Today.

Finish undecidability.

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Start counting.

Halting Problem.

Halting Problem.

$HALT(P, I)$

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P - program

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Notice:
Need a computer

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...with the notion of a stored program!!!!

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Program is a text string.

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Implement HALT?

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Run P on I and check!

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How long do you wait?

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Something about infinity here, maybe?

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Theorem: There is no program HALT.

Halt and Turing.

Proof:

Halt and Turing.

Proof: Assume there is a program $HALT(\cdot,\cdot)$.

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Turing(P)

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Contradiction. Program HALT does not exist!

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Questions?

Another view of proof: diagonalization.

Any program is a fixed length string.

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P_1	H	H	L	$\cdots$
P_2	L	L	H	$\cdots$
P_3	L	H	H	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$

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Halt does not exist!

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Wow.

Proof play by play.

Assumed HALT(P, I) existed.

Proof play by play.

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What is P?

Proof play by play.

Assumed HALT(P, I) existed.

What is P? Text.

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You have *Text* that is the program HALT(P, I).

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Turing “diagonalizes” on list of program.

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It is not a program!!!!

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$\Longrightarrow$ HALT is not a program.

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$\implies$ HALT is not a program.

Questions?

We are so smart!

Wow, that was easy!

We are so smart!

Wow, that was easy!

We should be famous!

No computers for Turing!

In Turing's time.

No computers for Turing!

In Turing's time.

No computers.

No computers for Turing!

In Turing’s time.

No computers.

Adding machines.

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Adding machines.
e.g., Babbage (from table of logarithms) 1812.

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Concept of program as data wasn't really there.

Turing machine.

Turing machine.

A Turing machine.
– an (infinite) tape with characters

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– an (infinite) tape with characters
– be in a state, and read a character

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Just a mathematician?

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Computer, assembly code, programming language, browser, html, javascript..

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We can't get enough of building more Turing machines.

Undecidable problems.

Does a program, P, print "Hello World"?

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Undecidability for Diophantine set of equations
$\Longrightarrow$ no program can take any set of integer equations and always correctly output whether it has an integer solution.

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No Turing Program

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This statement is a lie. Neither true nor false!

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Program is text, so we can pass it to itself, or refer to self.

Summary: decidability.

Computer Programs are an interesting thing.

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Computation is a lens for other action in the world.

Probability

What's to come?

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What's to come? Probability.

Probability

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A bag contains:

Probability

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A bag contains:

What is the chance that a ball taken from the bag is blue?

Probability

What's to come? Probability.

A bag contains:

What is the chance that a ball taken from the bag is blue?

Count blue.

Probability

What's to come? Probability.

A bag contains:

What is the chance that a ball taken from the bag is blue?

Count blue. Count total.

Probability

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Count blue. Count total. Divide.

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For now:

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For now: Counting!

Outline: basics

1. Counting.
2. Tree
3. Rules of Counting
4. Sample with/without replacement where order does/doesn’t matter.

Count?

How many outcomes possible for k coin tosses?
How many poker hands?
How many handshakes for n people?
How many diagonals in a convex polygon?
How many 10 digit numbers?
How many 10 digit numbers without repetition?

Using a tree..

How many 3-bit strings?

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How many different sequences of three bits from $\{0, 1\}$?

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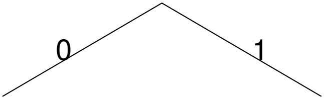

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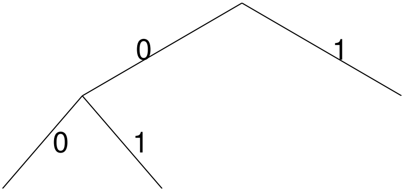

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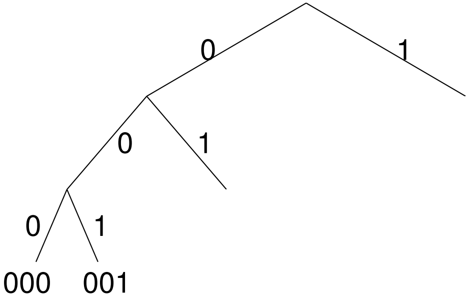

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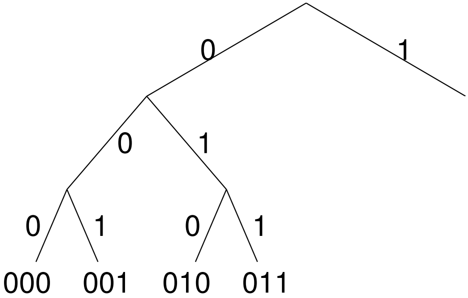

