Review.

After Midterm content.
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See lecture 13, for pre-Midterm review.
Counting.

First Rule: Enumerate objects with sequence of choices.

Number of Objects: \( n_1 \times n_2 \times \ldots \).

Example: Poker deals.

Second Rule: Divide out if by ordering of same objects.

Example: Poker hands. Orderings of ANAGRAM.

Sum Rule: If sets of objects disjoint add sizes.

Example: Hands with joker, hands without.

Inclusion/Exclusion: For arbitrary sets \( A, B \).

\[ |A \cup B| = |A| + |B| - |A \cap B| \]

Example: 10 digit numbers with 9 in the first or second digit.
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Combinatorial Proofs.

**Theorem:** \( \binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1} \).

**Proof:** How many size \( k \) subsets of \( n+1 \)?
Combinatorial Proofs.

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**Proof:** How many size $k$ subsets of $n+1$? $\binom{n+1}{k}$.

How many contain the first element? Choose first element, need to choose $k-1$ more from remaining $n$ elements.

$\Rightarrow \binom{n}{k-1}$

How many don't contain the first element? Need to choose $k$ elements from remaining $n$ elements.

$\Rightarrow \binom{n}{k}$

So, $\binom{n}{k-1} + \binom{n}{k} = \binom{n+1}{k}$. 

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Uncountability/Undecidability.

Natural Numbers are countable. Definition.
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Concept: Can programatically modify text of input program (to HALT).
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Concept: Can programatically modify text of input program (to HALT).
Concept: Can call program A.
CS70: Review of Probability.

Probability Review
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Discrete Probability

Probability Space: $\Omega$, $Pr[\omega] \geq 0$, $\sum_{\omega} Pr[\omega] = 1$. 
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Applications: compute expectations by decomposing.

Indicators: Empty bins, Fixed points.

Time to Coupon: Sum times to “next” coupon.

Geometric distribution vs. direct.

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$Y = f(X)$ is Random Variable.

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Linearity of Expectation: \( E[X + Y] = E[X] + E[Y]. \)

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\( \Pr[Y = y] = \sum_{x \in f^{-1}y} \Pr[X = x]. \)
Tail Bounds: WWLN

Variance:
\[ \text{Var}(X) = \mathbb{E}(X - \mathbb{E}[X])^2 = \mathbb{E}[X^2] - \mathbb{E}[X]^2 \]

Fact:
\[ \text{Var}(aX + b) = a^2 \text{Var}(X) \]

Sum:

Markov:
\[ \Pr(X \geq a) \leq \frac{\mathbb{E}[f(X)]}{f(a)} \]

Chebyshev:
\[ \Pr(|X - \mathbb{E}[X]| \geq a) \leq \frac{\text{Var}(X)}{a^2} \]

WLLN:
\[ X_m \text{ i.i.d.} \Rightarrow X_1 + \cdots + X_n \approx \mathbb{E}[X] \]
Tail Bounds: WWLN

Variance

Variance


Fact: \( \text{var}[aX + b] = a^2 \text{var}[X] \)
Tail Bounds: WWLN

- **Variance:** $\text{var}[X] := E[(X - E[X])^2] = E[X^2] - E[X]^2$
- **Fact:** $\text{var}[aX + b] = a^2 \text{var}[X]$
- **Sum:** $X, Y, Z$ pairwise ind. $\Rightarrow \text{var}[X + Y + Z] = \cdots$
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- **Chebyshev**: \( \Pr[|X - E[X]| \geq a] \leq \text{var}[X]/a^2 \)
Tail Bounds: WWLN

- **Variance:** \( \text{var}[X] := E[(X - E[X])^2] = E[X^2] - E[X]^2 \)
- **Fact:** \( \text{var}[aX + b] = a^2 \text{var}[X] \)
- **Sum:** \( X, Y, Z \) pairwise ind. \( \Rightarrow \text{var}[X + Y + Z] = \cdots \)
- **Markov:** \( Pr[X \geq a] \leq E[f(X)]/f(a) \) where ...
- **Chebyshev:** \( Pr[|X - E[X]| \geq a] \leq \text{var}[X]/a^2 \)
- **WLLN:** \( X_m \) i.i.d. \( \Rightarrow \frac{X_1 + \cdots + X_n}{n} \approx E[X] \)
Random Variables so far.

Probability Space: $\Omega$, $Pr : \Omega \rightarrow [0, 1]$, $\sum_{\omega \in \Omega} Pr(\omega) = 1$. 
Random Variables so far.

Probability Space: $\Omega$, $Pr : \Omega \rightarrow [0, 1]$, $\sum_{\omega \in \Omega} Pr(\omega) = 1$.
Random Variables: $X : \Omega \rightarrow R$. 
Random Variables so far.

Probability Space: $\Omega$, $Pr : \Omega \rightarrow [0, 1]$, $\sum_{\omega \in \Omega} Pr(\omega) = 1$.

Random Variables: $X : \Omega \rightarrow R$.

Associated event: $Pr[X = a] = \sum_{\omega : X(\omega) = a} Pr(\omega)$.
Random Variables so far.

Probability Space: $\Omega$, $Pr : \Omega \rightarrow [0, 1]$, $\sum_{\omega \in \Omega} Pr(w) = 1$.
Random Variables: $X : \Omega \rightarrow R$.

Associated event: $Pr[X = a] = \sum_{\omega : X(\omega) = a} Pr(\omega)$

$X$ and $Y$ independent $\iff$ all associated events are independent.
Random Variables so far.

Probability Space: $\Omega$, $Pr : \Omega \to [0, 1]$, $\sum_{\omega \in \Omega} Pr(\omega) = 1$.
Random Variables: $X : \Omega \to R$.

Associated event: $Pr[X = a] = \sum_{\omega : X(\omega) = a} Pr(\omega)$

$X$ and $Y$ independent $\iff$ all associated events are independent.

Expectation: $E[X] = \sum_{a} a Pr[X = a] = \sum_{\omega \in \Omega} X(\omega) Pr(\omega)$. 
Random Variables so far.

Probability Space: $\Omega$, $Pr : \Omega \rightarrow [0, 1]$, $\sum_{\omega \in \Omega} Pr(\omega) = 1$.
Random Variables: $X : \Omega \rightarrow R$.
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$X$ and $Y$ independent $\iff$ all associated events are independent.
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Random Variables: $X : \Omega \rightarrow R$.

Associated event: $Pr[X = a] = \sum_{\omega : X(\omega) = a} Pr(\omega)$

$X$ and $Y$ independent $\iff$ all associated events are independent.

Expectation: $E[X] = \sum_a a Pr[X = a] = \sum_{\omega \in \Omega} X(\omega) Pr(\omega)$.


Variance: $Var(X) = E[(X - E[X])^2] = E[X^2] - (E(X))^2$
Random Variables so far.

Probability Space: $\Omega$, $Pr : \Omega \rightarrow [0, 1]$, $\sum_{\omega \in \Omega} Pr(\omega) = 1$.
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Variance: $Var(X) = E[(X - E[X])^2] = E[X^2] - (E(X))^2$

For independent $X, Y$, $Var(X + Y) = Var(X) + Var(Y)$.
Random Variables so far.

Probability Space: $\Omega$, $Pr : \Omega \rightarrow [0, 1]$, $\sum_{\omega \in \Omega} Pr(\omega) = 1$.
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Variance: $Var(X) = E[(X - E[X])^2] = E[X^2] - (E(X))^2$

For independent $X, Y$, $Var(X + Y) = Var(X) + Var(Y)$.

Also: $Var(cX) = c^2 Var(X)$ and $Var(X + b) = Var(X)$. 

Poisson: $X \sim P(\lambda)$

$E(X) = \lambda$, $Var(X) = \lambda$.

Binomial: $X \sim B(n, p)$

$E(X) = np$, $Var(X) = np(1 - p)$.

Uniform: $X \sim U\{1, \ldots, n\}$

$E[X] = \frac{n + 1}{2}$, $Var(X) = \frac{n^2 - 1}{12}$.

Geometric: $X \sim G(p)$

$E(X) = \frac{1}{p}$, $Var(X) = \frac{1 - p}{p^2}$.
Random Variables so far.

Probability Space: \( \Omega, \Pr : \Omega \rightarrow [0, 1], \sum_{\omega \in \Omega} \Pr(\omega) = 1. \)

Random Variables: \( X : \Omega \rightarrow R. \)

Associated event: \( \Pr[X = a] = \sum_{\omega : X(\omega) = a} \Pr(\omega) \)

\( X \) and \( Y \) independent \( \iff \) all associated events are independent.

Expectation: \( E[X] = \sum_a a \Pr[X = a] = \sum_{\omega \in \Omega} X(\omega) \Pr(\omega). \)

Linearity: \( E[X + Y] = E[X] + E[Y]. \)

Variance: \( \Var(X) = E[(X - E[X])^2] = E[X^2] - (E[X])^2 \)

For independent \( X, Y, \ Var(X + Y) = Var(X) + Var(Y). \)

Also: \( \Var(cX) = c^2 \Var(X) \) and \( \Var(X + b) = \Var(X). \)

Poisson: \( X \sim P(\lambda) \)
Random Variables so far.

Probability Space: $\Omega$, $Pr : \Omega \rightarrow [0, 1]$, $\sum_{\omega \in \Omega} Pr(\omega) = 1$.
Random Variables: $X : \Omega \rightarrow R$.

Associated event: $Pr[X = a] = \sum_{\omega : X(\omega) = a} Pr(\omega)$

$X$ and $Y$ independent $\iff$ all associated events are independent.

Expectation: $E[X] = \sum_a a Pr[X = a] = \sum_{\omega \in \Omega} X(\omega) Pr(\omega)$.


Variance: $Var(X) = E[(X - E[X])^2] = E[X^2] - (E(X))^2$

For independent $X, Y$, $Var(X + Y) = Var(X) + Var(Y)$.

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Poisson: $X \sim P(\lambda)$ $E(X) = \lambda$, $Var(X) = \lambda$. 
Random Variables so far.

Probability Space: $\Omega$, $Pr : \Omega \rightarrow [0, 1]$, $\sum_{\omega \in \Omega} Pr(\omega) = 1$.

