2] When each unordered object corresponds equal numbers of ordered objects.

Counting sets..when order doesn't matter.

How many poker hands?

$52 \times 51 \times 50 \times 49 \times 48$

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Counting sets..when order doesn't matter.

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How many poker hands?

$52 \times 51 \times 50 \times 49 \times 48$???

Are $A, K, Q, 10, J$ of spades
and $10, J, Q, K, A$ of spades the same?

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Are $A, K, Q, 10, J$ of spades
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Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.[2]

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Number of orderings for a poker hand: "5!"

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(The "!" means factorial, not Exclamation.)

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Can write as...

$$\frac{52 \times 51 \times 50 \times 49 \times 48}{5!}$$

$$\frac{52!}{5! \times 47!}$$

---

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Number of orderings for a poker hand: "5!"

Can write as...
$$\frac{52 \times 51 \times 50 \times 49 \times 48}{5!}$$
$$\frac{52!}{5! \times 47!}$$

Generic: ways to choose 5 out of 52 possibilities.

[2] When each unordered object corresponds equal numbers of ordered objects.

Ordered to unordered.

Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.

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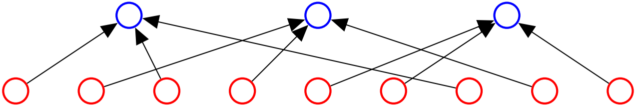

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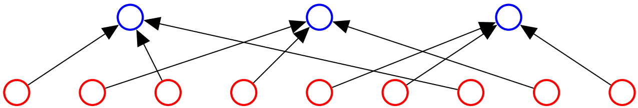

How many red nodes (ordered objects)?

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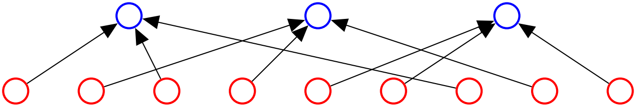

How many red nodes (ordered objects)? 9.

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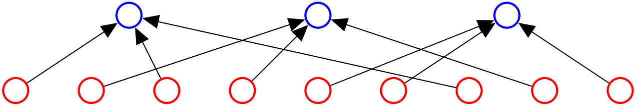

How many red nodes (ordered objects)? 9.

How many red nodes mapped to one blue node?

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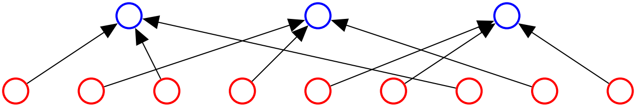

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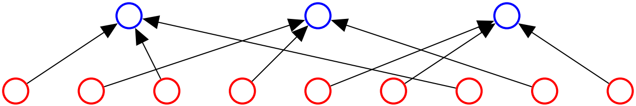

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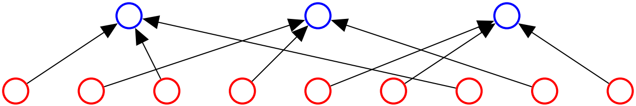

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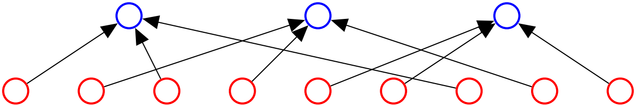

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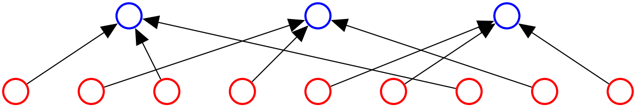

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How many poker deals?

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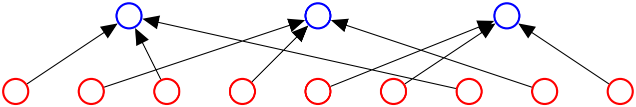

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How many poker deals? $52 \cdot 51 \cdot 50 \cdot 49 \cdot 48$.

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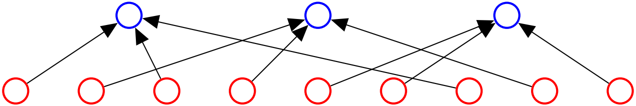

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How many poker deals per hand?

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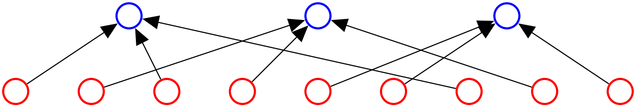

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How many poker deals per hand?
   Map each deal to ordered deal:

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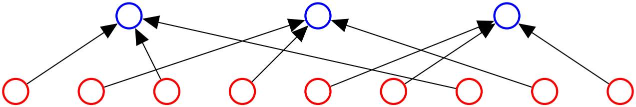

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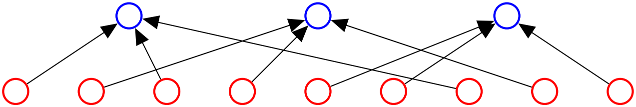
How many poker deals per hand?
Map each deal to ordered deal: $5!$

Ordered to unordered.

Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.



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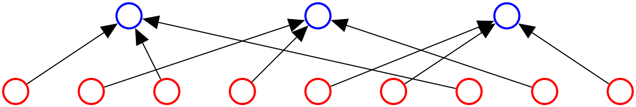

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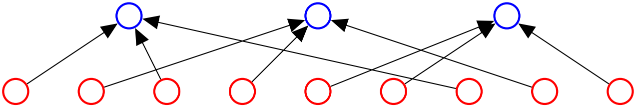

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Questions?

..order doesn't matter.

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Choose 2 out of n?

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$$\frac{n \times (n-1)}{}$$

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Choose 2 out of n?

$$\frac{n \times (n-1)}{2} = \frac{n!}{(n-2)! \times 2}$$

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Choose 3 out of n?

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Choose k **out of** n?

$$\frac{n!}{(n-k)!}$$

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Notation: $\binom{n}{k}$ **and pronounced "**n **choose** k**."**

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Familiar?

..order doesn't matter.

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Choose k **out of** n?

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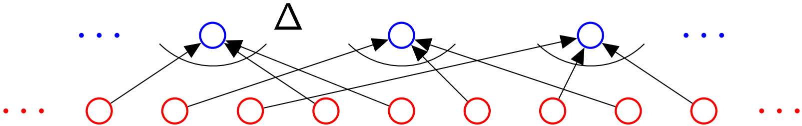
Familiar? Questions?

Example: Visualize the proof..

First rule: $n_1 \times n_2 \cdots \times n_3$**. Product Rule.**
Second rule: when order doesn't matter divide...



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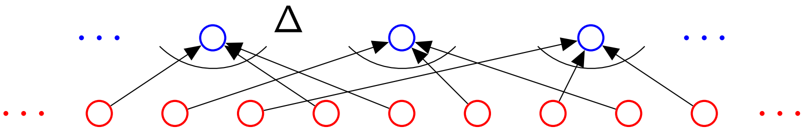

3 card Poker deals: 52

Example: Visualize the proof..

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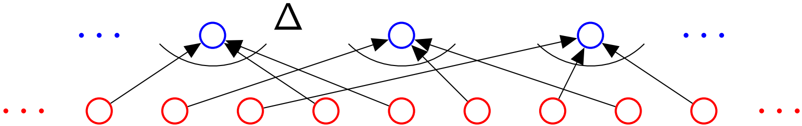

3 card Poker deals: 52×51

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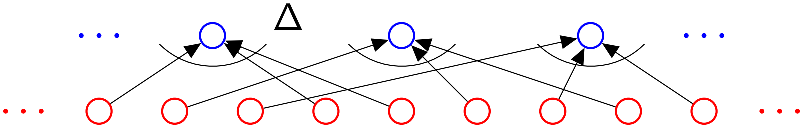

3 card Poker deals: $52 \times 51 \times 50$

Example: Visualize the proof..

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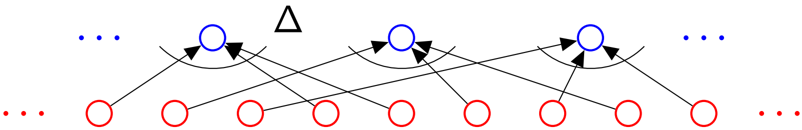

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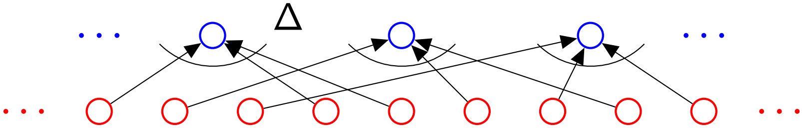

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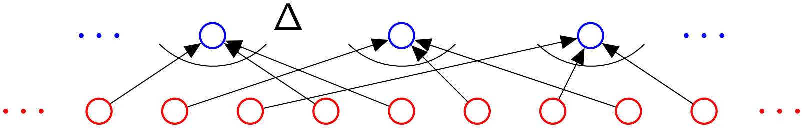

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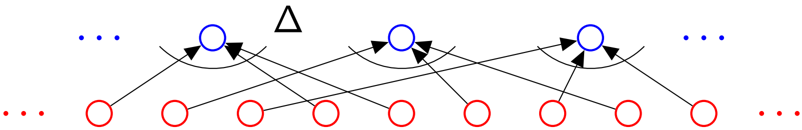

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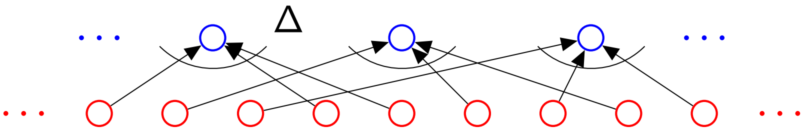