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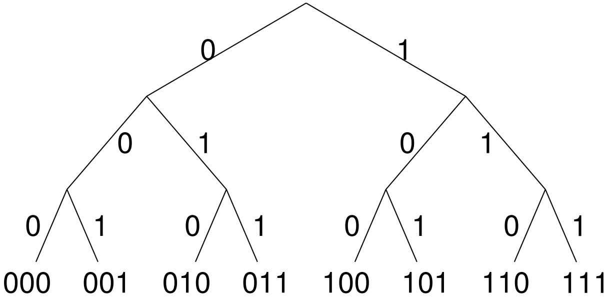

8 leaves which is $2 \times 2 \times 2$.

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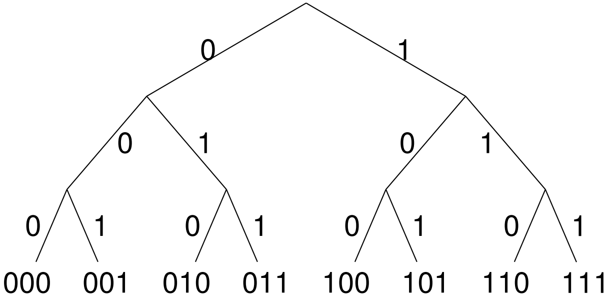

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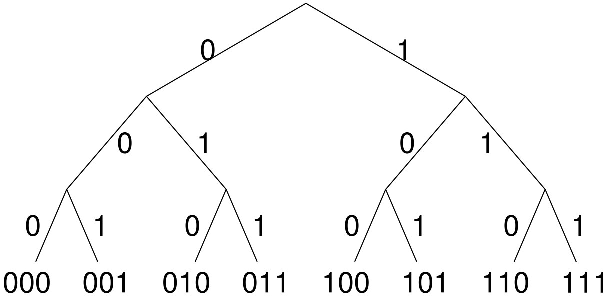

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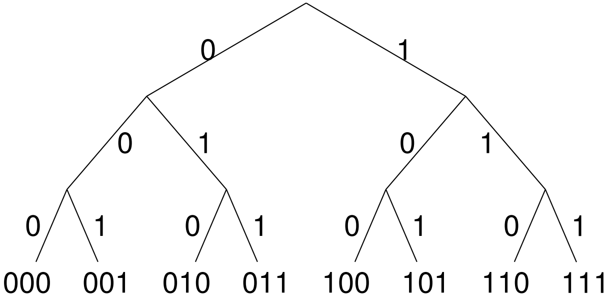

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First Rule of Counting: Product Rule

Objects made by choosing from n_1, then n_2, ..., then n_k
the number of objects is $n_1 \times n_2 \cdots \times n_k$.

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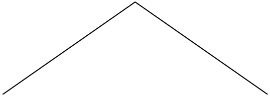

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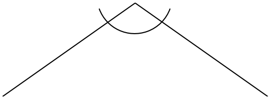

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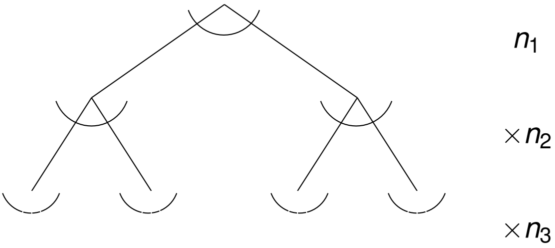

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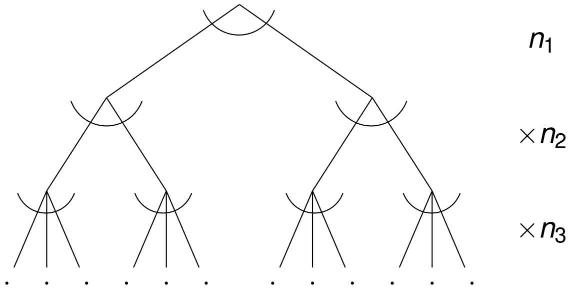

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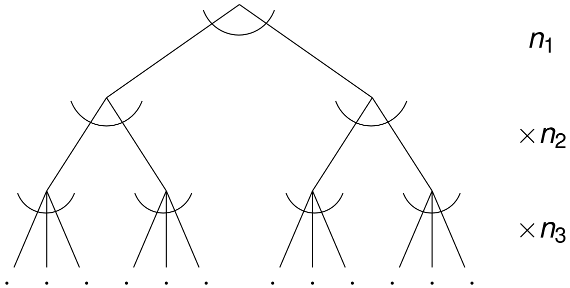

In picture, $2 \times 2 \times 3 = 12$!