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$X$ and $Y$ independent $\iff$ all associated events are independent.

Expectation: $E[X] = \sum_a a Pr[X = a] = \sum_{\omega \in \Omega} X(\omega) Pr(\omega)$.


Variance: $Var(X) = E[(X - E[X])^2] = E[X^2] - (E(X))^2$

For independent $X$, $Y$, $Var(X + Y) = Var(X) + Var(Y)$.

Also: $Var(cX) = c^2 Var(X)$ and $Var(X + b) = Var(X)$.

Poisson: $X \sim P(\lambda)$ $E(X) = \lambda$, $Var(X) = \lambda$.

Binomial: $X \sim B(n,p)$
Random Variables so far.

Probability Space: Ω, Pr : Ω → [0, 1], ∑ω∈Ω Pr(w) = 1.
Random Variables: X : Ω → R.

Associated event: Pr[X = a] = Σω:X(ω)=a Pr(ω)
X and Y independent ⇐⇒ all associated events are independent.
Expectation: E[X] = ∑a aPr[X = a] = Σω∈Ω X(ω) Pr(ω).

Variance: Var(X) = E[(X − E[X])^2] = E[X^2] − (E(X))^2
For independent X, Y, Var(X + Y) = Var(X) + Var(Y).
Also: Var(cX) = c^2 Var(X) and Var(X + b) = Var(X).

Poisson: X ∼ P(λ) E(X) = λ, Var(X) = λ.
Binomial: X ∼ B(n, p) E(X) = np, Var(X) = np(1 − p)
Random Variables so far.

Probability Space: $\Omega$, $Pr : \Omega \rightarrow [0, 1]$, $\sum_{\omega \in \Omega} Pr(\omega) = 1$.
Random Variables: $X : \Omega \rightarrow R$.

Associated event: $Pr[X = a] = \sum_{\omega : X(\omega) = a} Pr(\omega)$

$X$ and $Y$ independent $\iff$ all associated events are independent.

Expectation: $E[X] = \sum_{a} a Pr[X = a] = \sum_{\omega \in \Omega} X(\omega) Pr(\omega)$.


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Uniform: $X \sim U\{1, \ldots, n\}$
Probability Space: $\Omega$, $Pr : \Omega \rightarrow [0, 1]$, $\sum_{\omega \in \Omega} Pr(w) = 1$.

Random Variables: $X : \Omega \rightarrow R$.

Associated event: $Pr[X = a] = \sum_{\omega : X(\omega) = a} Pr(\omega)$

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Random Variables so far.

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Geometric: $X \sim G(p)$
Probability Space: $\Omega$, $Pr : \Omega \rightarrow [0, 1]$, $\sum_{\omega \in \Omega} Pr(\omega) = 1$.

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Geometric: $X \sim G(p)$ $E(X) = \frac{1}{p}$, $Var(X) = \frac{1-p}{p^2}$.
Continuous Probability

1. pdf:

\[ \Pr[X \in (x, x + \delta)] = f_X(x) \delta \]

2. CDF:

\[ \Pr[X \leq x] = F_X(x) = \int_{x-\infty}^{x} f_X(y) \, dy \]

3. Uniform pdf:

\[ f_X(x) = \frac{1}{b-a} \{ a \leq x \leq b \}; \quad F_X(x) = x - a \frac{b-a}{b-a} \text{ for } a \leq x \leq b. \]

4. Exponential pdf:

\[ f_X(x) = \lambda e^{-\lambda x} \frac{1}{x \geq 0}; \quad F_X(x) = 1 - e^{-\lambda x} \text{ for } x \geq 0. \]

5. Target:

\[ f_X(x) = 2x \frac{1}{x \geq 0}; \quad F_X(x) = x^2 \text{ for } 0 \leq x \leq 1. \]

6. Joint pdf:

\[ \Pr[X \in (x, x + \delta), Y = (y, y + \delta)] = f_{X,Y}(x, y) \delta^2 \]

6.1 Conditional Distribution:

\[ f_{X|Y}(x, y) = \frac{f_{X,Y}(x, y)}{f_Y(y)}. \]

6.2 Independence:

\[ f_{X|Y}(x, y) = f_X(x). \]
Continuous Probability

1. **pdf:**  \( Pr[\ X \in (x, x + \delta)] = f_X(x)\delta. \)
Continuous Probability

1. **pdf**: \( Pr[X \in (x, x + \delta)] = f_X(x)\delta. \)

2. **CDF**: 

---

6. **Joint pdf**: \( Pr[X \in (x, x + \delta), Y = (y, y + \delta)] = f_{X,Y}(x, y)\delta^2. \)

6.1 **Conditional Distribution**: 
\[
f_{X|Y}(x, y) = \frac{f_{X,Y}(x, y)}{f_Y(y)}.
\]

6.2 **Independence**: 
\[
f_{X|Y}(x, y) = f_X(x) \quad \text{if} \quad f_{X,Y}(x, y) \neq 0.
\]
Continuous Probability

1. pdf: \( \Pr[X \in (x, x + \delta)] = f_X(x)\delta. \)
2. CDF: \( \Pr[X \leq x] = F_X(x) = \int_{-\infty}^{x} f_X(y)dy. \)
Continuous Probability

1. pdf: \( Pr[X \in (x, x + \delta)] = f_X(x)\delta \).
2. CDF: \( Pr[X \leq x] = F_X(x) = \int_{-\infty}^{x} f_X(y)dy \).
3. \( U[a, b] \):
Continuous Probability

1. pdf: \( Pr[X \in (x, x + \delta)] = f_X(x) \delta \).
2. CDF: \( Pr[X \leq x] = F_X(x) = \int_{-\infty}^{x} f_X(y)dy \).
3. \( U[a, b] \): \( f_X(x) = \frac{1}{b-a} 1\{a \leq x \leq b\} \).
4. Expo(\( \lambda \)): \( f_X(x) = \lambda \exp\{-\lambda x\} 1\{x \geq 0\} \); \( F_X(x) = 1 - \exp\{-\lambda x\} \) for \( x \geq 0 \).
5. Target: \( f_X(x) = 2x 1\{0 \leq x \leq 1\} \); \( F_X(x) = x^2 \) for \( 0 \leq x \leq 1 \).
6. Joint pdf: \( Pr\left[X \in (x, x + \delta), Y = (y, y + \delta)\right] = f_{X,Y}(x, y) \delta^2 \).
6.1 Conditional Distribution: \( f_{X|Y}(x, y) = \frac{f_{X,Y}(x, y)}{f_Y(y)} \).
6.2 Independence: \( f_{X|Y}(x, y) = f_X(x) \).
Continuous Probability

1. pdf: $Pr[X \in (x, x + \delta)] = f_X(x)\delta$.
2. CDF: $Pr[X \leq x] = F_X(x) = \int_{-\infty}^{x} f_X(y)dy$.
3. $U[a, b]$: $f_X(x) = \frac{1}{b-a}1\{a \leq x \leq b\}; F_X(x) = \frac{x-a}{b-a}$ for $a \leq x \leq b$. 
Continuous Probability

1. **pdf:** \( Pr[X \in (x, x + \delta)] = f_X(x)\delta. \)
2. **CDF:** \( Pr[X \leq x] = F_X(x) = \int_{-\infty}^{x} f_X(y)dy. \)
3. **U[a, b]:** \( f_X(x) = \frac{1}{b-a} 1\{a \leq x \leq b\}; F_X(x) = \frac{x-a}{b-a} \) for \(a \leq x \leq b.\)
4. **Expo(\lambda):**
Continuous Probability

1. **pdf:** \( Pr[X \in (x, x + \delta)] = f_X(x)\delta. \)
2. **CDF:** \( Pr[X \leq x] = F_X(x) = \int_{-\infty}^{x} f_X(y)dy. \)
3. **U[a, b]:** \( f_X(x) = \frac{1}{b-a}1\{a \leq x \leq b\}; F_X(x) = \frac{x-a}{b-a} \) for \( a \leq x \leq b. \)
4. **Expo(\(\lambda\)):**
   \[ f_X(x) = \lambda \exp\{-\lambda x\}1\{x \geq 0\}; \]
Continuous Probability

1. pdf: $Pr[X \in (x, x + \delta)] = f_X(x)\delta$.
2. CDF: $Pr[X \leq x] = F_X(x) = \int_{-\infty}^{x} f_X(y)dy$.
3. $U[a, b]$: $f_X(x) = \frac{1}{b-a}1\{a \leq x \leq b\}$; $F_X(x) = \frac{x-a}{b-a}$ for $a \leq x \leq b$.
4. $Expo(\lambda)$:
   \[ f_X(x) = \lambda \exp\{-\lambda x\}1\{x \geq 0\}; F_X(x) = 1 - \exp\{-\lambda x\} \text{ for } x \geq 0. \]
Continuous Probability

1. pdf: $Pr[X \in (x, x + \delta)] = f_X(x)\delta$.

2. CDF: $Pr[X \leq x] = F_X(x) = \int_{-\infty}^{x} f_X(y)dy$.

3. $U[a, b]$: $f_X(x) = \frac{1}{b-a} 1\{a \leq x \leq b\}; F_X(x) = \frac{x-a}{b-a}$ for $a \leq x \leq b$.

4. $\text{Expo}(\lambda)$:
   $f_X(x) = \lambda \exp\{-\lambda x\} 1\{x \geq 0\}; F_X(x) = 1 - \exp\{-\lambda x\}$ for $x \geq 0$.

5. Target:
Continuous Probability

1. pdf: \( Pr[X \in (x, x + \delta)] = f_X(x)\delta. \)
2. CDF: \( Pr[X \leq x] = F_X(x) = \int_{-\infty}^{x} f_X(y)dy. \)
3. \( U[a, b] \): \( f_X(x) = \frac{1}{b-a} 1\{a \leq x \leq b\}; F_X(x) = \frac{x-a}{b-a} \) for \( a \leq x \leq b \).
4. \( \text{Expo}(\lambda) \):
   \( f_X(x) = \lambda \exp\{-\lambda x\} 1\{x \geq 0\}; F_X(x) = 1 - \exp\{-\lambda x\} \) for \( x \geq 0 \).
5. Target: \( f_X(x) = 2x 1\{0 \leq x \leq 1\}; \)
Continuous Probability

1. pdf: $Pr[X \in (x, x + \delta)] = f_X(x) \delta$.

