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4. Euclid's GCD Algorithm.
   A little tricky here!

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The sphere minimizes surface area to volume.

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Surface Area is roughly at least the volume!

Recursive Definition.

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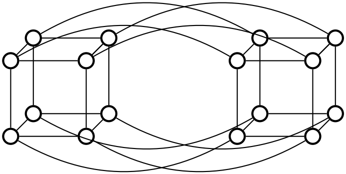

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Restatement: for any cut in the hypercube, the number of cut edges is at least the size of the small side.

Proof of Large Cuts.

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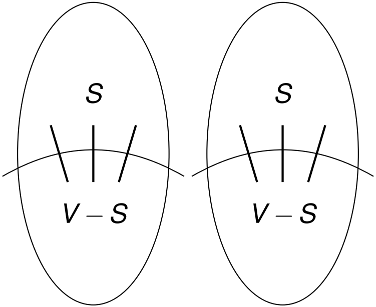

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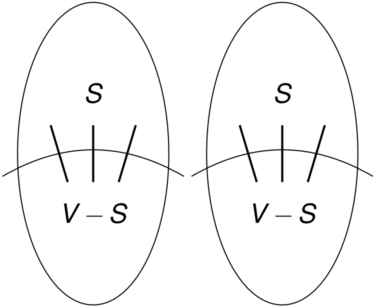

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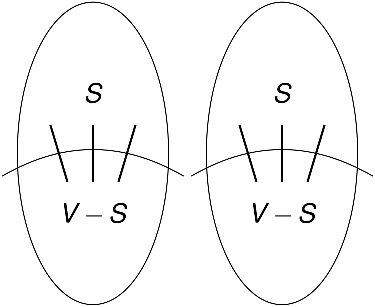

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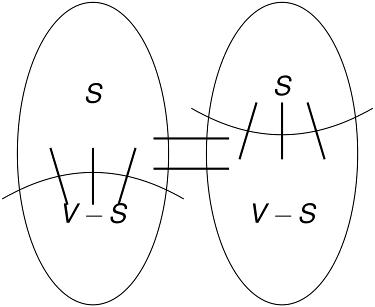

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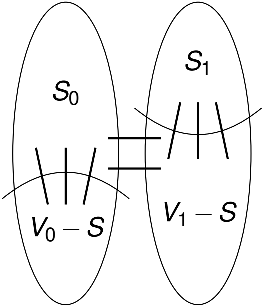
$$|S_0| \geq |V_0|/2.$$



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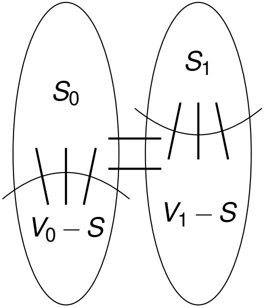

$|S_0| \geq |V_0|/2.$

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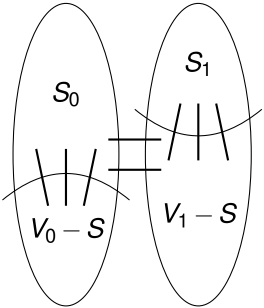

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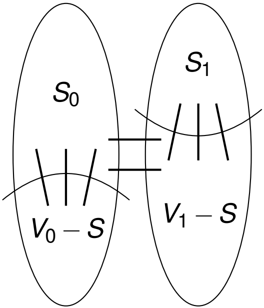

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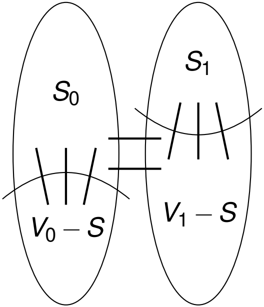

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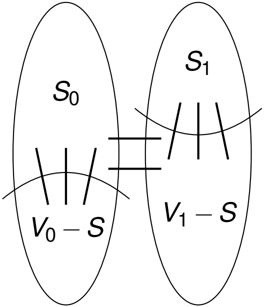

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Edges in E_x connect corresponding nodes.

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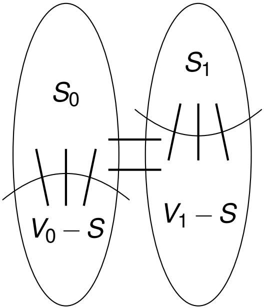

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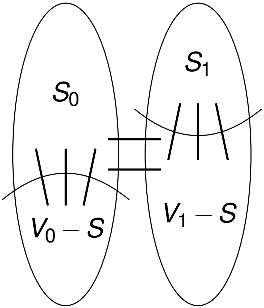

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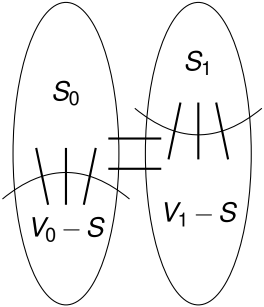

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Recall Case 1: $|S_0|, |S_1| \leq |V|/2$

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Edges in E_x connect corresponding nodes.