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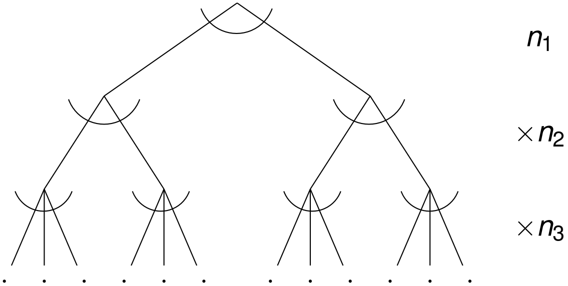

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2 ways for first choice,

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How many outcomes possible for k coin tosses?

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Using the first rule..

How many outcomes possible for k coin tosses?

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$2 \times 2 \cdots \times 2 = 2^k$

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Questions?

Permutations.

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How many 10 digit numbers **without repeating a digit**?

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n ways for first choice, $n-1$ ways for second,
$n-2$ choices for third, ...

... $n*(n-1)*(n-2) \cdot *(n-k+1) = \frac{n!}{(n-k)!}$.

How many orderings of n objects are there?
Permutations of n objects.

n ways for first, $n-1$ ways for second,
$n-2$ ways for third,

[1] By definition: $0! = 1$.

Permutations.

How many 10 digit numbers **without repeating a digit**?
(leading zeros are ok.)

10 ways for first, 9 ways for second, 8 ways for third, ...

... $10*9*8\cdots*1 = 10!$.[1]

How many different samples of size k from n numbers **without replacement.**

n ways for first choice, $n-1$ ways for second,
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Permutations.

How many 10 digit numbers **without repeating a digit**?
(leading zeros are ok.)

10 ways for first, 9 ways for second, 8 ways for third, ...

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n ways for first choice, $n-1$ ways for second,
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One-to-One Functions.

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How many one-to-one functions from $|S|$ to $|S|$.

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So total number is $|S| \times |S|-1 \cdots 1 = |S|!$
A one-to-one function is a permutation!

Counting sets..when order doesn't matter.

How many poker hands?

[2] When each unordered object corresponds equal numbers of ordered objects.

Counting sets..when order doesn't matter.

How many poker hands?

$52 \times 51 \times 50 \times 49 \times 48$

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Are $A, K, Q, 10, J$ of spades

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$52 \times 51 \times 50 \times 49 \times 48$???

Are $A, K, Q, 10, J$ of spades
and $10, J, Q, K, A$ of spades the same?

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Counting sets..when order doesn't matter.

How many poker hands?

$52 \times 51 \times 50 \times 49 \times 48$???

Are $A, K, Q, 10, J$ of spades
and $10, J, Q, K, A$ of spades the same?

Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.[2]

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Counting sets..when order doesn't matter.

How many poker hands?

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Number of orderings for a poker hand: "5!"

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Number of orderings for a poker hand: "5!"
(The "!" means factorial, not Exclamation.)

---

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Can write as...

$$\frac{52!}{5! \times 47!}$$

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Counting sets..when order doesn't matter.

How many poker hands?

$52 \times 51 \times 50 \times 49 \times 48$???

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Number of orderings for a poker hand: "5!"

Can write as...

$$\frac{52 \times 51 \times 50 \times 49 \times 48}{5!}$$

$$\frac{52!}{5! \times 47!}$$

Generic: ways to choose 5 out of 52 possibilities.

[2] When each unordered object corresponds equal numbers of ordered objects.

Ordered to unordered.

Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.

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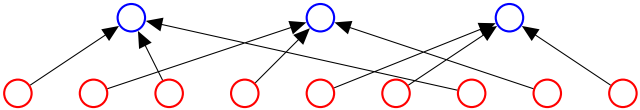

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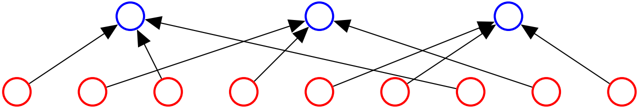
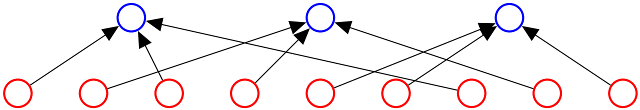


How many red nodes (ordered objects)?