2. CDF: $Pr[X \leq x] = F_X(x) = \int_{-\infty}^{x} f_X(y) dy$.

3. $U[a, b]$: $f_X(x) = \frac{1}{b-a} 1\{a \leq x \leq b\}; F_X(x) = \frac{x-a}{b-a}$ for $a \leq x \leq b$.

4. Expo($\lambda$):
   $f_X(x) = \lambda \exp\{-\lambda x\} 1\{x \geq 0\}; F_X(x) = 1 - \exp\{-\lambda x\}$ for $x \geq 0$.

5. Target: $f_X(x) = 2x1\{0 \leq x \leq 1\}; F_X(x) = x^2$ for $0 \leq x \leq 1$.

6. Joint pdf: $Pr[X \in (x, x + \delta), Y = (y, y + \delta)] = f_{X,Y}(x, y) \delta^2$.

6.1 Conditional Distribution: $f_{X|Y}(x, y) = \frac{f_{X,Y}(x, y)}{f_Y(y)}$.

6.2 Independence: $f_{X|Y}(x, y) = f_X(x)$.
Continuous Probability: Moments.

- Expectation.

\[
E[X] = \int_{-\infty}^{\infty} x f(x) \, dx
\]

- Variance.

\[
E[(X - E[X])^2]
\]

Same as discrete.

- Markov?

Sure.

- Chebyshev?

Sure.
Continuous Probability: Moments.

- **Expectation.**
  \[ E[X] = \int_{-\infty}^{\infty} xf(x) \, dx. \]
- **Variance.**
Continuous Probability: Moments.

- **Expectation.**
  \[ E[X] = \int_{-\infty}^{\infty} xf(x) \, dx. \]

- **Variance.**
  \[ E[(X - E[X])^2] \]
Continuous Probability: Moments.

- **Expectation.**
  \[ E[X] = \int_{-\infty}^{\infty} xf(x) \, dx. \]

- **Variance.**
  \[ E[(X - E[X])^2] \text{ Same as discrete.} \]

- **Markov?**

- **Chebyshev?** Yes.
Continuous Probability: Moments.

- **Expectation.**
  \[ E[X] = \int_{-\infty}^{\infty} xf(x) \, dx. \]

- **Variance.**
  \[ E[(X - E[X])^2] \text{ Same as discrete.} \]

- **Markov?** Sure.

- **Chebyshev?**
Continuous Probability: Moments.

- **Expectation.**
  \[ E[X] = \int_{-\infty}^{\infty} x f(x) \, dx. \]

- **Variance.**
  \[ E[(X - E[X])^2] \text{ Same as discrete.} \]

- **Markov?** Sure.

- **Chebyshev?** Sure.
Distributions.

- $X \sim U[a, b]$  
  $f_X(x) = \frac{1}{(b-a)} 1\{x \in [a, b]\}$.  
  $F(x) = \min\left(\frac{x-a}{b-a} 1\{x \in [a, b]\}, 1.0\right)$
Distributions.

- $X \sim U[a, b]$
  
  \[
  f_X(x) = \frac{1}{b - a} 1 \{x \in [a, b]\}. \quad F(x) = \min\left(\frac{x-a}{b-a} 1 \{x \in [a, b]\}, 1.0\right)
  \]
  \[
  E[X] = \frac{b - a}{2}.
  \]
Distributions.

- $X \sim U[a, b]$
  
  $f_X(x) = \frac{1}{b-a} 1\{x \in [a, b]\}$.  
  
  $F(x) = \min\left(\frac{x-a}{b-a} 1\{x \in [a, b]\}, 1.0\right)$

  $E[X] = \frac{b-a}{2}$.

  $Var(X) = \frac{(b-a)^2}{12}$.
Distributions.

- \( X \sim U[a, b] \)
  \[
  f_X(x) = \frac{1}{(b-a)} 1\{x \in [a, b]\}. \quad F(x) = \min\left(\frac{x-a}{b-a} 1\{x \in [a, b]\}, 1.0\right)
  \]
  \[
  E[X] = \frac{b-a}{2}.
  \]
  \[
  Var(X) = \frac{(b-a)^2}{12}.
  \]

- \( X \sim \text{Expo}(\lambda) \)
  \[
  f_X(x) = \lambda e^{-\lambda x} 1\{x \geq 0\}. \quad F_X(x) = 1 - e^{\lambda x}.
  \]
  \[
  E[X] = \frac{1}{\lambda}.
  \]
  \[
  Var[X] = \frac{1}{\lambda^2}
  \]
  \[
  X = \lim_{n \to \infty} \frac{1}{n} X \quad G(\lambda / n)
  \]
  \[
  Pr[X > s + t | X > s] = Pr[X > t]: \text{Memoryless.}
  \]
Distributions.

- **$X \sim U[a, b]$**
  
  \( f_X(x) = \frac{1}{b-a} 1\{x \in [a, b]\} \).
  
  \( F(x) = \min(\frac{x-a}{b-a} 1\{x \in [a, b]\}, 1.0) \)
  
  \( E[X] = \frac{b-a}{2} \).
  
  \( Var(X) = \frac{(b-a)^2}{12} \).

- **$X \sim Expo(\lambda)$**
  
  \( f_X(x) = \lambda e^{-\lambda x} 1\{x \geq 0\} \)
  \( F_X(x) = 1 - e^{-\lambda x} . \)
  
  \( E[X] = \frac{1}{\lambda} \).
  
  \( Var[X] = \frac{1}{\lambda^2} \).
  
  \( X = \lim_{n \to \infty} \frac{1}{n} X \ G(\lambda / n) \)
  
  \( Pr[X > s + t | X > s] = Pr[X > t] \): Memoryless.

- **$X \sim N(\mu, \sigma)$**
  
  \( f(x) = \frac{1}{\sqrt{2\pi}} e^{-(x-\mu)^2 / 2} \)
  \( F(x) = \Phi(x) = \int_{-\infty}^{x} f(x) dx \)
  
  CLT: \( A_n = \frac{X_1 + \ldots + X_n}{n} \), \( E[X] = \mu \), and \( Var(X) = \sigma^2 \).
  
  \( \lim_{n \to \infty} A_n \to N(\mu, \sigma) \)
  \( \frac{X-\mu}{\sigma} \sim N(0, 1) \).
Markov Chains

- Markov Chain:

\[
\Pr[X_{n+1} = j | X_0, \ldots, X_n = i] = P(i, j)
\]

- FSE:

\[
\alpha(i) = \sum_j P(i, j) \alpha(j);
\]
\[
\beta(i) = 1 + \sum_j P(i, j) \beta(j);
\]

- \(\pi_n = \pi_0 P^n\)

- \(\pi\) is invariant iff \(\pi P = \pi\)

- Irreducible \(\Rightarrow\) one and only one invariant distribution \(\pi\)

- Irreducible \(\Rightarrow\) fraction of time in state \(i\) approaches \(\pi(i)\)

- Irreducible + Aperiodic \(\Rightarrow\) \(\pi_n \rightarrow \pi\).

- Calculating \(\pi\): One finds \(P \pi = \pi\) and \(\pi\) is distribution.
Markov Chains

- Markov Chain: \( Pr[X_{n+1} = j | X_0, \ldots, X_n = i] = P(i, j) \)
Markov Chains

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Markov Chains

- **Markov Chain**: $Pr[X_{n+1} = j|X_0, \ldots, X_n = i] = P(i,j)$
- **FSE**: $\beta(i) = 1 + \sum_j P(i,j)\beta(j); \alpha(i) = \sum_j P(i,j)\alpha(j)$.
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Markov Chains

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Markov Chains

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- \( \pi \) is invariant iff \( \pi P = \pi \)
- Irreducible \( \Rightarrow \) one and only one invariant distribution \( \pi \)
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Markov Chains

- Markov Chain: $Pr[X_{n+1} = j | X_0, \ldots, X_n = i] = P(i, j)$
- FSE: $\beta(i) = 1 + \sum_j P(i, j)\beta(j); \alpha(i) = \sum_j P(i, j)\alpha(j)$.
- $\pi_n = \pi_0 P^n$
- $\pi$ is invariant iff $\pi P = \pi$
- Irreducible $\Rightarrow$ one and only one invariant distribution $\pi$
- Irreducible $\Rightarrow$ fraction of time in state $i$ approaches $\pi(i)$
- Irreducible + Aperiodic $\Rightarrow \pi_n \rightarrow \pi$. 
Markov Chains

- Markov Chain: \( Pr[X_{n+1} = j|X_0, \ldots, X_n = i] = P(i,j) \)
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- \( \pi_n = \pi_0 P^n \)
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- Calculating \( \pi \): One finds \( P\pi = \pi \) and \( \pi \) is distribution.
Some exercise.