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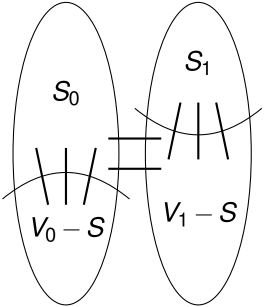
Total edges cut:

Induction Step. Case 2.

Thm: For any cut $(S, V - S)$ in the hypercube, the number of cut edges is at least the size of the small side, $|S|$.

Proof: Induction Step. Case 2.



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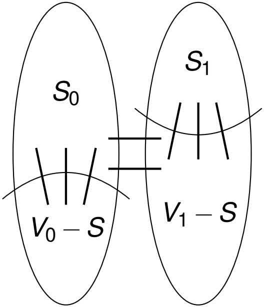

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Total edges cut:

$\geq |S_1|$

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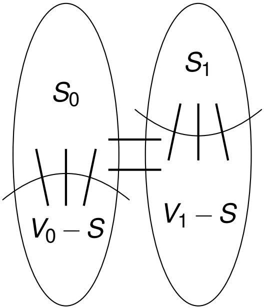

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Total edges cut:

$\geq \; |S_1| + |V_0| - |S_0|$

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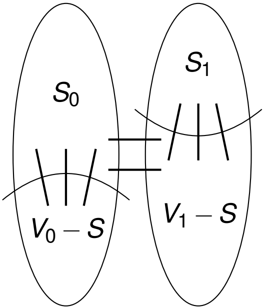

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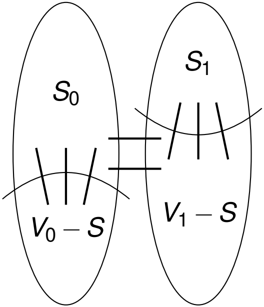

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Total edges cut:

$\geq |S_1| + |V_0| - |S_0| + |S_0| - |S_1| = |V_0|$

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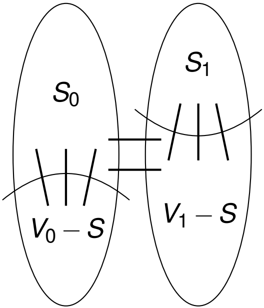
Proof: Induction Step. Case 2.



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$|V_0|$

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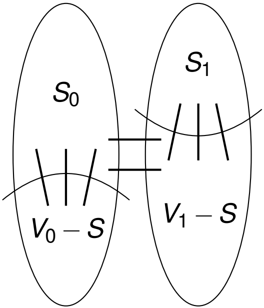

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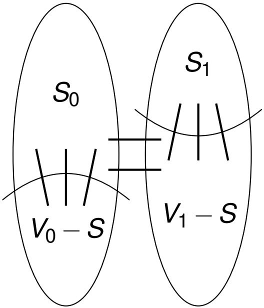

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$|V_0| = |V|/2 \geq |S|$. □

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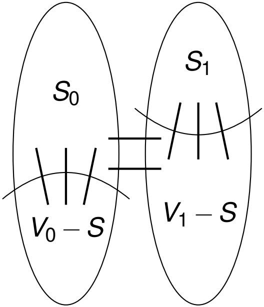

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Total edges cut:
$\geq \ |S_1| + |V_0| - |S_0| + |S_0| - |S_1| = |V_0|$
$|V_0| = |V|/2 \geq |S|$. □
Also, case 3 where $|S_1| \geq |V|/2$ is symmetric.

Hypercubes and Boolean Functions.

The cuts in the hypercubes are exactly the transitions from 0 sets to 1 set on boolean functions on $\{0,1\}^n$.

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Yes/No Computer Programs $\equiv$ Boolean function on $\{0,1\}^n$

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The cuts in the hypercubes are exactly the transitions from 0 sets to 1 set on boolean functions on $\{0,1\}^n$.

Central area of study in computer science!

Yes/No Computer Programs $\equiv$ Boolean function on $\{0,1\}^n$

Central object of study.

Next Up.

Modular Arithmetic.

Clock Math

If it is 1:00 now.

Clock Math

If it is 1:00 now.
What time is it in 2 hours?

Clock Math

If it is 1:00 now.
What time is it in 2 hours? 3:00!

Clock Math

If it is 1:00 now.
What time is it in 2 hours? 3:00!
What time is it in 5 hours?