Ordered to unordered.

Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.



How many red nodes (ordered objects)? 9.

Ordered to unordered.

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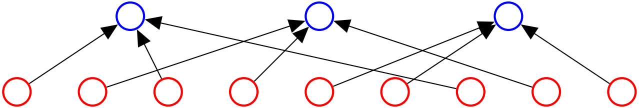

How many red nodes (ordered objects)? 9.

How many red nodes mapped to one blue node?

Ordered to unordered.

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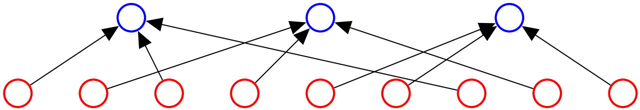

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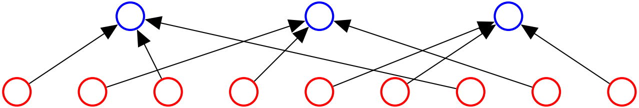

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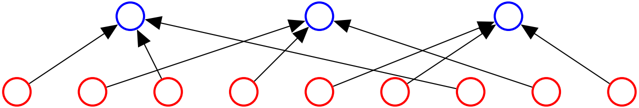

How many red nodes (ordered objects)? 9.

How many red nodes mapped to one blue node? 3.

How many blue nodes (unordered objects)? $\frac{9}{3}$

Ordered to unordered.

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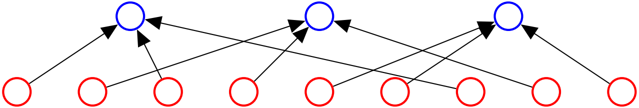

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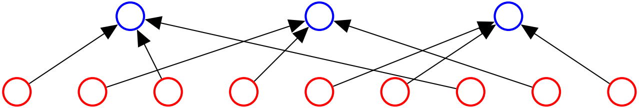

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How many poker deals?

Ordered to unordered.

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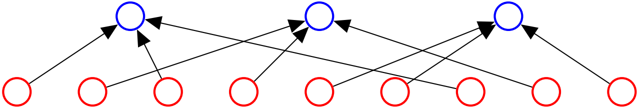

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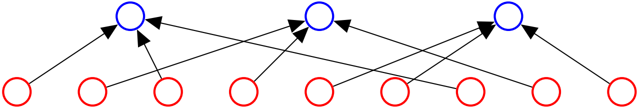
How many poker deals? $52 \cdot 51 \cdot 50 \cdot 49 \cdot 48$.

Ordered to unordered.

Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.



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How many poker deals per hand?

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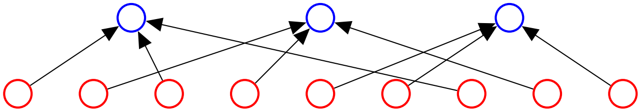
Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.



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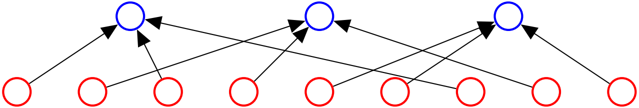
How many poker deals? $52 \cdot 51 \cdot 50 \cdot 49 \cdot 48$.

How many poker deals per hand?
Map each deal to ordered deal:

Ordered to unordered.

Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.



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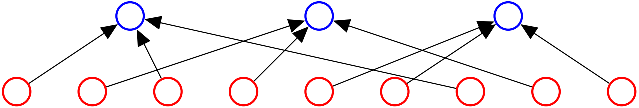
How many poker deals? $52 \cdot 51 \cdot 50 \cdot 49 \cdot 48$.

How many poker deals per hand?
Map each deal to ordered deal: 5!

Ordered to unordered.

Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.



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How many poker hands?

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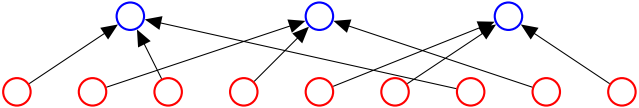
Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.



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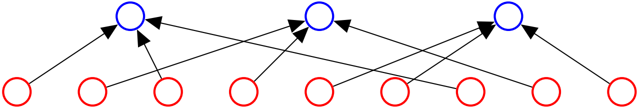
How many poker deals? $52 \cdot 51 \cdot 50 \cdot 49 \cdot 48$.