1. True or False
Some exercise.

1. True or False
2. Some Key Results
Some exercise.

1. True or False
2. Some Key Results
3. Quiz 1: G
Some exercise.

1. True or False
2. Some Key Results
3. Quiz 1: G
4. Quiz 2: PG
Some exercise.

1. True or False
2. Some Key Results
3. Quiz 1: G
4. Quiz 2: PG
5. Quiz 3: R
Some exercise.

1. True or False
2. Some Key Results
3. Quiz 1: G
4. Quiz 2: PG
5. Quiz 3: R
6. Common Mistakes
Some exercise.

1. True or False
2. Some Key Results
3. Quiz 1: G
4. Quiz 2: PG
5. Quiz 3: R
6. Common Mistakes
True or False

- $\Omega$ and $A$ are independent.
True or False

- $\Omega$ and $A$ are independent.  True

- $\Pr[A \cap B] = \Pr[A] + \Pr[B] - \Pr[A \cup B]$.  True

- $\Pr[A \setminus B] \geq \Pr[A] - \Pr[B]$.  True

- $X_1, \ldots, X_n \text{i.i.d.} \Rightarrow \text{var}(X_1 + \cdots + X_n) = \text{var}(X_1)$.  False

- $\Pr[|X - a| \geq b] \leq \mathbb{E}[(X - a)^2]^{b/2}$.  True

- $X_1, \ldots, X_n \text{i.i.d.} \Rightarrow X_1 + \cdots + X_n - n\mathbb{E}[X_1] \xrightarrow{\text{d}} N(0, 1)$

- $\sqrt{n} \Rightarrow X \equiv \text{Expo}(\lambda) \Rightarrow \Pr[X > 5 | X > 3] = \Pr[X > 2]$.  True

- $\exp(-\lambda^5) \exp(-\lambda^3) = \exp(-\lambda^2)$.  True
True or False

- \(\Omega\) and \(A\) are independent. True
- \(Pr[A \cap B] = Pr[A] + Pr[B] - Pr[A \cup B]\).
True or False

- Ω and $A$ are independent. True
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- $X_1, \ldots, X_n$ i.i.d. $\implies var\left(\frac{X_1 + \ldots + X_n}{n}\right) = var(X_1)$. False
True or False

- $\Omega$ and $A$ are independent. True
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- $X_1, \ldots, X_n$ i.i.d. $\implies var\left(\frac{X_1 + \cdots + X_n}{n}\right) = var(X_1)$. False: $\times \frac{1}{n}$
True or False

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- $Pr[|X - a| \geq b] \leq \frac{E[(X-a)^2]}{b^2}$. True
True or False

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- \( X_1, \ldots, X_n \text{ i.i.d.} \Rightarrow \frac{X_1 + \ldots + X_n - nE[X_1]}{n\sigma(X_1)} \rightarrow \mathcal{N}(0, 1) \).
True or False

- $\Omega$ and $A$ are independent. True
- $Pr[A \cap B] = Pr[A] + Pr[B] - Pr[A \cup B]$. True
- $Pr[A \setminus B] \geq Pr[A] - Pr[B]$. True
- $X_1, \ldots, X_n$ i.i.d. $\implies \var\left(\frac{X_1 + \cdots + X_n}{n}\right) = \var(X_1)$. False: $\times \frac{1}{n}$
- $Pr[|X - a| \geq b] \leq \frac{E[(X-a)^2]}{b^2}$. True
- $X_1, \ldots, X_n$ i.i.d. $\implies \frac{X_1 + \cdots + X_n - n\mu}{n\sigma(X_1)} \rightarrow N(0, 1)$. False: $\sqrt{n}$
True or False

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- \( X_1, \ldots, X_n \text{ i.i.d.} \implies \text{var}\left(\frac{X_1 + \cdots + X_n}{n}\right) = \text{var}(X_1) \). False: \( \times \frac{1}{n} \)
- \( Pr[|X - a| \geq b] \leq \frac{E[(X-a)^2]}{b^2} \). True
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- \( X = \text{Expo}(\lambda) \implies Pr[X > 5|X > 3] = Pr[X > 2] \).
True or False

- \( \Omega \) and \( A \) are independent.  \( \text{True} \)
- \( Pr[A \cap B] = Pr[A] + Pr[B] - Pr[A \cup B] \).  \( \text{True} \)
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- \( X_1, \ldots, X_n \) i.i.d.  \( \implies \text{var} \left( \frac{X_1 + \cdots + X_n}{n} \right) = \text{var}(X_1). \)  \( \text{False: } \times \frac{1}{n} \)
- \( \text{Pr}[|X - a| \geq b] \leq \frac{E[(X-a)^2]}{b^2} \).  \( \text{True} \)
- \( X_1, \ldots, X_n \) i.i.d.  \( \implies \frac{X_1 + \cdots + X_n - nE[X_1]}{n \sigma(X_1)} \to N(0, 1). \)  \( \text{False: } \sqrt{n} \)
- \( X = \text{ Expo}(\lambda) \implies \text{Pr}[X > 5 | X > 3] = \text{Pr}[X > 2]. \)  \( \text{True} \)
True or False

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- $Pr[A \cap B] = Pr[A] + Pr[B] - Pr[A \cup B]$. True
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- $Pr[|X - a| \geq b] \leq \frac{E[(X-a)^2]}{b^2}$. True
- $X_1, \ldots, X_n$ i.i.d. $\implies \frac{X_1 + \cdots + X_n - nE[X_1]}{n\sigma(X_1)} \to \mathcal{N}(0, 1)$. False: $\sqrt{n}$
- $X = Expo(\lambda) \implies Pr[X > 5 | X > 3] = Pr[X > 2]$. True: 
  $$\frac{\exp\{-\lambda 5\}}{\exp\{-\lambda 3\}} = \exp\{-\lambda 2\}.$$
Correct or not?

\[ n = \sum_{i=0}^{n} X_i \]

X_i are i.i.d, mean \( \mu \) and variance \( \sigma \).

When \( n \gg 1 \), one has

\[ A_n \in [\mu - 2\sigma, \mu + 2\sigma] \]

with prob \( \geq 95\% \).

Yes

If \( 0.3 < \sigma < 3 \), then

\[ A_n \in [\mu - 0.61\sqrt{n}, \mu + 0.61\sqrt{n}] \]

with prob \( \geq 95\% \).

No

If \( 0.3 < \sigma < 3 \), then

\[ A_n \in [\mu - 6.1\sqrt{n}, \mu + 6.1\sqrt{n}] \]

with prob \( \geq 95\% \).

Yes
Correct or not?

\[ A_n = \frac{\sum_{i=0}^{n} X_i}{n}, \text{ } X_i \text{ i.i.d, mean } \mu \text{ and variance } \sigma. \]
Correct or not?

\[ A_n = \frac{\sum_{i=0}^{n} X_i}{n}, \, X_i \text{ i.i.d, mean } \mu \text{ and variance } \sigma. \]

When \( n \gg 1 \), one has

\[ A_n \in [\mu - 6\frac{\sigma}{\sqrt{n}}, \mu + 6\frac{\sigma}{\sqrt{n}}] \text{ with prob } \geq 95\%. \]
Correct or not?

\[ A_n = \frac{\sum_{i=0}^{n} X_i}{n}, \ X_i \text{ i.i.d, mean } \mu \text{ and variance } \sigma. \]

When \( n \gg 1 \), one has

- \( A_n \in [\mu - 2\sigma \frac{1}{n}, \mu + 2\sigma \frac{1}{n}] \) with prob \( \geq 95\% \).
Correct or not?

\[ A_n = \frac{\sum_{i=0}^{n} X_i}{n}, \ X_i \ \text{i.i.d, mean} \ \mu \ \text{and variance} \ \sigma. \]

When \( n \gg 1 \), one has

- \( A_n \in [\mu - 2\sigma \frac{1}{n}, \mu + 2\sigma \frac{1}{n}] \) with prob \( \geq 95\% \).  **No**
Correct or not?

\( A_n = \frac{\sum_{i=0}^{n} X_i}{n} \), \( X_i \) i.i.d, mean \( \mu \) and variance \( \sigma \).

When \( n \gg 1 \), one has

- \( A_n \in [\mu - 2\sigma \frac{1}{n}, \mu + 2\sigma \frac{1}{n}] \) with prob \( \geq 95\% \). No
- \( A_n \in [\mu - 2\sigma \frac{1}{\sqrt{n}}, \mu + 2\sigma \frac{1}{\sqrt{n}}] \) with prob \( \geq 95\% \). Yes
Correct or not?

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- \( A_n \in [\mu - 2\sigma \frac{1}{\sqrt{n}}, \mu + 2\sigma \frac{1}{\sqrt{n}}] \) with prob \( \geq 95\% \). Yes
- If \( 0.3 < \sigma < 3 \), then
  \[ A_n \in [\mu - 0.6 \frac{1}{\sqrt{n}}, \mu + 0.6 \frac{1}{\sqrt{n}}] \text{ with prob } \geq 95\%. \]
Correct or not?

\[ A_n = \frac{\sum_{i=0}^{n} X_i}{n}, \text{ } X_i \text{ i.i.d, mean } \mu \text{ and variance } \sigma. \]

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  \[ A_n \in [\mu - 0.6 \frac{1}{\sqrt{n}}, \mu + 0.6 \frac{1}{\sqrt{n}}] \text{ with prob } \geq 95\%. \text{ No} \]
Correct or not?

\[ A_n = \frac{\sum_{i=0}^{n} X_i}{n}, \] for \( i.i.d. \), mean \( \mu \) and variance \( \sigma \).

When \( n \gg 1 \), one has

- \( A_n \in [\mu - 2\sigma \frac{1}{n}, \mu + 2\sigma \frac{1}{n}] \) with prob \( \geq 95\% \). No

- \( A_n \in [\mu - 2\sigma \frac{1}{\sqrt{n}}, \mu + 2\sigma \frac{1}{\sqrt{n}}] \) with prob \( \geq 95\% \). Yes

- If \( 0.3 < \sigma < 3 \), then
  \[ A_n \in [\mu - 0.6 \frac{1}{\sqrt{n}}, \mu + 0.6 \frac{1}{\sqrt{n}}] \] with prob \( \geq 95\% \). No

- If \( 0.3 < \sigma < 3 \), then
  \[ A_n \in [\mu - 6 \frac{1}{\sqrt{n}}, \mu + 6 \frac{1}{\sqrt{n}}] \] with prob \( \geq 95\% \).
Correct or not?

\[ A_n = \frac{\sum_{i=0}^{n} X_i}{n}, \quad X_i \text{ i.i.d, mean } \mu \text{ and variance } \sigma. \]

When \( n \gg 1 \), one has

- \( A_n \in [\mu - 2\sigma \frac{1}{\sqrt{n}}, \mu + 2\sigma \frac{1}{\sqrt{n}}] \) with prob \( \geq 95\% \). No
- \( A_n \in [\mu - 2\sigma \frac{1}{\sqrt{n}}, \mu + 2\sigma \frac{1}{\sqrt{n}}] \) with prob \( \geq 95\% \). Yes
- If \( 0.3 < \sigma < 3 \), then
  \[ A_n \in [\mu - 0.6 \frac{1}{\sqrt{n}}, \mu + 0.6 \frac{1}{\sqrt{n}}] \] with prob \( \geq 95\% \). No
- If \( 0.3 < \sigma < 3 \), then
  \[ A_n \in [\mu - 6 \frac{1}{\sqrt{n}}, \mu + 6 \frac{1}{\sqrt{n}}] \] with prob \( \geq 95\% \). Yes
$A_n = \frac{\sum_{i=0}^{n} X_i}{n}$, $X_i$ i.i.d, mean $\mu$ and variance $\sigma$.