Clock Math

If it is 1:00 now.
What time is it in 2 hours? 3:00!
What time is it in 5 hours? 6:00!

Clock Math

If it is 1:00 now.

What time is it in 2 hours? 3:00!

What time is it in 5 hours? 6:00!

What time is it in 15 hours?

Clock Math

If it is 1:00 now.
What time is it in 2 hours? 3:00!
What time is it in 5 hours? 6:00!
What time is it in 15 hours? 16:00!

Clock Math

If it is 1:00 now.
- What time is it in 2 hours? 3:00!
- What time is it in 5 hours? 6:00!
- What time is it in 15 hours? 16:00!
  - Actually 4:00.

Clock Math

If it is 1:00 now.

- What time is it in 2 hours? 3:00!
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- What time is it in 15 hours? 16:00!
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16 is the “same as 4” with respect to a 12 hour clock system.

Clock Math

If it is 1:00 now.

- What time is it in 2 hours? 3:00!
- What time is it in 5 hours? 6:00!
- What time is it in 15 hours? 16:00!
  - Actually 4:00.

16 is the "same as 4" with respect to a 12 hour clock system.
Clock time equivalent up to to addition/subtraction of 12.

Clock Math

If it is 1:00 now.
- What time is it in 2 hours? 3:00!
- What time is it in 5 hours? 6:00!
- What time is it in 15 hours? 16:00!
  - Actually 4:00.

16 is the “same as 4” with respect to a 12 hour clock system.
- Clock time equivalent up to to addition/subtraction of 12.

Clock Math

If it is 1:00 now.
What time is it in 2 hours? 3:00!
What time is it in 5 hours? 6:00!
What time is it in 15 hours? 16:00!
Actually 4:00.

16 is the “same as 4” with respect to a 12 hour clock system.
Clock time equivalent up to to addition/subtraction of 12.

What time is it in 100 hours?

Clock Math

If it is 1:00 now.

What time is it in 2 hours? 3:00!

What time is it in 5 hours? 6:00!

What time is it in 15 hours? 16:00!

Actually 4:00.

16 is the “same as 4” with respect to a 12 hour clock system.

Clock time equivalent up to to addition/subtraction of 12.

What time is it in 100 hours? 101:00!

Clock Math

If it is 1:00 now.

What time is it in 2 hours? 3:00!

What time is it in 5 hours? 6:00!

What time is it in 15 hours? 16:00!

Actually 4:00.

16 is the “same as 4” with respect to a 12 hour clock system.

Clock time equivalent up to to addition/subtraction of 12.

What time is it in 100 hours? 101:00! or 5:00.

Clock Math

If it is 1:00 now.
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  What time is it in 5 hours? 6:00!
  What time is it in 15 hours? 16:00!
    Actually 4:00.

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  Clock time equivalent up to to addition/subtraction of 12.

What time is it in 100 hours? 101:00! or 5:00.
  $101 = 12 \times 8 + 5$.

Clock Math

If it is 1:00 now.
What time is it in 2 hours? 3:00!
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Actually 4:00.

16 is the "same as 4" with respect to a 12 hour clock system.
Clock time equivalent up to to addition/subtraction of 12.

What time is it in 100 hours? 101:00! or 5:00.
$101 = 12 \times 8 + 5$.
5 is the same as 101 for a 12 hour clock system.

Clock Math

If it is 1:00 now.

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What time is it in 5 hours? 6:00!

What time is it in 15 hours? 16:00!

Actually 4:00.

16 is the "same as 4" with respect to a 12 hour clock system.

Clock time equivalent up to to addition/subtraction of 12.

What time is it in 100 hours? 101:00! or 5:00.

$101 = 12 \times 8 + 5$.

5 is the same as 101 for a 12 hour clock system.

Clock time equivalent up to addition of any integer multiple of 12.

Clock Math

If it is 1:00 now.
  What time is it in 2 hours? 3:00!
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Custom is only to use the representative in $\{12, 1, \ldots, 11\}$

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5 is the same as 101 for a 12 hour clock system.

Clock time equivalent up to addition of any integer multiple of 12.

Custom is only to use the representative in $\{12, 1, \ldots, 11\}$

(Almost remainder, except for 12 and 0 are equivalent.)

Day of the week.

Today is Monday.

Day of the week.

Today is Monday.
What day is it a year from now?

Day of the week.

Today is Monday.

What day is it a year from now? on February 6, 2018?

Day of the week.

Today is Monday.

What day is it a year from now? on February 6, 2018?

Number days.

Day of the week.

Today is Monday.
What day is it a year from now? on February 6, 2018?
Number days.
0 for Sunday, 1 for Monday, . . . , 6 for Saturday.

Day of the week.