How many poker deals per hand?
Map each deal to ordered deal: 5!

How many poker hands? $\frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48}{5!}$

Ordered to unordered.

Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.



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Questions?

..order doesn’t matter.

..order doesn't matter.

Choose 2 out of n?

..order doesn't matter.

Choose 2 out of n?

$$\frac{n \times (n-1)}{}$$

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Choose 2 out of n?

$$\frac{n \times (n-1)}{2}$$

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Choose 2 out of n?

$$\frac{n \times (n-1)}{2} = \frac{n!}{(n-2)! \times 2}$$

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Choose 3 out of n?

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Choose 3 out of n?

$$\frac{n \times (n-1) \times (n-2)}{}$$

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Choose k **out of** n?

$$\frac{n!}{(n-k)!}$$

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Notation: $\binom{n}{k}$ **and pronounced "n choose k."**

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Familiar?

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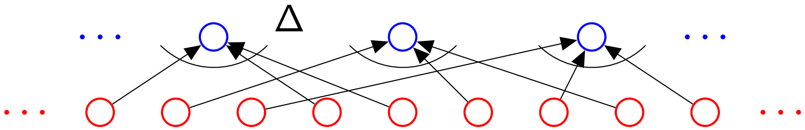
Familiar? Questions?

Example: Visualize the proof..

First rule: $n_1 \times n_2 \cdots \times n_3$**. Product Rule.**

Second rule: when order doesn't matter divide...



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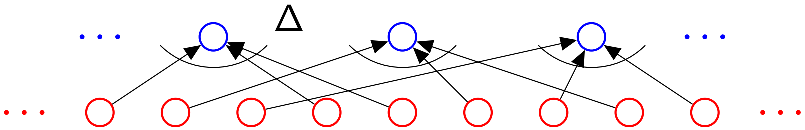
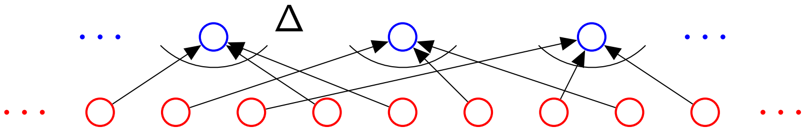


3 card Poker deals: 52

Example: Visualize the proof..

First rule: $n_1 \times n_2 \cdots \times n_3$**. Product Rule.**
Second rule: when order doesn't matter divide...

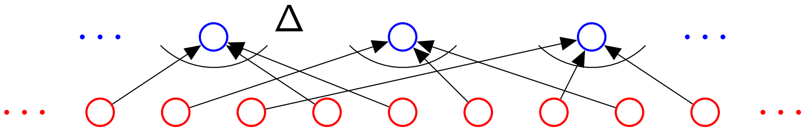


3 card Poker deals: 52×51

Example: Visualize the proof..

First rule: $n_1 \times n_2 \cdots \times n_3$**. Product Rule.**

Second rule: when order doesn't matter divide...

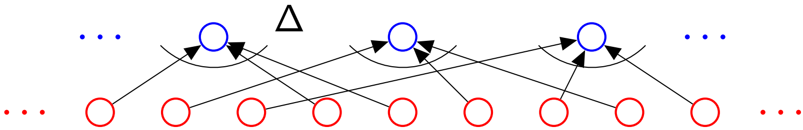


3 card Poker deals: $52 \times 51 \times 50$

Example: Visualize the proof..

First rule: $n_1 \times n_2 \cdots \times n_3$. **Product Rule.**

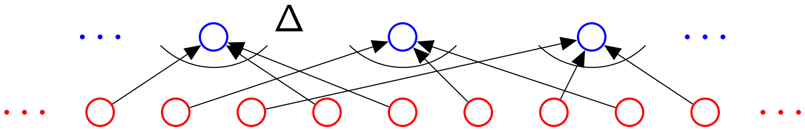
Second rule: when order doesn't matter divide...



3 card Poker deals: $52 \times 51 \times 50 = \frac{52!}{49!}$.

Example: Visualize the proof..

First rule: $n_1 \times n_2 \cdots \times n_3$**. Product Rule.**
Second rule: when order doesn't matter divide...



3 card Poker deals: $52 \times 51 \times 50 = \frac{52!}{49!}$. First rule.