When $n \gg 1$, one has

- $A_n \in [\mu - 2\sigma \frac{1}{n}, \mu + 2\sigma \frac{1}{n}]$ with prob $\geq 95\%$. No
- $A_n \in [\mu - 2\sigma \frac{1}{\sqrt{n}}, \mu + 2\sigma \frac{1}{\sqrt{n}}]$ with prob $\geq 95\%$. Yes

- If $0.3 < \sigma < 3$, then
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- If $0.3 < \sigma < 3$, then
  $A_n \in [\mu - 6\frac{1}{\sqrt{n}}, \mu + 6\frac{1}{\sqrt{n}}]$ with prob $\geq 95\%$. Yes
[1] $Pr[X \geq z] \leq \frac{E[f(X)]}{f(a)}$.

[2] $Pr[|X - E[X]| \geq z] \leq \frac{\text{Var}(X)}{z^2}$.

[3] $\sum_y y Pr[Y = y | X = x]$.

[4] $Pr[|\frac{X_1 + X_2 + \cdots + X_n}{n} - E[X_1]| \geq \varepsilon] \to 0$

[5] $Pr[|\frac{X_1 + X_2 + \cdots + X_n}{n} - E[X_1]| \leq \varepsilon] \to 1$
[1] \( Pr[X \geq z] \leq \frac{E[f(X)]}{f(a)} \).

[2] \( Pr[|X - E[X]| \geq z] \leq \frac{\text{Var}(X)}{z^2} \).

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[4] \( Pr[\left| \frac{X_1 + X_2 + \cdots + X_n}{n} - E[X_1] \right| \geq \varepsilon] \to 0 \)

[5] \( Pr[\left| \frac{X_1 + X_2 + \cdots + X_n}{n} - E[X_1] \right| \leq \varepsilon] \to 1 \)

▶ WLLN
[1] $Pr[X \geq z] \leq \frac{E[f(X)]}{f(a)}$.

[2] $Pr[|X - E[X]| \geq z] \leq \frac{\text{Var}(X)}{z^2}$.

[3] $\sum_y yPr[Y = y|X = x]$.

[4] $Pr[|\frac{X_1 + X_2 + \cdots + X_n}{n} - E[X_1]| \geq \varepsilon] \rightarrow 0$

[5] $Pr[|\frac{X_1 + X_2 + \cdots + X_n}{n} - E[X_1]| \leq \varepsilon] \rightarrow 1$

▶ WLLN (4)
[1] $Pr[X \geq z] \leq \frac{E[f(X)]}{f(a)}$.

[2] $Pr[|X - E[X]| \geq z] \leq \frac{\text{Var}(X)}{z^2}$.

[3] $\sum_y yPr[Y = y|X = x]$.

[4] $Pr[|\frac{X_1 + X_2 + \cdots + X_n}{n} - E[X_1]| \geq \varepsilon] \rightarrow 0$

[5] $Pr[|\frac{X_1 + X_2 + \cdots + X_n}{n} - E[X_1]| \leq \varepsilon] \rightarrow 1$

- WLLN (4) and (5)
- Chebyshev
[1] \( Pr[X \geq z] \leq \frac{E[f(X)]}{f(a)} \).

[2] \( Pr[|X - E[X]| \geq z] \leq \frac{Var(X)}{z^2} \).

[3] \( \sum_y yPr[Y = y|X = x] \).

[4] \( Pr[|\frac{X_1 + X_2 + \cdots + X_n}{n} - E[X_1]| \geq \varepsilon] \rightarrow 0 \)

[5] \( Pr[|\frac{X_1 + X_2 + \cdots + X_n}{n} - E[X_1]| \leq \varepsilon] \rightarrow 1 \)

▶ WLLN (4) and (5)

▶ Chebyshev (2)
[1] \( \Pr[X \geq z] \leq \frac{E[f(X)]}{f(a)} \).

[2] \( \Pr[|X - E[X]| \geq z] \leq \frac{\text{Var}(X)}{z^2} \).

[3] \( \sum_y y \Pr[Y = y | X = x] \).

[4] \( \Pr[\left| \frac{X_1 + X_2 + \cdots + X_n}{n} - E[X_1] \right| \geq \varepsilon] \to 0 \)

[5] \( \Pr[\left| \frac{X_1 + X_2 + \cdots + X_n}{n} - E[X_1] \right| \leq \varepsilon] \to 1 \)

- WLLN (4) and (5)
- Chebyshev (2)
- Markov’s inequality
[1] $\Pr[X \geq z] \leq \frac{E[f(X)]}{f(a)}$.

[2] $\Pr[|X - E[X]| \geq z] \leq \frac{\text{Var}(X)}{z^2}$.

[3] $\sum_y y \Pr[Y = y | X = x]$.

[4] $\Pr[|\frac{X_1 + X_2 + \cdots + X_n}{n} - E[X_1]| \geq \varepsilon] \to 0$

[5] $\Pr[|\frac{X_1 + X_2 + \cdots + X_n}{n} - E[X_1]| \leq \varepsilon] \to 1$

▶ WLLN (4) and (5)

▶ Chebyshev (2)

▶ Markov’s inequality (1)

▶ $E[Y | X = x]$
[1] \( \Pr[X \geq z] \leq \frac{E[f(X)]}{f(a)} \).

[2] \( \Pr[|X - E[X]| \geq z] \leq \frac{\text{Var}(X)}{z^2} \).

[3] \( \sum_y y \Pr[Y = y | X = x] \).

[4] \( \Pr[|\frac{X_1 + X_2 + \cdots + X_n}{n} - E[X_1]| \geq \varepsilon] \to 0 \)

[5] \( \Pr[|\frac{X_1 + X_2 + \cdots + X_n}{n} - E[X_1]| \leq \varepsilon] \to 1 \)

- WLLN (4) and (5)
- Chebyshev (2)
- Markov’s inequality (1)
- \( E[Y | X = x] \) (3)
Quiz 1: G

1. What is $\Pr[A|B]$?
   $\Pr[A|B] = \frac{\Pr[A \cap B]}{\Pr[B]} = 0.407$.

2. What is $\Pr[B|A]$?
   $\Pr[B|A] = \frac{\Pr[A \cap B]}{\Pr[A]} = 0.406 \times 0.7$.

3. Are $A$ and $B$ positively correlated?
   No.
   $\Pr[A \cap B] = 0.4 < \Pr[A] \Pr[B] = 0.407 \times 0.607$. 
Quiz 1: G

1. What is $\Pr[A \mid B]$?

$$
\Pr[A \mid B] = \frac{\Pr[A \cap B]}{\Pr[B]} = 0.40
$$

2. What is $\Pr[B \mid A]$?

$$
\Pr[B \mid A] = \frac{\Pr[A \cap B]}{\Pr[A]} = 0.60 	imes 0.7
$$

3. Are $A$ and $B$ positively correlated?

No.

$$
\Pr[A \cap B] = 0.4 < \Pr[A] \Pr[B] = 0.6 \times 0.7
$$
1. What is $P[A|B]$?
1. What is $P[A|B]$?

$Pr[A|B] =$

\[ Pr[A|B] = \frac{Pr[A \cap B]}{Pr[B]} \]

\[ = \frac{0.12}{0.37} \approx 0.3243 \]
1. What is $P[A|B]$?

$$Pr[A|B] = \frac{Pr[A \cap B]}{Pr[B]} =$$
1. What is $P[A|B]$?

$$Pr[A|B] = \frac{Pr[A \cap B]}{Pr[B]} = \frac{0.4}{0.7}$$
1. What is $P[A|B]$?

$$Pr[A|B] = \frac{Pr[A \cap B]}{Pr[B]} = \frac{0.4}{0.7}$$

2. What is $Pr[B|A]$?
Quiz 1: G

1. What is $P[A|B]$?

$$Pr[A|B] = \frac{Pr[A \cap B]}{Pr[B]} = \frac{0.4}{0.7}$$

2. What is $Pr[B|A]$?

$$Pr[B|A] =$$
1. What is \( P[A|B] \)?

\[
Pr[A|B] = \frac{Pr[A \cap B]}{Pr[B]} = \frac{0.4}{0.7}
\]

2. What is \( Pr[B|A] \)?

\[
Pr[B|A] = \frac{Pr[A \cap B]}{Pr[A]} =
\]
Quiz 1: G

1. What is $P[A|B]$?

$$Pr[A|B] = \frac{Pr[A \cap B]}{Pr[B]} = \frac{0.4}{0.7}$$

2. What is $Pr[B|A]$?

$$Pr[B|A] = \frac{Pr[A \cap B]}{Pr[A]} = \frac{0.4}{0.6}$$
1. What is $P[A|B]$?

$$Pr[A|B] = \frac{Pr[A \cap B]}{Pr[B]} = \frac{0.4}{0.7}$$

2. What is $Pr[B|A]$?

$$Pr[B|A] = \frac{Pr[A \cap B]}{Pr[A]} = \frac{0.4}{0.6}$$

3. Are $A$ and $B$ positively correlated?
1. What is $P[A|B]$?

\[ Pr[A|B] = \frac{Pr[A \cap B]}{Pr[B]} = \frac{0.4}{0.7} \]

2. What is $Pr[B|A]$?

\[ Pr[B|A] = \frac{Pr[A \cap B]}{Pr[A]} = \frac{0.4}{0.6} \]

3. Are $A$ and $B$ positively correlated?

No.
1. What is $P[A|B]$?

$$Pr[A|B] = \frac{Pr[A \cap B]}{Pr[B]} = \frac{0.4}{0.7}$$

2. What is $Pr[B|A]$?

$$Pr[B|A] = \frac{Pr[A \cap B]}{Pr[A]} = \frac{0.4}{0.6}$$

3. Are $A$ and $B$ positively correlated?

No. $Pr[A \cap B] = 0.4 < Pr[A]Pr[B] = 0.6 \times 0.7$. 
Quiz 1: G

4. What is $\text{cov}(X, Y)$?

$$\text{cov}(X, Y) = E[XY] - E[X]E[Y] = 0.8 - 0.6 \times 1.4 = -0.04$$
Quiz 1: G

4. What is $\text{cov}(X, Y)$?

$\text{cov}(X, Y) = E[XY] - E[X]E[Y] = 0.8 - 0.6 \times 1.4 = -0.04$
4. What is \( \text{cov}(X, Y) \)?
4. What is \( \text{cov}(X, Y) \)?