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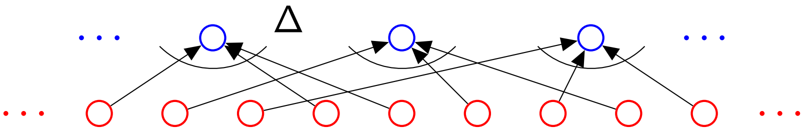

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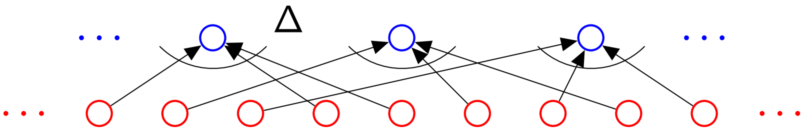

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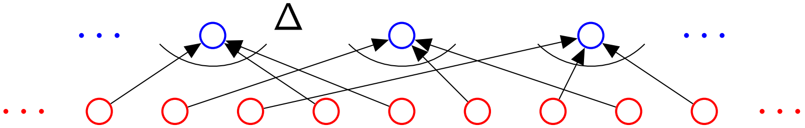

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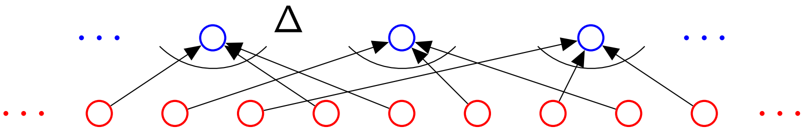

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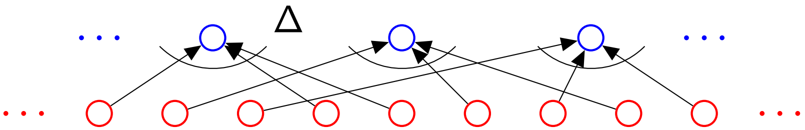

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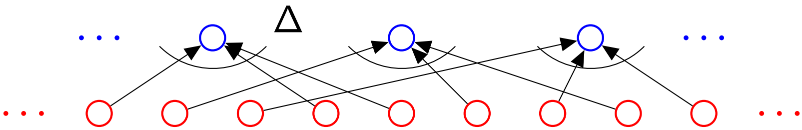

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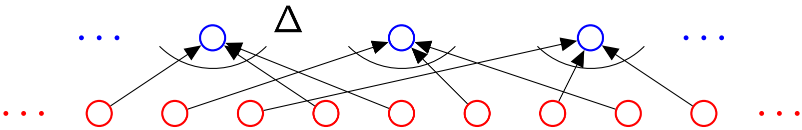

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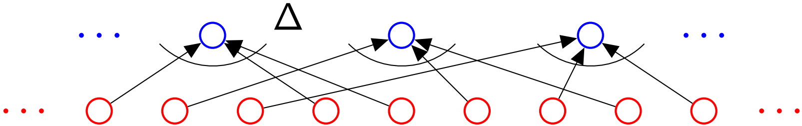

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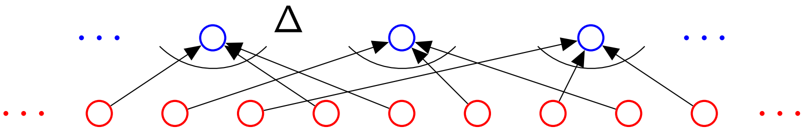

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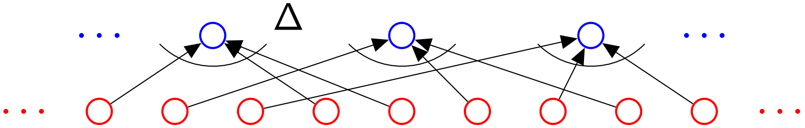

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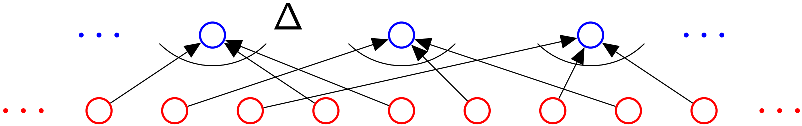

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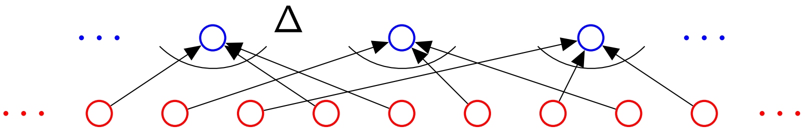

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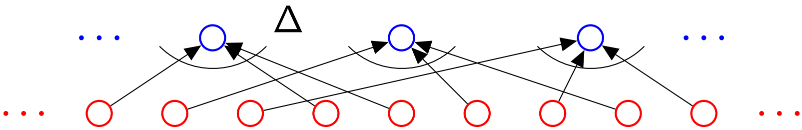