Today is Monday.
- What day is it a year from now? on February 6, 2018?
  - Number days.
    - 0 for Sunday, 1 for Monday, . . . , 6 for Saturday.

Day of the week.

Today is Monday.

- What day is it a year from now? on February 6, 2018?
  - Number days.
    - 0 for Sunday, 1 for Monday, . . . , 6 for Saturday.

Today: day 2.

Day of the week.

Today is Monday.

- What day is it a year from now? on February 6, 2018?
  - Number days.
    - 0 for Sunday, 1 for Monday, ..., 6 for Saturday.

Today: day 2.

- 5 days from now.

Day of the week.

Today is Monday.
- What day is it a year from now? on February 6, 2018?
  - Number days.
    - 0 for Sunday, 1 for Monday, ..., 6 for Saturday.

Today: day 2.
- 5 days from now. day 7

Day of the week.

Today is Monday.

What day is it a year from now? on February 6, 2018?

Number days.

0 for Sunday, 1 for Monday, . . . , 6 for Saturday.

Today: day 2.

5 days from now. day 7 or day 0

Day of the week.

Today is Monday.

What day is it a year from now? on February 6, 2018?

Number days.

0 for Sunday, 1 for Monday, . . . , 6 for Saturday.

Today: day 2.

5 days from now. day 7 or day 0 or Sunday.

Day of the week.

Today is Monday.

What day is it a year from now? on February 6, 2018?

Number days.

0 for Sunday, 1 for Monday, . . . , 6 for Saturday.

Today: day 2.

5 days from now. day 7 or day 0 or Sunday.

25 days from now.

Day of the week.

Today is Monday.

- What day is it a year from now? on February 6, 2018?
  - Number days.
    - 0 for Sunday, 1 for Monday, . . . , 6 for Saturday.

Today: day 2.

- 5 days from now. day 7 or day 0 or Sunday.
- 25 days from now. day 27

Day of the week.

Today is Monday.

What day is it a year from now? on February 6, 2018?

Number days.

0 for Sunday, 1 for Monday, . . . , 6 for Saturday.

Today: day 2.

5 days from now. day 7 or day 0 or Sunday.

25 days from now. day 27 or day 6.

Day of the week.

Today is Monday.
  What day is it a year from now? on February 6, 2018?
    Number days.
      0 for Sunday, 1 for Monday, . . . , 6 for Saturday.

Today: day 2.
  5 days from now. day 7 or day 0 or Sunday.
  25 days from now. day 27 or day 6. $27 = (7)3 + 6$

Day of the week.

Today is Monday.
- What day is it a year from now? on February 6, 2018?
  - Number days.
    - 0 for Sunday, 1 for Monday, ..., 6 for Saturday.

Today: day 2.
- 5 days from now. day 7 or day 0 or Sunday.
- 25 days from now. day 27 or day 6. $27 = (7)3 + 6$
  - two days are equivalent up to addition/subtraction of multiple of 7.

Day of the week.

Today is Monday.
- What day is it a year from now? on February 6, 2018?
  - Number days.
    - 0 for Sunday, 1 for Monday, . . . , 6 for Saturday.

Today: day 2.
- 5 days from now. day 7 or day 0 or Sunday.
- 25 days from now. day 27 or day 6. $27 = (7)3 + 6$
  - two days are equivalent up to addition/subtraction of multiple of 7.
  - 11 days from now

Day of the week.

Today is Monday.

What day is it a year from now? on February 6, 2018?

Number days.

0 for Sunday, 1 for Monday, . . . , 6 for Saturday.

Today: day 2.

5 days from now. day 7 or day 0 or Sunday.

25 days from now. day 27 or day 6. $27 = (7)3 + 6$

two days are equivalent up to addition/subtraction of multiple of 7.

11 days from now is day 6

Day of the week.

Today is Monday.
What day is it a year from now? on February 6, 2018?
Number days.
0 for Sunday, 1 for Monday, . . . , 6 for Saturday.

Today: day 2.
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25 days from now. day 27 or day 6. $27 = (7)3 + 6$
two days are equivalent up to addition/subtraction of multiple of 7.
11 days from now is day 6 which is Saturday!

Day of the week.

Today is Monday.

What day is it a year from now? on February 6, 2018?

Number days.

0 for Sunday, 1 for Monday, . . . , 6 for Saturday.

Today: day 2.

5 days from now. day 7 or day 0 or Sunday.

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two days are equivalent up to addition/subtraction of multiple of 7.

11 days from now is day 6 which is Saturday!

What day is it a year from now?

Day of the week.

Today is Monday.
- What day is it a year from now? on February 6, 2018?
  - Number days.
    - 0 for Sunday, 1 for Monday, ..., 6 for Saturday.