\[
\text{cov}(X, Y) = \frac{\sum_{ij} (x_i - \mu_X)(y_j - \mu_Y) p_{ij}}{\sqrt{\sum_{i} (x_i - \mu_X)^2 p_i} \sqrt{\sum_{i} (y_j - \mu_Y)^2 p_j}}
\]
4. What is $\text{cov}(X, Y)$?

$$
\text{cov}(X, Y) = E[XY] - E[X]E[Y]
$$

= 
4. What is \( \text{cov}(X, Y) \)?

\[
= 0.8 - 0.6 \times 1.4 =
\]
4. What is $\text{cov}(X, Y)$?

$$\text{cov}(X, Y) = E[XY] - E[X]E[Y]$$

$$= 0.8 - 0.6 \times 1.4 = -0.04$$
4. What is $\text{cov}(X, Y)$?

\[
\text{cov}(X, Y) = E[XY] - E[X]E[Y]
\]

\[
= 0.8 - 0.6 \times 1.4 = -0.04
\]
Quiz 1: G

7. Is this Markov chain irreducible? Yes.

8. Is this Markov chain periodic? No. The return times to 3 are $\{3, 5, \ldots\}$: coprime!

9. Does $\pi_n$ converge to a value independent of $\pi_0$? Yes!

10. Does $\frac{1}{n} \sum_{m=1}^{n-1} \mathbb{1}_{X_m = 3}$ converge as $n \to \infty$? Yes!

11. Calculate $\pi$.

Let $a = \pi(1)$. Then $a = \pi(5)$, $\pi(2) = 0 \cdot a$, $\pi(4) = \pi(2) = 0 \cdot a$, $\pi(3) = 0 \cdot a$, $\pi(1) + \pi(4) = a$. Thus, $\pi = [a, 0.5a, a, 0.5a, a] = [1, 0.5, 1, 0.5, 1]$, so $a = \frac{1}{4}$.
Quiz 1: G

7. Is this Markov chain irreducible?
Yes.

8. Is this Markov chain periodic?
No. The return times to 3 are {3, 5}: coprime!

9. Does $\pi_n$ converge to a value independent of $\pi_0$?
Yes!

10. Does $\frac{1}{n} \sum_{m=1}^{n-1} X_m = 1$ converge as $n \to \infty$?
Yes!

11. Calculate $\pi$.
Let $a = \pi(1)$. Then

\[
\begin{align*}
a &= \pi(5), \\
\pi(2) &= 0, \\
\pi(4) &= \pi(2) = 0, \\
\pi(3) &= 0, \\
\pi(1) + \pi(4) &= a.
\end{align*}
\]

Thus, $\pi = [a, 0, 0.5a, a, 0.5a] = [1, 0, 0, 0]$, so $a = \frac{1}{4}$. 
7. Is this Markov chain irreducible?
7. Is this Markov chains irreducible? Yes.
Quiz 1: G

7. Is this Markov chains irreducible? Yes.

8. Is this Markov chain periodic?
Quiz 1: G

7. Is this Markov chain irreducible? Yes.
8. Is this Markov chain periodic?
   No.
7. Is this Markov chains irreducible? Yes.

8. Is this Markov chain periodic?
   No. The return times to 3 are
7. Is this Markov chains irreducible? Yes.

8. Is this Markov chain periodic?

No. The return times to 3 are \{3, 5, \ldots\}:
7. Is this Markov chain irreducible? Yes.

8. Is this Markov chain periodic?
   No. The return times to 3 are \{3, 5, \ldots\}: coprime!
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   No. The return times to 3 are \( \{3, 5, \ldots\} \): coprime!
9. Does \( \pi_n \) converge to a value independent of \( \pi_0 \)?
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8. Is this Markov chain periodic?
   No. The return times to 3 are \( \{3, 5, \ldots\} \): coprime!
9. Does \( \pi_n \) converge to a value independent of \( \pi_0 \)? Yes!
Quiz 1: G

7. Is this Markov chains irreducible? Yes.
8. Is this Markov chain periodic?
   No. The return times to 3 are \{3, 5, ..\}: coprime!
9. Does \( \pi_n \) converge to a value independent of \( \pi_0 \)? Yes!
10. Does \( \frac{1}{n} \sum_{m=1}^{n-1} 1\{X_m = 3\} \) converge as \( n \to \infty \)?
Quiz 1: G

7. Is this Markov chains irreducible? Yes.
8. Is this Markov chain periodic?
   No. The return times to 3 are \{3,5,..\}: coprime!
9. Does \( \pi_n \) converge to a value independent of \( \pi_0 \)? Yes!
10. Does \( \frac{1}{n} \sum_{m=1}^{n-1} 1\{X_m = 3\} \) converge as \( n \to \infty \)? Yes!
7. Is this Markov chains irreducible? Yes.
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   No. The return times to 3 are \{3,5,..\}: coprime!
9. Does \(\pi_n\) converge to a value independent of \(\pi_0\)? Yes!
10. Does \(\frac{1}{n} \sum_{m=1}^{n-1} 1\{X_m = 3\}\) converge as \(n \to \infty\)? Yes!
11. Calculate \(\pi\).
7. Is this Markov chains irreducible? **Yes.**
8. Is this Markov chain periodic?
   
   No. The return times to 3 are \{3, 5, ..\}: coprime!
9. Does \( \pi_n \) converge to a value independent of \( \pi_0 \)? **Yes!**
10. Does \( \frac{1}{n} \sum_{m=1}^{n-1} 1\{X_m = 3\} \) converge as \( n \to \infty \)? **Yes!**
11. Calculate \( \pi \).
    
    Let \( a = \pi(1) \).
7. Is this Markov chains irreducible? Yes.

8. Is this Markov chain periodic?

   No. The return times to 3 are \{3, 5, ..\}: coprime!

9. Does \( \pi_n \) converge to a value independent of \( \pi_0 \)? Yes!

10. Does \( \frac{1}{n} \sum_{m=1}^{n-1} 1\{X_m = 3\} \) converge as \( n \to \infty \)? Yes!

11. Calculate \( \pi \).

    Let \( a = \pi(1) \). Then \( a = \pi(5) \),
7. Is this Markov chain irreducible? **Yes.**

8. Is this Markov chain periodic?
   
   **No.** The return times to 3 are \{3, 5, ..\}: coprime!

9. Does \(\pi_n\) converge to a value independent of \(\pi_0\)? **Yes!**

10. Does \(\frac{1}{n} \sum_{m=1}^{n-1} 1\{X_m = 3\}\) converge as \(n \to \infty\)? **Yes!**

11. Calculate \(\pi\).

   Let \(a = \pi(1)\). Then \(a = \pi(5), \pi(2) = 0.5a,\)
Quiz 1: G

7. Is this Markov chains irreducible? **Yes.**

8. Is this Markov chain periodic?
   
   **No.** The return times to 3 are {3, 5,..}: coprime!

9. Does $\pi_n$ converge to a value independent of $\pi_0$? **Yes!**

10. Does $\frac{1}{n} \sum_{m=1}^{n-1} 1\{X_m = 3\}$ converge as $n \to \infty$? **Yes!**

11. Calculate $\pi$.

   Let $a = \pi(1)$. Then $a = \pi(5), \pi(2) = 0.5a,
   
   $\pi(4) = \pi(2) = 0.5a,$
7. Is this Markov chains irreducible? Yes.

8. Is this Markov chain periodic?
   No. The return times to 3 are \{3, 5, ..\}: coprime!

9. Does \( \pi_n \) converge to a value independent of \( \pi_0 \)? Yes!

10. Does \( \frac{1}{n} \sum_{m=1}^{n-1} 1\{X_m = 3\} \) converge as \( n \to \infty \)? Yes!

11. Calculate \( \pi \).
    
    Let \( a = \pi(1) \). Then \( a = \pi(5) \), \( \pi(2) = 0.5a \),
    \( \pi(4) = \pi(2) = 0.5a \), \( \pi(3) = 0.5\pi(1) + \pi(4) = a \).
7. Is this Markov chains irreducible? Yes.
8. Is this Markov chain periodic?
   No. The return times to 3 are \{3, 5, \ldots\}: coprime!
9. Does \( \pi_n \) converge to a value independent of \( \pi_0 \)? Yes!
10. Does \( \frac{1}{n} \sum_{m=1}^{n-1} 1\{X_m = 3\} \) converge as \( n \to \infty \)? Yes!
11. Calculate \( \pi \).

   Let \( a = \pi(1) \). Then \( a = \pi(5), \pi(2) = 0.5a, \)
   \( \pi(4) = \pi(2) = 0.5a, \pi(3) = 0.5\pi(1) + \pi(4) = a. \)
   Thus, \( \pi = [a, 0.5a, a, 0.5a, a] = [1, 0.5, 1, 0.5, 1]a, \) so \( a = \)}
7. Is this Markov chain irreducible? Yes.

8. Is this Markov chain periodic?

   No. The return times to 3 are \(\{3, 5, \ldots\}\): coprime!

9. Does \(\pi_n\) converge to a value independent of \(\pi_0\)? Yes!

10. Does \(\frac{1}{n} \sum_{m=1}^{n-1} 1\{X_m = 3\}\) converge as \(n \to \infty\)? Yes!

11. Calculate \(\pi\).

   Let \(a = \pi(1)\). Then \(a = \pi(5), \pi(2) = 0.5a, \pi(4) = \pi(2) = 0.5a, \pi(3) = 0.5\pi(1) + \pi(4) = a\).