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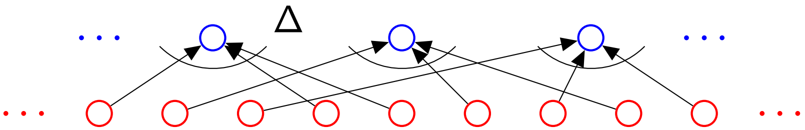

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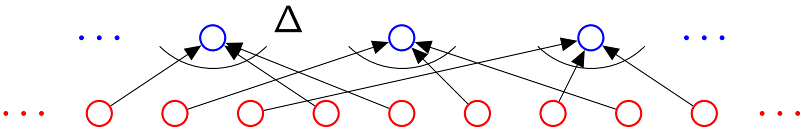

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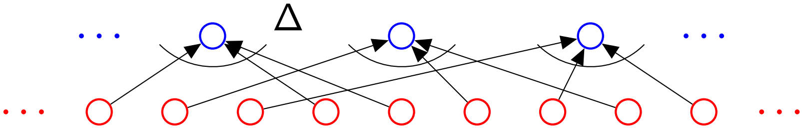

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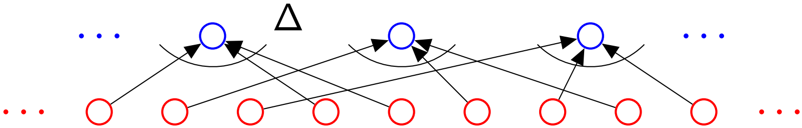

Orderings of ANAGRAM?

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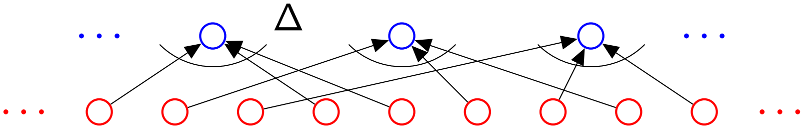

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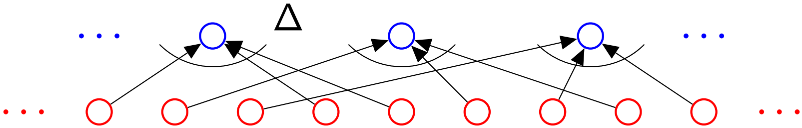

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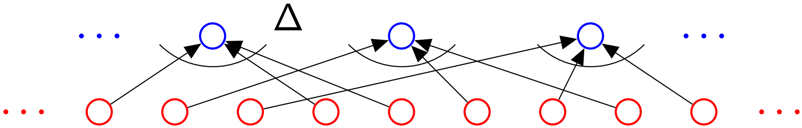

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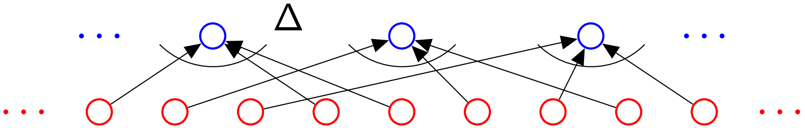

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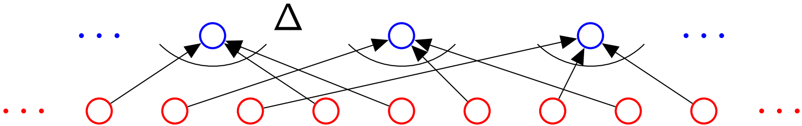

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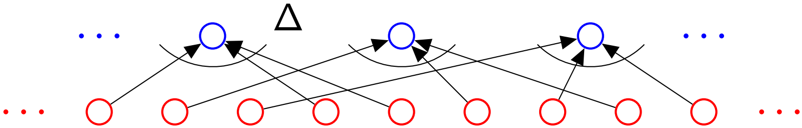

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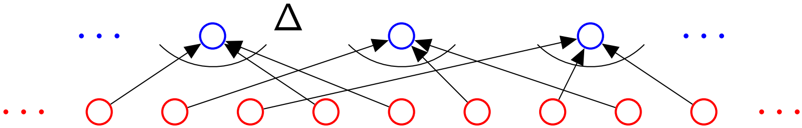

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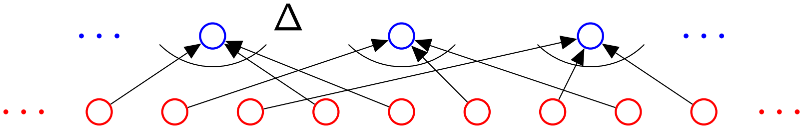

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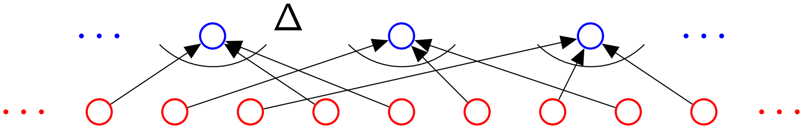