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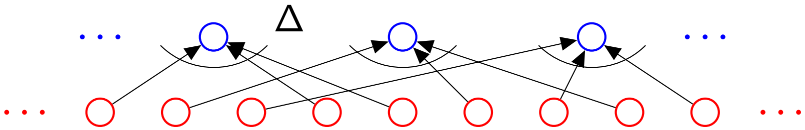
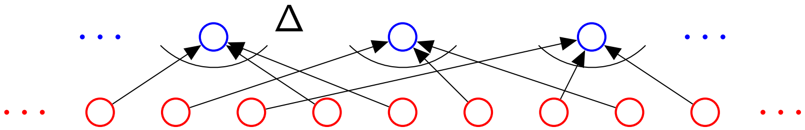


3 card Poker deals: $52 \times 51 \times 50 = \frac{52!}{49!}$. First rule.
Poker hands: Δ?

Example: Visualize the proof..

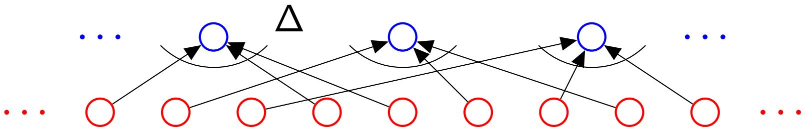
First rule: $n_1 \times n_2 \cdots \times n_3$. **Product Rule.**
Second rule: when order doesn't matter divide...



3 card Poker deals: $52 \times 51 \times 50 = \frac{52!}{49!}$. First rule.
Poker hands: Δ?
Hand: Q, K, A.

Example: Visualize the proof..

First rule: $n_1 \times n_2 \cdots \times n_3$. **Product Rule.**
Second rule: when order doesn't matter divide...



3 card Poker deals: $52 \times 51 \times 50 = \frac{52!}{49!}$. First rule.
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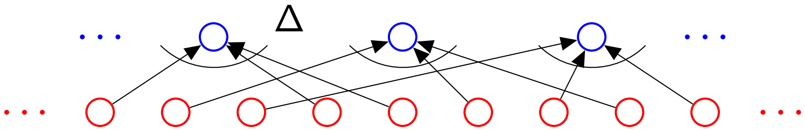

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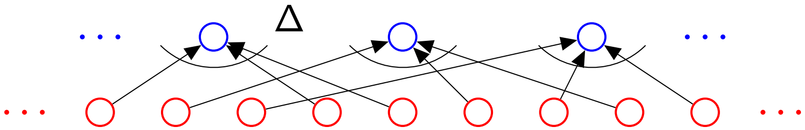

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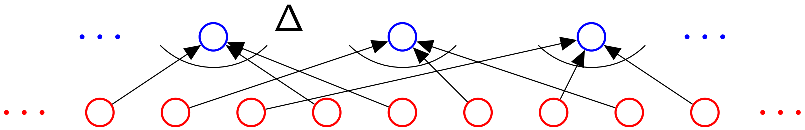

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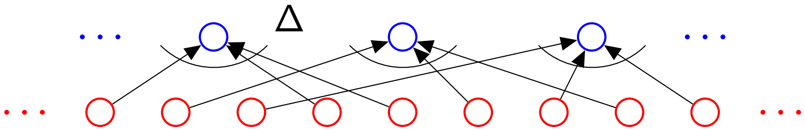

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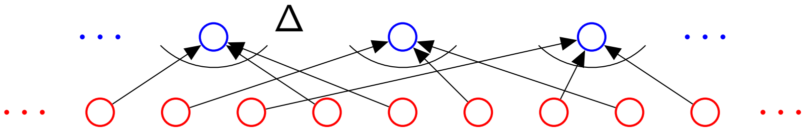

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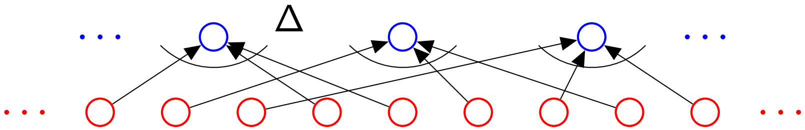

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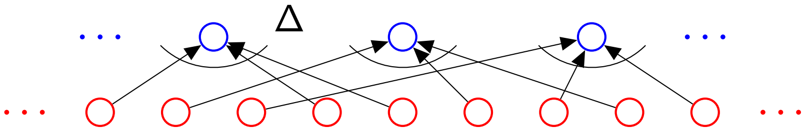