Today: day 2.
- 5 days from now. day 7 or day 0 or Sunday.
- 25 days from now. day 27 or day 6. $27 = (7)3 + 6$
  - two days are equivalent up to addition/subtraction of multiple of 7.
  - 11 days from now is day 6 which is Saturday!

What day is it a year from now?
- This year is not a leap year.

Day of the week.

Today is Monday.
What day is it a year from now? on February 6, 2018?
Number days.
0 for Sunday, 1 for Monday, ..., 6 for Saturday.

Today: day 2.
5 days from now. day 7 or day 0 or Sunday.
25 days from now. day 27 or day 6. $27 = (7)3 + 6$
two days are equivalent up to addition/subtraction of multiple of 7.
11 days from now is day 6 which is Saturday!

What day is it a year from now?
This year is not a leap year. So 365 days from now.

Day of the week.

Today is Monday.
  What day is it a year from now? on February 6, 2018?
    Number days.
      0 for Sunday, 1 for Monday, ..., 6 for Saturday.

Today: day 2.
  5 days from now. day 7 or day 0 or Sunday.
  25 days from now. day 27 or day 6. $27 = (7)3 + 6$
    two days are equivalent up to addition/subtraction of multiple of 7.
    11 days from now is day 6 which is Saturday!

What day is it a year from now?
  This year is not a leap year. So 365 days from now.
  Day 2+365 or day 367.

Day of the week.

Today is Monday.
What day is it a year from now? on February 6, 2018?
Number days.
0 for Sunday, 1 for Monday, ..., 6 for Saturday.

Today: day 2.
5 days from now. day 7 or day 0 or Sunday.
25 days from now. day 27 or day 6. $27 = (7)3 + 6$
two days are equivalent up to addition/subtraction of multiple of 7.
11 days from now is day 6 which is Saturday!

What day is it a year from now?
This year is not a leap year. So 365 days from now.
Day 2+365 or day 367.
Smallest representation:

Day of the week.

Today is Monday.
  What day is it a year from now? on February 6, 2018?
    Number days.
      0 for Sunday, 1 for Monday, . . . , 6 for Saturday.

Today: day 2.
  5 days from now. day 7 or day 0 or Sunday.
  25 days from now. day 27 or day 6. $27 = (7)3 + 6$
    two days are equivalent up to addition/subtraction of multiple of 7.
    11 days from now is day 6 which is Saturday!

What day is it a year from now?
  This year is not a leap year. So 365 days from now.
  Day 2+365 or day 367.
Smallest representation:
  subtract 7 until smaller than 7.

Day of the week.

Today is Monday.
What day is it a year from now? on February 6, 2018?
Number days.
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11 days from now is day 6 which is Saturday!

What day is it a year from now?
This year is not a leap year. So 365 days from now.
Day 2+365 or day 367.
Smallest representation:
subtract 7 until smaller than 7.
divide and get remainder.

Day of the week.

Today is Monday.

What day is it a year from now? on February 6, 2018?

Number days.

0 for Sunday, 1 for Monday, . . . , 6 for Saturday.

Today: day 2.

5 days from now. day 7 or day 0 or Sunday.

25 days from now. day 27 or day 6. $27 = (7)3 + 6$

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subtract 7 until smaller than 7.

divide and get remainder.

367/7

Day of the week.

Today is Monday.
- What day is it a year from now? on February 6, 2018?
  - Number days.
    - 0 for Sunday, 1 for Monday, . . . , 6 for Saturday.

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- 25 days from now. day 27 or day 6. $27 = (7)3 + 6$
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  - 11 days from now is day 6 which is Saturday!

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- This year is not a leap year. So 365 days from now.
- Day 2+365 or day 367.

Smallest representation:
- subtract 7 until smaller than 7.
- divide and get remainder.

367/7 leaves quotient of 52 and remainder 3.

Day of the week.

Today is Monday.
What day is it a year from now? on February 6, 2018?
Number days.
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Today: day 2.
5 days from now. day 7 or day 0 or Sunday.
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two days are equivalent up to addition/subtraction of multiple of 7.
11 days from now is day 6 which is Saturday!

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This year is not a leap year. So 365 days from now.
Day 2+365 or day 367.
Smallest representation:
subtract 7 until smaller than 7.
divide and get remainder.
367/7 leaves quotient of 52 and remainder 3. $365 = 7(52) + 3$

Day of the week.

Today is Monday.

What day is it a year from now? on February 6, 2018?

Number days.

0 for Sunday, 1 for Monday, ..., 6 for Saturday.

Today: day 2.

5 days from now. day 7 or day 0 or Sunday.

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two days are equivalent up to addition/subtraction of multiple of 7.