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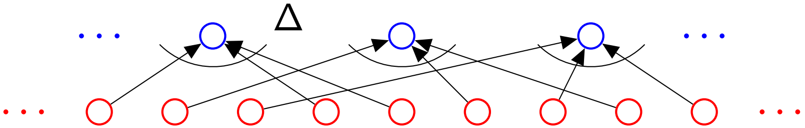

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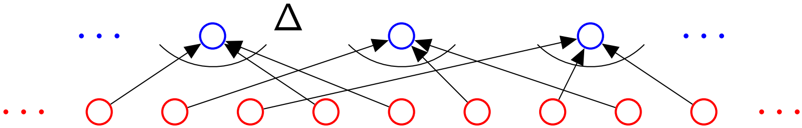

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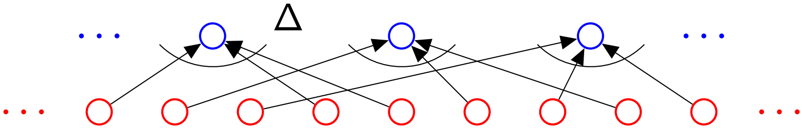

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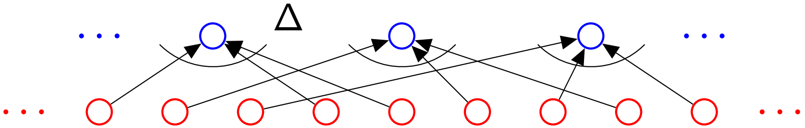

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What is Δ?

ANAGRAM

$A_1NA_2GRA_3M$, $A_2NA_1GRA_3M$, ...

$\Delta = 3 \times 2 \times 1 = 3!$ First rule!

$\Longrightarrow \frac{7!}{3!}$ Second rule!

Some Practice.

How many orderings of letters of CAT?

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3 ways to choose first letter, 2 ways for second, 1 for last.

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$\implies 3 \times 2 \times 1$

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Ordered, except for A!

Some Practice.

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total orderings of 7 letters.

Some Practice.

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How many orderings of the letters in ANAGRAM?

Ordered, except for A!

total orderings of 7 letters. 7!
total "extra counts" or orderings of three A's?

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11 letters total.

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Sampling...

Sample k items out of n

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Without replacement:

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Without replacement:

Order matters:

Sampling...

Sample k items out of n

Without replacement:

Order matters: $n \times$

Sampling...

Sample k items out of n

Without replacement:

Order matters: $n \times n-1 \times n-2 \ldots$

Sampling...

Sample k items out of n

Without replacement:

Order matters: $n \times n-1 \times n-2 \ldots \times n-k+1$

Sampling...

Sample k items out of n

Without replacement:

Order matters: $n \times n-1 \times n-2 \ldots \times n-k+1 = \frac{n!}{(n-k)!}$

Order does not matter:

Sampling...

Sample k items out of n

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Order matters: $n \times n-1 \times n-2 \ldots \times n-k+1 = \frac{n!}{(n-k)!}$

Order does not matter:

Second Rule: divide by number of orders

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"n choose k"

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With Replacement.

Order matters: $n \times n$

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Order matters: $n \times n \times \ldots n$

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Order matters: $n \times n \times \ldots n = n^k$

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Problem: depends on how many of each item we chose!

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Different number of unordered elts map to each unordered elt.

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Unordered elt: $1, 2, 3$

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Unordered elt: $1,2,3$ 3! ordered elts map to it.

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Different number of unordered elts map to each unordered elt.

Unordered elt: $1,2,3$ — $3!$ ordered elts map to it.

Unordered elt: $1,2,2$ — $\frac{3!}{2!}$ ordered elts map to it.

How do we deal with this mess??

Splitting up some money....

How many ways can Bob and Alice split 5 dollars?

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For each of 5 dollars pick Bob or Alice(2^5), divide out order

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5 dollars for Bob and 0 for Alice:
one ordered set: (B, B, B, B, B).

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4 for Bob and 1 for Alice:

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“Sorted” way to specify, first Alice’s dollars, then Bob’s.

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How many ways can Bob and Alice split 5 dollars?
For each of 5 dollars pick Bob or Alice(2^5), divide out order ???

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one ordered set: (B, B, B, B, B).

4 for Bob and 1 for Alice:
5 ordered sets: (A, B, B, B, B) ; (B, A, B, B, B); ...

"Sorted" way to specify, first Alice's dollars, then Bob's.

- (B, B, B, B, B):
- (A, B, B, B, B):
- (A, A, B, B, B):

and so on.

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For each of 5 dollars pick Bob or Alice(2^5), divide out order ???