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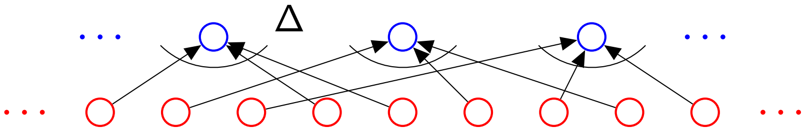

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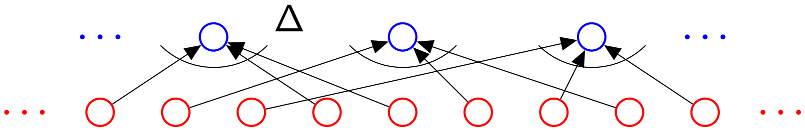

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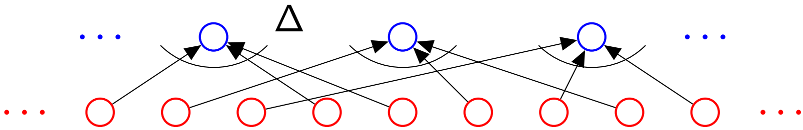

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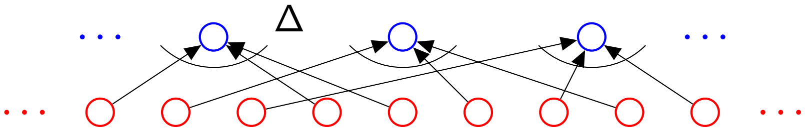

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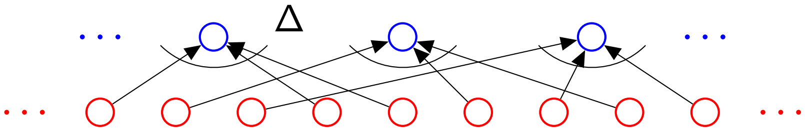

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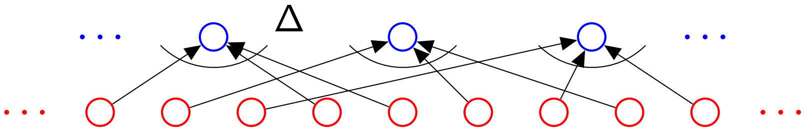

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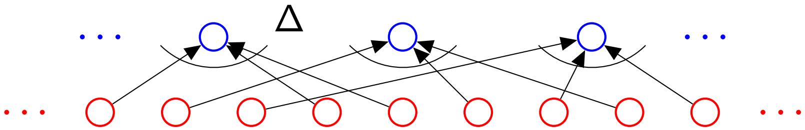

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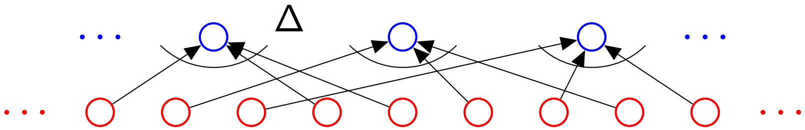

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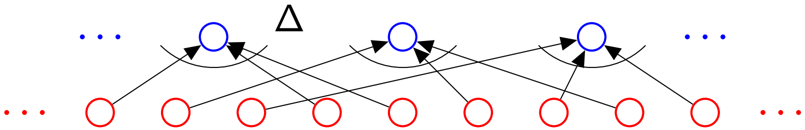

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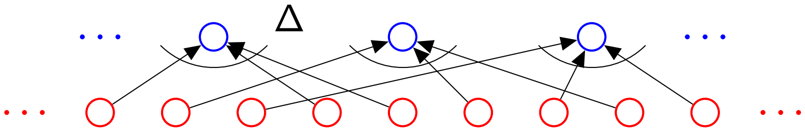

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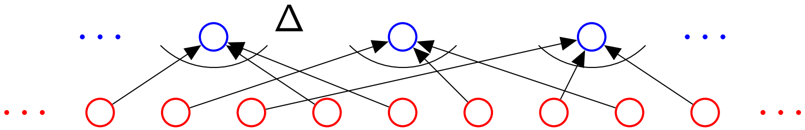

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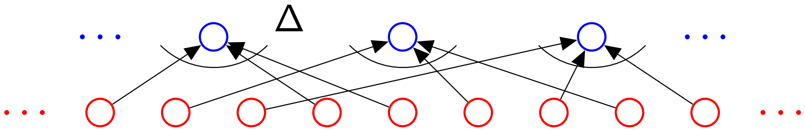