11 days from now is day 6 which is Saturday!

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This year is not a leap year. So 365 days from now.

Day 2+365 or day 367.

Smallest representation:

subtract 7 until smaller than 7.

divide and get remainder.

367/7 leaves quotient of 52 and remainder 3. $365 = 7(52) + 3$

or February 6, 2018 is a Wednesday.

Day of the week.

Today is Monday.
What day is it a year from now? on February 6, 2018?
Number days.
0 for Sunday, 1 for Monday, ..., 6 for Saturday.

Today: day 2.
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25 days from now. day 27 or day 6. $27 = (7)3 + 6$
two days are equivalent up to addition/subtraction of multiple of 7.
11 days from now is day 6 which is Saturday!

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This year is not a leap year. So 365 days from now.
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Smallest representation:
subtract 7 until smaller than 7.
divide and get remainder.
367/7 leaves quotient of 52 and remainder 3. $365 = 7(52) + 3$
or February 6, 2018 is a Wednesday.

Years and years...

80 years from now?

Years and years...

80 years from now? 20 leap years.

Years and years...

80 years from now?   20 leap years. 366×20 days

Years and years...

80 years from now? 20 leap years. 366×20 days
60 regular years.

Years and years...

80 years from now? 20 leap years. 366×20 days
60 regular years. 365×60 days

Years and years...

80 years from now? 20 leap years. 366×20 days
60 regular years. 365×60 days
Today is day 2.

Years and years...

80 years from now? 20 leap years. 366×20 days
60 regular years. 365×60 days
Today is day 2.
It is day $2 + 366 \times 20 + 365 \times 60$.

Years and years...

80 years from now? 20 leap years. 366×20 days
60 regular years. 365×60 days
Today is day 2.
It is day $2 + 366 \times 20 + 365 \times 60$. Equivalent to?

Years and years...

80 years from now?   20 leap years. 366×20 days
  60 regular years. 365×60 days
Today is day 2.
It is day $2 + 366 \times 20 + 365 \times 60$. Equivalent to?

Hmm.

Years and years...

80 years from now? 20 leap years. 366×20 days
60 regular years. 365×60 days
Today is day 2.
It is day $2 + 366 \times 20 + 365 \times 60$. Equivalent to?

Hmm.
What is remainder of 366 when dividing by 7?

Years and years...

80 years from now? 20 leap years. 366×20 days
60 regular years. 365×60 days
Today is day 2.
It is day $2 + 366 \times 20 + 365 \times 60$. Equivalent to?

Hmm.
What is remainder of 366 when dividing by 7? $52 \times 7 + 2$.

Years and years...

80 years from now? 20 leap years. 366×20 days
60 regular years. 365×60 days
Today is day 2.
It is day $2 + 366 \times 20 + 365 \times 60$. Equivalent to?

Hmm.
What is remainder of 366 when dividing by 7? $52 \times 7 + 2$.
What is remainder of 365 when dividing by 7?

Years and years...

80 years from now? 20 leap years. 366×20 days
60 regular years. 365×60 days
Today is day 2.
It is day $2 + 366 \times 20 + 365 \times 60$. Equivalent to?

Hmm.
What is remainder of 366 when dividing by 7? $52 \times 7 + 2$.
What is remainder of 365 when dividing by 7? 1

Years and years...

80 years from now? 20 leap years. 366×20 days

60 regular years. 365×60 days

Today is day 2.

It is day $2 + 366 \times 20 + 365 \times 60$. Equivalent to?

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What is remainder of 365 when dividing by 7? 1

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Years and years...

80 years from now? 20 leap years. 366×20 days

60 regular years. 365×60 days

Today is day 2.

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Hmm.

What is remainder of 366 when dividing by 7? $52 \times 7 + 2$.

What is remainder of 365 when dividing by 7? 1

Today is day 2.

Get Day: $2 + 2 \times 20 + 1 \times 60$

Years and years...

80 years from now? 20 leap years. 366×20 days

60 regular years. 365×60 days

Today is day 2.

It is day $2 + 366 \times 20 + 365 \times 60$. Equivalent to?

Hmm.

What is remainder of 366 when dividing by 7? $52 \times 7 + 2$.

What is remainder of 365 when dividing by 7? 1

Today is day 2.

Get Day: $2 + 2 \times 20 + 1 \times 60 = 102$

Years and years...

80 years from now? 20 leap years. 366×20 days
60 regular years. 365×60 days
Today is day 2.
It is day $2 + 366 \times 20 + 365 \times 60$. Equivalent to?

Hmm.
What is remainder of 366 when dividing by 7? $52 \times 7 + 2$.
What is remainder of 365 when dividing by 7? 1
Today is day 2.
Get Day: $2 + 2 \times 20 + 1 \times 60 = 102$
Remainder when dividing by 7?