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(B, B, B, B, B):
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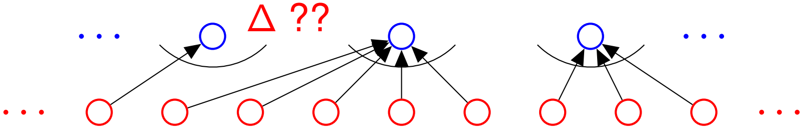

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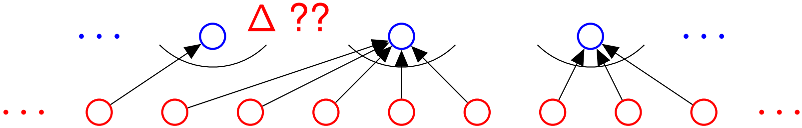

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How many ways can Bob and Alice split 5 dollars?
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5 dollars for Bob and 0 for Alice:
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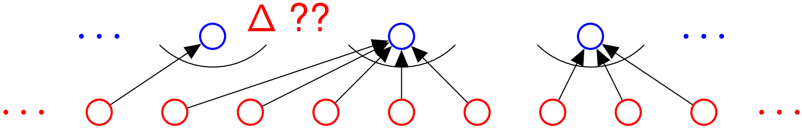
4 for Bob and 1 for Alice:
5 ordered sets: (A, B, B, B, B) ; (B, A, B, B, B); ...

"Sorted" way to specify, first Alice's dollars, then Bob's.
(B, B, B, B, B): 1: (B,B,B,B,B)
(A, B, B, B, B): 5: (A,B,B,B,B),(B,A,B,B,B),(B,B,A,B,B),...
(A, A, B, B, B):
and so on.



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For each of 5 dollars pick Bob or Alice(2^5), divide out order ???

5 dollars for Bob and 0 for Alice:
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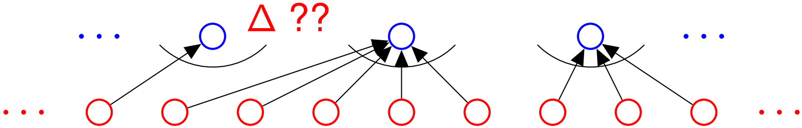
5 ordered sets: (A,B,B,B,B) ; (B,A,B,B,B); ...

"Sorted" way to specify, first Alice's dollars, then Bob's.

(B,B,B,B,B): 1: (B,B,B,B,B)
(A,B,B,B,B): 5: (A,B,B,B,B),(B,A,B,B,B),(B,B,A,B,B),...
(A,A,B,B,B): $\binom{5}{2}$;(A,A,B,B,B),(A,B,A,B,B),(A,B,B,A,B),...

and so on.



Second rule of counting is no good here!

Splitting 5 dollars..

How many ways can Alice, Bob, and Eve split 5 dollars.

Splitting 5 dollars..

How many ways can Alice, Bob, and Eve split 5 dollars.

Alice gets 3, Bob gets 1, Eve gets 1: (A, A, A, B, E).

Splitting 5 dollars..

How many ways can Alice, Bob, and Eve split 5 dollars.

Alice gets 3, Bob gets 1, Eve gets 1: (A,A,A,B,E).

Separate Alice's dollars from Bob's and then Bob's from Eve's.

Splitting 5 dollars..

How many ways can Alice, Bob, and Eve split 5 dollars.

Alice gets 3, Bob gets 1, Eve gets 1: (A, A, A, B, E).

Separate Alice's dollars from Bob's and then Bob's from Eve's.

Five dollars are five stars: $\star\star\star\star\star$.

Splitting 5 dollars..

How many ways can Alice, Bob, and Eve split 5 dollars.

Alice gets 3, Bob gets 1, Eve gets 1: (A, A, A, B, E).

Separate Alice's dollars from Bob's and then Bob's from Eve's.

Five dollars are five stars: $\star\star\star\star\star$.

Alice: 2, Bob: 1, Eve: 2.

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$\binom{7}{2}$ ways to split 5 dollars among 3 people.

Stars and Bars.

Ways to add up n numbers to sum to k?

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Or: k unordered choices from set of n possibilities with replacement.
Sample with replacement where order doesn't matter.

Summary.

First rule: $n_1 \times n_2 \cdots \times n_3$.

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Sample k times from n objects with replacement and order doesn't matter: $\binom{k+n-1}{n-1}$.

Quick review of the basics.

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Balls in bins.

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Example: Poker hands.

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5 indistinguishable balls into 52 bins only one ball in each bin
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Example: Poker hands.

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5 samples from 3 possibilities with replacement and no order

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5 samples from 52 possibilities without replacement
Example: Poker hands.

5 indistinguishable balls into 3 bins
5 samples from 3 possibilities with replacement and no order
Dividing 5 dollars among Alice, Bob and Eve.

Sum Rule

Two indistinguishable jokers in 54 card deck.
How many 5 card poker hands?

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Combinatorial Proofs.