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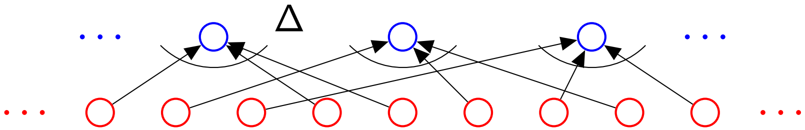

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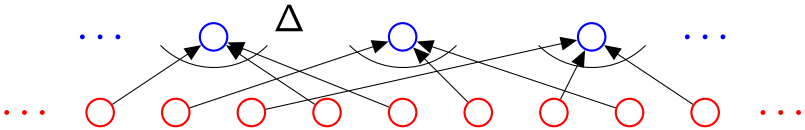

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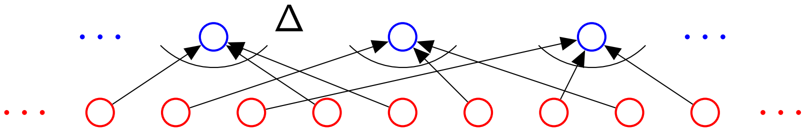

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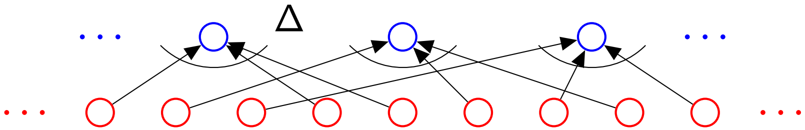

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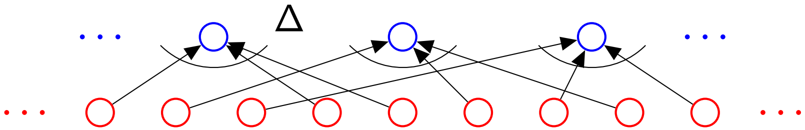

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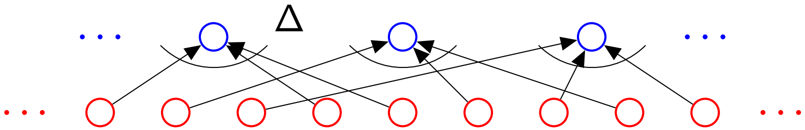

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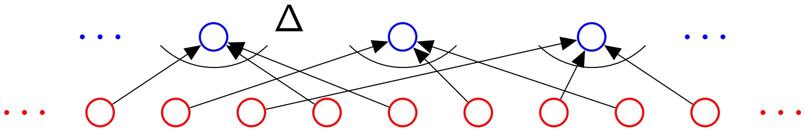

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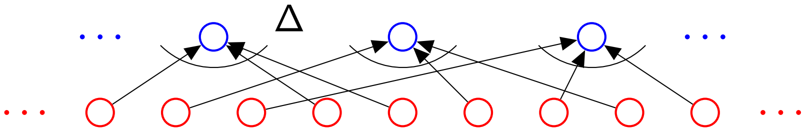

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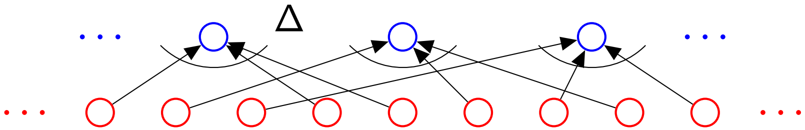

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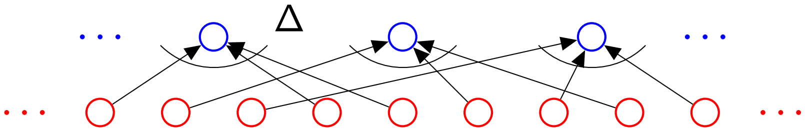

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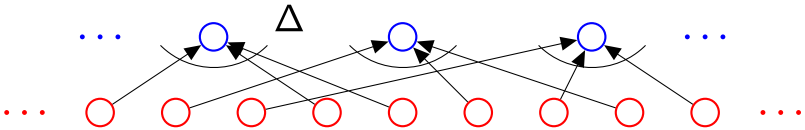

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3 ways to choose first letter, 2 ways for second, 1 for last.

$\implies 3 \times 2 \times 1 = 3!$ orderings

How many orderings of the letters in ANAGRAM?

Ordered, except for A!

total orderings of 7 letters. 7!
total "extra counts" or orderings of three A's? 3!

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4 S's, 4 I's, 2 P's.
11 letters total.

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$\Longrightarrow \frac{11!}{4!4!2!}$.

More counting on Monday.