   Thus, \(\pi = [a, 0.5a, a, 0.5a, a] = [1, 0.5, 1, 0.5, 1]a\), so \(a = 1/4\).
Quiz 1: G

12. Write the first step equations for calculating the mean time from 1 to 4.

\[ \beta(1) = 1 + 0.5 \times 1 \]
\[ \beta(2) = 1 + 0.5 \times (1 + \beta(1)) \]
\[ \beta(3) = 1 + 0.5 \times (1 + \beta(1)) \]
\[ \beta(5) = 1 + 0.5 \times (1 + \beta(1)) \]

13. Solve these equations.

\[ \beta(1) = 1 + 0.5 \times 1 \]
\[ \beta(1) = 2.5 + 0.5 \beta(1) \]
\[ \beta(1) = 5 \]
Quiz 1: G

12. Write the first step equations for calculating the mean time from 1 to 4.

\[ \beta(1) = \frac{1}{2} \]

\[ \beta(2) + \frac{1}{2} \beta(3) = \beta(1) \]

\[ \beta(3) = \frac{1}{2} + \beta(5) \]

\[ \beta(5) = \frac{1}{2} + \beta(1) \]

13. Solve these equations.

\[ \beta(1) = \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times (1 + \beta(1)) = 2.5 \times \frac{1}{2} + \frac{1}{2} \beta(1) \]

Hence, \[ \beta(1) = 0.5 \times 5 = 2.5 \]
12. Write the first step equations for calculating the mean time from 1 to 4.
12. Write the first step equations for calculating the mean time from 1 to 4.

\[ \beta(1) = 1 + 0.5\beta(2) + 0.5\beta(3) \]
12. Write the first step equations for calculating the mean time from 1 to 4.

\[ \beta(1) = 1 + 0.5\beta(2) + 0.5\beta(3) \]
\[ \beta(2) = 1 \]
12. Write the first step equations for calculating the mean time from 1 to 4.

\[ \beta(1) = 1 + 0.5\beta(2) + 0.5\beta(3) \]
\[ \beta(2) = 1 \]
\[ \beta(3) = 1 + \beta(5) \]
12. Write the first step equations for calculating the mean time from 1 to 4.

\[ \beta(1) = 1 + 0.5\beta(2) + 0.5\beta(3) \]
\[ \beta(2) = 1 \]
\[ \beta(3) = 1 + \beta(5) \]
\[ \beta(5) = 1 + \beta(1). \]
12. Write the first step equations for calculating the mean time from 1 to 4.

\[
\begin{align*}
\beta(1) &= 1 + 0.5\beta(2) + 0.5\beta(3) \\
\beta(2) &= 1 \\
\beta(3) &= 1 + \beta(5) \\
\beta(5) &= 1 + \beta(1).
\end{align*}
\]

13. Solve these equations.
12. Write the first step equations for calculating the mean time from 1 to 4.

\[ \beta(1) = 1 + 0.5\beta(2) + 0.5\beta(3) \]
\[ \beta(2) = 1 \]
\[ \beta(3) = 1 + \beta(5) \]
\[ \beta(5) = 1 + \beta(1). \]

13. Solve these equations.

\[ \beta(1) = 1 + 0.5 \times 1 + 0.5 \times (1 + (1 + \beta(1))) \]
12. Write the first step equations for calculating the mean time from 1 to 4.

\[ \beta(1) = 1 + 0.5\beta(2) + 0.5\beta(3) \]
\[ \beta(2) = 1 \]
\[ \beta(3) = 1 + \beta(5) \]
\[ \beta(5) = 1 + \beta(1). \]

13. Solve these equations.

\[ \beta(1) = 1 + 0.5 \times 1 + 0.5 \times (1 + (1 + \beta(1))) \]
\[ = 2.5 + 0.5\beta(1). \]
12. Write the first step equations for calculating the mean time from 1 to 4.

\[ \beta(1) = 1 + 0.5\beta(2) + 0.5\beta(3) \]
\[ \beta(2) = 1 \]
\[ \beta(3) = 1 + \beta(5) \]
\[ \beta(5) = 1 + \beta(1). \]

13. Solve these equations.

\[ \beta(1) = 1 + 0.5 \times 1 + 0.5 \times (1 + (1 + \beta(1))) \]
\[ = 2.5 + 0.5\beta(1). \]

Hence, \( \beta(1) = 5. \)
14. Which is $E[Y|X]$? Blue, red or green?
Quiz 1: G

14. Which is $E[Y|X]$? Blue, red or green?

Given $X=x$, $Y = U[a(x), b(x)]$.

Thus, $E[Y|X=x] = a(x) + b(x)$.
14. Which is $E[Y|X]$? Blue, red or green?

Answer: Red.
14. Which is $E[Y|X]$? Blue, red or green?

Answer: Red.
Given $X = x$, $Y = U[a(x), b(x)]$. 

Thus, $E[Y|X] = x \rightarrow a(x) + b(x)$. 

Quiz 1: G
14. Which is $E[Y|X]$? Blue, red or green?

Answer: Red.

Given $X = x$, $Y = U[a(x), b(x)]$. Thus, $E[Y|X = x] = \frac{a(x)+b(x)}{2}$. 
Quiz 2: PG

1. Find \((x, y)\) so that \(A\) and \(B\) are independent.

   We need \(\Pr[A \cap B] = \Pr[A] \cdot \Pr[B]\)

   That is, \(0.2 = (y + 0.2) \times 0.5\)

   Hence, \(y = 0.2\) and \(x = 0.3\).

2. Find the value of \(x\) that maximizes \(\Pr[B|A]\).

   When \(x = 0.5\), \(\Pr[B|A] = 1\).
Quiz 2: PG

1. Find \((x, y)\) so that \(A\) and \(B\) are independent.

We need \(\Pr[A \cap B] = \Pr[A] \cdot \Pr[B]\).

That is, 

\[
0 \cdot 0.2 = (y + 0.2) \cdot 0.5 = 0.5 y + 0.1.
\]

Hence, 

\[y = 0.2\] and \[x = 0.3\].

2. Find the value of \(x\) that maximizes \(\Pr[B | A]\).

When \(x = 0.5\), \(\Pr[B | A] = 1\).
1. Find \((x, y)\) so that \(A\) and \(B\) are independent.
1. Find \((x, y)\) so that \(A\) and \(B\) are independent. We need

\[
Pr[A \cap B] = Pr[A]Pr[B]
\]
1. Find \((x, y)\) so that \(A\) and \(B\) are independent.

We need

\[
Pr[A \cap B] = Pr[A] \cdot Pr[B]
\]

That is,

\[
0.2 = (y + 0.2) \times 0.5 =
\]
1. Find \((x, y)\) so that \(A\) and \(B\) are independent.

We need

\[ Pr[A \cap B] = Pr[A] \cdot Pr[B] \]

That is,

\[ 0.2 = (y + 0.2) \times 0.5 = 0.5y + 0.1 \]
1. Find \((x, y)\) so that \(A\) and \(B\) are independent.

We need

\[ Pr[A \cap B] = Pr[A]Pr[B] \]

That is,

\[ 0.2 = (y + 0.2) \times 0.5 = 0.5y + 0.1 \]

Hence,

\[ y = 0.2 \]
1. Find \((x, y)\) so that \(A\) and \(B\) are independent.

We need

\[
Pr[A \cap B] = Pr[A]Pr[B]
\]

That is,

\[
0.2 = (y + 0.2) \times 0.5 = 0.5y + 0.1
\]

Hence,

\[y = 0.2\] and \(x = \]
Quiz 2: PG

1. Find \((x, y)\) so that \(A\) and \(B\) are independent.

   We need
   \[
   Pr[A \cap B] = Pr[A]Pr[B]
   \]
   That is,
   \[
   0.2 = (y + 0.2) \times 0.5 = 0.5y + 0.1
   \]
   Hence,
   \[
   y = 0.2 \text{ and } x = 0.3.
   \]
Quiz 2: PG

1. Find \((x, y)\) so that \(A\) and \(B\) are independent.
   
   We need
   \[
   Pr[A \cap B] = Pr[A]Pr[B]
   \]
   That is,
   \[
   0.2 = (y + 0.2) \times 0.5 = 0.5y + 0.1
   \]
   Hence,
   \[
   y = 0.2 \text{ and } x = 0.3.
   \]

2. Find the value of \(x\) that maximizes \(Pr[B|A]\).
1. Find \((x, y)\) so that \(A\) and \(B\) are independent.

We need

\[
Pr[A \cap B] = Pr[A]Pr[B]
\]

That is,

\[
0.2 = (y + 0.2) \times 0.5 = 0.5y + 0.1
\]

Hence,

\[
y = 0.2 \text{ and } x = 0.3.
\]

2. Find the value of \(x\) that maximizes \(Pr[B|A]\).

When \(x = \)
1. Find \((x, y)\) so that \(A\) and \(B\) are independent.

We need 
\[
Pr[A \cap B] = Pr[A]Pr[B]
\]
That is, 
\[
0.2 = (y + 0.2) \times 0.5 = 0.5y + 0.1
\]
Hence, 
\[
y = 0.2\text{ and } x = 0.3.
\]

2. Find the value of \(x\) that maximizes \(Pr[B|A]\).

When \(x = 0.5\), \(Pr[B|A] = 1\).
Quiz 2: PG

3. Find $\alpha$ so that $X$ and $Y$ are independent. We need $\Pr[X = 0, Y = 0] = \Pr[X = 0] \Pr[Y = 0].$

Hence, $\alpha = 0.233.$

Typically, check $\Pr[X = 2, Y = 0] = \Pr[X = 2] \Pr[Y = 0]$ and so on.

But: $A$ and $B$ independent $\iff A, B$ independent. Take: $A = \text{"}X = 0\text{"}$ and $B = \text{"}Y = 0\text{"},$ since only two values for $X, Y.$
Find $\alpha$ so that $X$ and $Y$ are independent.

We need $\Pr[X = 0, Y = 0] = \Pr[X = 0] \Pr[Y = 0]$. That is,

$$0.1 \times (0.1 + \alpha) \times (0.1 + 0.2) = 0.03 + 0.3 \alpha$$

Hence, $\alpha = 0.233$.

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$A$ and $B$ independent $\iff$ $A$, $B$ independent.

Take: $A = \{X = 0\}$ and $B = \{Y = 0\}$, since only two values for $X, Y$. 