Theorem: $\binom{n}{k} = \binom{n}{n-k}$

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Choose a subset of size $n-k$

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Proof: How many subsets of size k? $\binom{n}{k}$

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Choose a subset of size $n-k$
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Choosing a subset of size k is same

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$\square$

Pascal's Triangle

Pascal's Triangle

0

1 1

Pascal's Triangle

		0		
	1		1	
1		2		1

Pascal's Triangle

			0			
		1		1		
	1		2		1	
1		3		3		1

Pascal's Triangle

0

1 1

1 2 1

1 3 3 1

1 4 6 4 1

Pascal's Triangle

				0				
			1		1			
		1		2		1		
	1		3		3		1	
1		4		6		4		1

Pascal's Triangle

0
1 1
1 2 1
1 3 3 1
1 4 6 4 1

Row n: coefficients of $(1+x)^n = (1+x)(1+x)\cdots(1+x)$.

Pascal's Triangle

$$\begin{array}{ccccccccc} & & & & 0 & & & & \\ & & & 1 & & 1 & & & \\ & & 1 & & 2 & & 1 & & \\ & 1 & & 3 & & 3 & & 1 & \\ 1 & & 4 & & 6 & & 4 & & 1 \end{array}$$

Row n: coefficients of $(1+x)^n = (1+x)(1+x)\cdots(1+x)$.

Foil (4 terms)

Pascal's Triangle

$$\begin{array}{ccccccccc} & & & & 0 & & & & \\ & & & 1 & & 1 & & & \\ & & 1 & & 2 & & 1 & & \\ & 1 & & 3 & & 3 & & 1 & \\ 1 & & 4 & & 6 & & 4 & & 1 \end{array}$$

Row n: coefficients of $(1+x)^n = (1+x)(1+x)\cdots(1+x)$.

Foil (4 terms) on steroids:

Pascal's Triangle

$$\begin{array}{ccccccccc} &&&& 0 &&&& \\ &&& 1 && 1 &&& \\ && 1 && 2 && 1 && \\ & 1 && 3 && 3 && 1 & \\ 1 && 4 && 6 && 4 && 1 \end{array}$$

Row n: coefficients of $(1+x)^n = (1+x)(1+x)\cdots(1+x)$.

Foil (4 terms) on steroids:

2^n terms:

Pascal's Triangle

$$
\begin{array}{ccccccccc}
 & & & & 0 & & & & \\
 & & & 1 & & 1 & & & \\
 & & 1 & & 2 & & 1 & & \\
 & 1 & & 3 & & 3 & & 1 & \\
1 & & 4 & & 6 & & 4 & & 1
\end{array}
$$

Row n: coefficients of $(1+x)^n = (1+x)(1+x)\cdots(1+x)$.

Foil (4 terms) on steroids:
2^n terms: choose 1 or x from each factor of $(1+x)$.

Pascal's Triangle

				0				
			1		1			
		1		2		1		
	1		3		3		1	
1		4		6		4		1

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Row n: coefficients of $(1+x)^n = (1+x)(1+x)\cdots(1+x)$.

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Simplify: collect all terms corresponding to x^k.

Pascal's Triangle

$$\begin{array}{ccccccccc} &&&& 0 &&&& \\ &&& 1 && 1 &&& \\ && 1 && 2 && 1 && \\ & 1 && 3 && 3 && 1 & \\ 1 && 4 && 6 && 4 && 1 \end{array}$$

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Foil (4 terms) on steroids:
2^n terms: choose 1 or x from each factor of $(1+x)$.

Simplify: collect all terms corresponding to x^k.
Coefficient of x^k is $\binom{n}{k}$: choose k factors where x is in product.

Pascal's Triangle

$$\begin{array}{ccccccccc} & & & & 0 & & & & \\ & & & 1 & & 1 & & & \\ & & 1 & & 2 & & 1 & & \\ & 1 & & 3 & & 3 & & 1 & \\ 1 & & 4 & & 6 & & 4 & & 1 \end{array}$$

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$$\begin{array}{ccc} & \binom{0}{0} & \\ \binom{1}{0} & & \binom{1}{1} \end{array}$$

Pascal's Triangle

0
1 1
1 2 1
1 3 3 1
1 4 6 4 1

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$$\binom{0}{0}$$
$$\binom{1}{0} \quad \binom{1}{1}$$
$$\binom{2}{0} \quad \binom{2}{1} \quad \binom{2}{2}$$

Pascal's Triangle

$$
\begin{array}{ccccccccc}
 & & & & 0 & & & & \\
 & & & 1 & & 1 & & & \\
 & & 1 & & 2 & & 1 & & \\
 & 1 & & 3 & & 3 & & 1 & \\
1 & & 4 & & 6 & & 4 & & 1
\end{array}
$$

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$$
\begin{array}{ccccccc}
 & & & \binom{0}{0} & & & \\
 & & \binom{1}{0} & & \binom{1}{1} & & \\
 & \binom{2}{0} & & \binom{2}{1} & & \binom{2}{2} & \\
\binom{3}{0} & & \binom{3}{1} & & \binom{3}{2} & & \binom{3}{3}
\end{array}
$$

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$$
\begin{array}{ccccccccc}
 & & & & 0 & & & & \\
 & & & 1 & & 1 & & & \\
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\end{array}
$$

Pascal's rule $\implies \binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$.