Years and years...

80 years from now? 20 leap years. 366×20 days

60 regular years. 365×60 days

Today is day 2.

It is day $2 + 366 \times 20 + 365 \times 60$. Equivalent to?

Hmm.

What is remainder of 366 when dividing by 7? $52 \times 7 + 2$.

What is remainder of 365 when dividing by 7? 1

Today is day 2.

Get Day: $2 + 2 \times 20 + 1 \times 60 = 102$

Remainder when dividing by 7? $102 = 14 \times 7$

Years and years...

80 years from now? 20 leap years. 366×20 days
60 regular years. 365×60 days
Today is day 2.
It is day $2 + 366 \times 20 + 365 \times 60$. Equivalent to?

Hmm.
What is remainder of 366 when dividing by 7? $52 \times 7 + 2$.
What is remainder of 365 when dividing by 7? 1
Today is day 2.
Get Day: $2 + 2 \times 20 + 1 \times 60 = 102$
Remainder when dividing by 7? $102 = 14 \times 7 + 4$.

Years and years...

80 years from now? 20 leap years. 366×20 days
60 regular years. 365×60 days
Today is day 2.
It is day $2 + 366 \times 20 + 365 \times 60$. Equivalent to?

Hmm.
What is remainder of 366 when dividing by 7? $52 \times 7 + 2$.
What is remainder of 365 when dividing by 7? 1
Today is day 2.
Get Day: $2 + 2 \times 20 + 1 \times 60 = 102$
Remainder when dividing by 7? $102 = 14 \times 7 + 4$.
Or February 7, 2096 is Thursday!

Years and years...

80 years from now? 20 leap years. 366×20 days
60 regular years. 365×60 days
Today is day 2.
It is day $2 + 366 \times 20 + 365 \times 60$. Equivalent to?

Hmm.
What is remainder of 366 when dividing by 7? $52 \times 7 + 2$.
What is remainder of 365 when dividing by 7? 1
Today is day 2.
Get Day: $2 + 2 \times 20 + 1 \times 60 = 102$
Remainder when dividing by 7? $102 = 14 \times 7 + 4$.
Or February 7, 2096 is Thursday!

Further Simplify Calculation:

Years and years...

80 years from now? 20 leap years. 366×20 days
60 regular years. 365×60 days
Today is day 2.
It is day $2 + 366 \times 20 + 365 \times 60$. Equivalent to?

Hmm.
What is remainder of 366 when dividing by 7? $52 \times 7 + 2$.
What is remainder of 365 when dividing by 7? 1
Today is day 2.
Get Day: $2 + 2 \times 20 + 1 \times 60 = 102$
Remainder when dividing by 7? $102 = 14 \times 7 + 4$.
Or February 7, 2096 is Thursday!

Further Simplify Calculation:
20 has remainder 6 when divided by 7.

Years and years...

80 years from now? 20 leap years. 366×20 days
60 regular years. 365×60 days
Today is day 2.
It is day $2 + 366 \times 20 + 365 \times 60$. Equivalent to?

Hmm.
What is remainder of 366 when dividing by 7? $52 \times 7 + 2$.
What is remainder of 365 when dividing by 7? 1
Today is day 2.
Get Day: $2 + 2 \times 20 + 1 \times 60 = 102$
Remainder when dividing by 7? $102 = 14 \times 7 + 4$.
Or February 7, 2096 is Thursday!

Further Simplify Calculation:
20 has remainder 6 when divided by 7.
60 has remainder 4 when divided by 7.

Years and years...

80 years from now? 20 leap years. 366×20 days
60 regular years. 365×60 days
Today is day 2.
It is day $2 + 366 \times 20 + 365 \times 60$. Equivalent to?

Hmm.
What is remainder of 366 when dividing by 7? $52 \times 7 + 2$.
What is remainder of 365 when dividing by 7? 1
Today is day 2.
Get Day: $2 + 2 \times 20 + 1 \times 60 = 102$
Remainder when dividing by 7? $102 = 14 \times 7 + 4$.
Or February 7, 2096 is Thursday!

Further Simplify Calculation:
20 has remainder 6 when divided by 7.
60 has remainder 4 when divided by 7.
Get Day: $2 + 2 \times 6 + 1 \times 4 = 18$.

Years and years...

80 years from now? 20 leap years. 366×20 days
60 regular years. 365×60 days
Today is day 2.
It is day $2 + 366 \times 20 + 365 \times 60$. Equivalent to?