<table>
<thead>
<tr>
<th>$\omega$</th>
<th>$X(\omega)$</th>
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<tr>
<td>a</td>
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Quiz 2: PG

A great student solves each question correctly with a probability of 0.8 whereas a good student does it with a probability of 0.6. One student got right 70% of the 10 questions on Midterm 1 and 70% of the 10 questions on Midterm 2. What is the expected score of the student on the final?

\[ p = \Pr[\text{great} | \text{solves}] = 0.3 (0.8) + 0.7 (0.6) \approx 0.27 \]

Expected score = \[ 80\% + (1-p) 60\% \approx 65\% \]
Quiz 2: PG

4. A CS70 student is great w.p. 0.3 and good w.p. 0.7.
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\[ p := Pr[\text{great} | \text{scores}] = \]

\[
\begin{array}{c}
\binom{20}{14} 0.8^{14} 0.2^6 \\
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\end{array}
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Expected score \(= p80\% + (1 - p)60\% \approx 65\%\).
You roll a balanced six-sided die 20 times. Use CLT to upper-bound the probability that the total number of dots exceeds 85.

Let \( X = X_1 + \cdots + X_{20} \) be the total number of dots. Then

\[
X - 70 \approx N(0, 1)
\]

where

\[
\sigma^2 = \text{var}(X_1) = \frac{1}{6} \cdot 6 \sum_{m=1}^{6} m^2 - 3.5^2 \approx 2.9.
\]

Now,

\[
\Pr[X > 85] = \Pr[X - 70 > 15]
\]

\[
= \Pr[X - 70 > 1.7 \times 4.5]
\]

\[
= \Pr[X - 70 > 2]\]

\[
\approx 2.5\%.
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Now,

$$Pr[X > 85] = Pr[X - 70 > 15]$$
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\[
\Pr[X > 85] = \Pr[X - 70 > 15] = \Pr\left[ \frac{X - 70}{1.7 \times 4.5} > \frac{15}{1.7 \times 4.5} \right]
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Quiz 2: PG

7. You roll a balanced six-sided die 20 times. Use Chebyshev to upper-bound the probability that the total number of dots exceeds 85. Let $X = X_1 + \cdots + X_{20}$ be the total number of dots. Then

$$\Pr[X > 85] = \Pr[X - 70 > 15] \leq \Pr[|X - 70| > 15] \leq \frac{\text{var}(X)}{15^2}.$$ 

Now, $\text{var}(X) = 20 \times \text{var}(X_1) = 20 \times 2.9 = 58$. Hence,

$$\Pr[X > 85] \leq \frac{58}{15^2} \approx 0.26.$$
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Hence,

$$Pr[X > 85] \leq \frac{58}{15^2} \approx 0.26.$$
1. The lifespans of good lightbulbs are exponentially distributed with mean 1 year. Those of defective bulbs are exponentially distributed with mean 0.8 year.

All the bulbs in one batch are equally likely to be good or defective.

You test one bulb and note that it burns out after 0.6 year.

(a) What is the probability you got a batch of good bulbs?

(b) What is the expected lifespan of another bulb in that batch? Hint: If $X = \text{Expo}(\lambda)$, $f_X(x) = \lambda e^{-\lambda x}1\{x > 0\}$, $E[X] = 1/\lambda$.

Let $X$ be the lifespan of a bulb, $G$ the event that it is good, and $B$ the event that it is bad.

(a) $p := \Pr\{G | X \in (0.6, 0.6 + \delta)\} = 0.5$

(b) $E[\text{lifespan of other bulb}] = p \times 1 + (1 - p) \times 0.8 \approx 0.9$. 

\[e^{-0.6 \delta}e^{-0.6 \delta} + (0.8) - 1 e^{-0.8 - 1}0.6 \delta \approx 0.488.\]
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Quiz 3: R

1. The lifespans of good lightbulbs are exponentially distributed with mean 1 year. Those of defective bulbs are exponentially distributed with mean 0.8. All the bulbs in one batch are equally likely to be good or defective.

(a) What is the probability you got a batch of good bulbs?

(b) What is the expected lifespan of another bulb in that batch?

Hint: If $X = \text{Exp}(\lambda)$, $f_X(x) = \lambda e^{-\lambda x} 1\{x > 0\}$, $E[X] = 1/\lambda$.

Let $X$ be the lifespan of a bulb, $G$ the event that it is good, and $B$ the event that it is bad.

(a) $p := \Pr[G | X \in (0.6, 0.6 + \delta)] = 0.5$.

$b \Pr[X \in (0.6, 0.6 + \delta) | G] + 0.5 \Pr[X \in (0.6, 0.6 + \delta) | D] = e^{-0.6 \delta} e^{-0.6 \delta} + (0.8) - 1 e^{-0.8 \delta} - 1 0.6 \delta \approx 0.488$.

(b) $E[\text{lifespan of other bulb}] = p \times 1 + (1 - p) \times 0.8 \approx 0.9$. 
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Let $X$ be the lifespan of a bulb, $G$ the event that it is good, and $B$ the event that it is bad.

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If \( X = \text{Expo}(\lambda) \), then 
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Hint: If $X = \text{Expo}(\lambda)$, $f_X(x) = \lambda e^{-\lambda x} 1\{x > 0\}$, $E[X] = 1/\lambda$.

Let $X$ be the lifespan of a bulb, $G$ the event that it is good,
Quiz 3: R

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\[ = \frac{0.5 Pr[X \in (0.6, 0.6 + \delta)|G]}{0.5 Pr[X \in (0.6, 0.6 + \delta)|G] + 0.5 Pr[X \in (0.6, 0.6 + \delta)|D]} \]
Quiz 3: R

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Quiz 3: R

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(b) $E[ \text{lifespan of other bulb } ] = p \times 1 + (1 - p) \times 0.8 \approx 0.9$. 

Quiz 3: R

2. In the figure, 1, 2, 3, 4 are links that fail after i.i.d. times that are $U[0, 1]$. Find the average time until A and B are disconnected.

Let $X_k$ be the lifespan of link $k$, for $k = 1, \ldots, 4$. We are looking for $E[Z]$ where $Z = \max\{Y_1, Y_2\}$ with $Y_1 = \min\{X_1, X_2\}$ and $Y_2 = \min\{X_3, X_4\}$. 

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\Pr[Y_1 > t] = \Pr[X_1 > t] \Pr[X_2 > t] = (1 - t)^2
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f_Z(t) = 8t - 12t^2 + 4t^3
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E[Z] = \int_0^1 tf_Z(t)\,dt = \frac{8}{3} - \frac{12}{4} + \frac{4}{5} \approx 0.4667.
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$E[Z] = \int_0^1 tf_Z(t) \, dt = \frac{81}{3} - \frac{121}{4} + \frac{41}{5} \approx 0.4667$. 
2. In the figure, 1, 2, 3, 4 are links that fail after i.i.d. times that are $U[0,1]$. 

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\begin{align*}
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&= 4t^2 - 4t^3 + t^4 \\
\int_0^1 tf[Z](t) \, dt &= 8t - 12t^2 + 4t^3 \\
&= 8t^3 - 12t^4 + 4t^5 \\
&\approx 0.4667.
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Quiz 3: R

3. We are given \( \pi_0 \). Find \( \lim_{n \to \infty} \pi_n \).

With probability \( \alpha = 0.2 \pi_0 (1) + \pi_0 (2) + \pi_0 (3) \), the MC ends up in \{2, 3\}.

With probability \( 1 - \alpha \), it ends up in state 4.

If it is in \{2, 3\}, the probability that it is in state 2 converges to \( 0.2 + 0.6 = 0.25 \).

Hence, the limiting distribution is \( [0, 0.25 \alpha, 0, 0.75 \alpha, 1 - \alpha] \).
Quiz 3: R

We are given $\pi_0$. Find $\lim_{n \to \infty} \pi_n$.

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Hence, the limiting distribution is

$$[0, 0.25\alpha, 0.75\alpha, 1 - \alpha].$$
4. A bag has \(n\) red and \(n\) blue balls. You pick two balls (no replacement).

Let \(X = 1\) if ball 1 is red and \(X = -1\) otherwise.

Define \(Y\) likewise for ball 2.

Are \(X\) and \(Y\) positively, negatively, or uncorrelated?

Clearly, negatively.

5. Calculate \(\text{cov}(X, Y)\).

\[
\text{cov}(X, Y) = \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y],
\]

by symmetry \(\mathbb{E}[X] = \mathbb{E}[Y] = 0\).

\[
\mathbb{E}[XY] = \text{Pr}[X = Y] - \text{Pr}[X \neq Y] = \frac{n-1}{2n-1}.
\]

E.g., if \(X = +1\) = red, then \(Y\) is red w.p. \(\frac{n-1}{2n-1}\).

\[
\mathbb{E}[XY] = \frac{2(n-1)}{2n-1} - \frac{1}{2n-1} = -\frac{1}{2n-1}.
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\[ \rightarrow \] Are \( X \) and \( Y \) positively, negatively, or uncorrelated?

Clearly, negatively.

5. Calculate \( \text{cov}(X, Y) \).
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\text{cov}(X, Y) = E[XY] - E[X]E[Y]
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E[X] = E[Y],
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\( Pr[X = Y] = (n - 1)/(2n - 1) \)
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E.g., if $X = +1 = \text{red}$, then $Y$ is red w.p. $(n - 1)/(2n - 1)$
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\[Pr[X = Y] = \frac{(n-1)}{(2n-1)}\]

E.g., if \( X = +1 = \text{red} \), then \( Y \) is red w.p. \( \frac{(n-1)}{(2n-1)} \)

\[E[XY] = \frac{2(n-1)}{(2n-1)} - 1 = \frac{-1}{(2n-1)} = \text{cov}(X, Y).\]
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- $\Omega = \{1, 2, 3\}$. Define $X, Y$ with $\text{cov}(X, Y) = 0$ and $X, Y$ not independent.
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Let
\[
\begin{align*}
X(1) &= -1, \\
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X(1) &= 1, \\
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\end{align*}
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- $3 \times 3.5 = 12.5$. 

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- $X = B(n, p) \implies \text{var}(X) = np(1 - p)$. No.

- $Pr[X = a] = Pr[X = a|A] + Pr[X = a|\overline{A}]$. No.
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Thanks and Best Wishes!
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