Combinatorial Proofs.

Theorem: $\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$.

Proof: How many size k subsets of $n+1$?

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How many size k subsets of $n+1$?
How many contain the first element?

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How many size k subsets of $n+1$?
How many contain the first element?
Chose first element, need to choose $k-1$ more from remaining n elements.

Combinatorial Proofs.

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How many size k subsets of $n+1$?
How many contain the first element?
Chose first element, need to choose $k-1$ more from remaining n elements.
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How many don't contain the first element ?

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How many don't contain the first element ?
Need to choose k elements from remaining n elts.

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So, $\binom{n}{k-1} + \binom{n}{k}$

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Need to choose k elements from remaining n elts.
$\implies \binom{n}{k}$

So, $\binom{n}{k-1} + \binom{n}{k} = \binom{n+1}{k}$. $\square$

Combinatorial Proof.

Theorem: $\binom{n}{k} = \binom{n-1}{k-1} + \cdots + \binom{k-1}{k-1}$.

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$$\{1, \ldots, i, \underline{\ldots, n}\}$$

Must choose $k-1$ elements from $n-i$ remaining elements.

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Add them up to get the total number of subsets of size k

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Add them up to get the total number of subsets of size k
which is also $\binom{n+1}{k}$.

$\square$

Binomial Theorem: $x = 1$

Theorem: $2^n = \binom{n}{n} + \binom{n}{n-1} + \cdots + \binom{n}{0}$

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Construct a subset with sequence of n choices:

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Construct a subset with sequence of n choices:
element i **is in** or **is not** in the subset: 2 poss.

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First rule of counting: $2 \times 2 \cdots \times 2 = 2^n$ subsets.

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Sum over i to get total number of subsets..

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Sum over i to get total number of subsets..which is also 2^n. $\square$

Simple Inclusion/Exclusion

Sum Rule: For disjoint sets S **and** T**,** $|S \cup T| = |S| + |T|$
Used to reason about all subsets
by adding number of subsets of size 1, 2, 3,...

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Inclusion/Exclusion Rule:
For any S and T, $|S \cup T| = |S| + |T| - |S \cap T|$.

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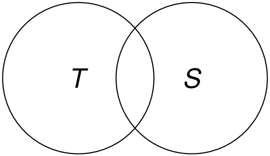

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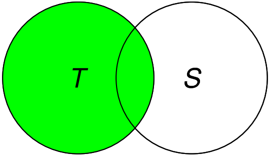

In T. $\implies |T|$

Simple Inclusion/Exclusion

Sum Rule: For disjoint sets S and T, $|S \cup T| = |S| + |T|$
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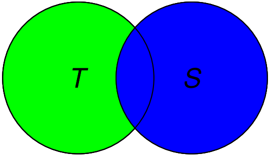

In T. $\implies$ $|T|$
In S. $\implies +$ $|S|$

Simple Inclusion/Exclusion

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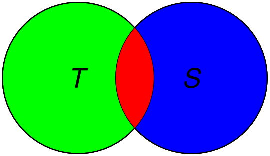

$S \cap T$

In T. $\implies$ $|T|$
In S. $\implies$ $+$ $|S|$
Elements in $S \cap T$ are counted twice.

Simple Inclusion/Exclusion

Sum Rule: For disjoint sets S and T, $|S \cup T| = |S| + |T|$
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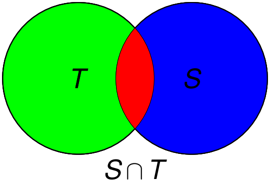

In T. $\implies |T|$
In S. $\implies + \ |S|$
Elements in $S \cap T$ are counted twice.
Subtract. $\implies -|S \cap T|$

Simple Inclusion/Exclusion

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or didn't contain an element. (E.g., first element, first i elements.)

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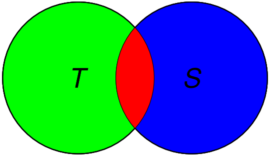
For any S and T, $|S \cup T| = |S| + |T| - |S \cap T|$.



$S \cap T$

In T. $\implies |T|$
In S. $\implies + \ |S|$
Elements in $S \cap T$ are counted twice.
Subtract. $\implies -|S \cap T|$

$$|S \cup T| = |S| + |T| - |S \cap T|$$

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Answer: $|S| + |T| - |S \cap T| = 10^9 + 10^9 - 10^8$.

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Disjoint – so add!