Hmm.
What is remainder of 366 when dividing by 7? $52 \times 7 + 2$.
What is remainder of 365 when dividing by 7? 1
Today is day 2.
Get Day: $2 + 2 \times 20 + 1 \times 60 = 102$
Remainder when dividing by 7? $102 = 14 \times 7 + 4$.
Or February 7, 2096 is Thursday!

Further Simplify Calculation:
20 has remainder 6 when divided by 7.
60 has remainder 4 when divided by 7.
Get Day: $2 + 2 \times 6 + 1 \times 4 = 18$.
Or Day 4.

Years and years...

80 years from now? 20 leap years. 366×20 days
60 regular years. 365×60 days
Today is day 2.
It is day $2 + 366 \times 20 + 365 \times 60$. Equivalent to?

Hmm.
What is remainder of 366 when dividing by 7? $52 \times 7 + 2$.
What is remainder of 365 when dividing by 7? 1
Today is day 2.
Get Day: $2 + 2 \times 20 + 1 \times 60 = 102$
Remainder when dividing by 7? $102 = 14 \times 7 + 4$.
Or February 7, 2096 is Thursday!

Further Simplify Calculation:
20 has remainder 6 when divided by 7.
60 has remainder 4 when divided by 7.
Get Day: $2 + 2 \times 6 + 1 \times 4 = 18$.
Or Day 4. February 6, 2095 is Thursday.

Years and years...

80 years from now? 20 leap years. 366×20 days
60 regular years. 365×60 days
Today is day 2.
It is day $2 + 366 \times 20 + 365 \times 60$. Equivalent to?

Hmm.
What is remainder of 366 when dividing by 7? $52 \times 7 + 2$.
What is remainder of 365 when dividing by 7? 1
Today is day 2.
Get Day: $2 + 2 \times 20 + 1 \times 60 = 102$
Remainder when dividing by 7? $102 = 14 \times 7 + 4$.
Or February 7, 2096 is Thursday!

Further Simplify Calculation:
20 has remainder 6 when divided by 7.
60 has remainder 4 when divided by 7.
Get Day: $2 + 2 \times 6 + 1 \times 4 = 18$.
Or Day 4. February 6, 2095 is Thursday.

"Reduce" at any time in calculation!

Modular Arithmetic: refresher.

x **is congruent to** y **modulo** m or "$x \equiv y \pmod{m}$"
if and only if $(x - y)$ is divisible by m.

Modular Arithmetic: refresher.

x **is congruent to** y **modulo** m or "$x \equiv y \pmod{m}$"
if and only if $(x - y)$ is divisible by m.
...or x and y have the same remainder w.r.t. m.

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if and only if $(x - y)$ is divisible by m.
...or x and y have the same remainder w.r.t. m.
...or $x = y + km$ for some integer k.

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Mod 7 equivalence classes:

Modular Arithmetic: refresher.

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...or x and y have the same remainder w.r.t. m.
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Mod 7 equivalence classes:
$\{\ldots, -7, 0, 7, 14, \ldots\}$

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Mod 7 equivalence classes:
$\{\ldots,-7,0,7,14,\ldots\}$ $\{\ldots,-6,1,8,15,\ldots\}$

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...or $x = y + km$ for some integer k.

Mod 7 equivalence classes:
$\{\ldots,-7,0,7,14,\ldots\}$ $\{\ldots,-6,1,8,15,\ldots\}$ ...

Useful Fact: Addition, subtraction, multiplication can be done with any equivalent x and y.

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Mod 7 equivalence classes:
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or " $a \equiv c \pmod{m}$ and $b \equiv d \pmod{m}$

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Mod 7 equivalence classes:
$\{\ldots, -7, 0, 7, 14, \ldots\}$ $\{\ldots, -6, 1, 8, 15, \ldots\}$ ...

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or " $a \equiv c \pmod{m}$ and $b \equiv d \pmod{m}$
$\implies a + b \equiv c + d \pmod{m}$ and $a \cdot b = c \cdot d \pmod{m}$"

Proof: If $a \equiv c \pmod{m}$, then $a = c + km$ for some integer k.

Modular Arithmetic: refresher.

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$\implies a + b \equiv c + d \pmod{m}$ and $a \cdot b = c \cdot d \pmod{m}$"

Proof: If $a \equiv c \pmod{m}$, then $a = c + km$ for some integer k.
If $b \equiv d \pmod{m}$, then $b = d + jm$ for some integer j.

Modular Arithmetic: refresher.

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Proof: If $a \equiv c \pmod{m}$, then $a = c + km$ for some integer k.
If $b \equiv d \pmod{m}$, then $b = d + jm$ for some integer j.
Therefore,

Modular Arithmetic: refresher.

x **is congruent to** y **modulo** m or "$x \equiv y \pmod{m}$"
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$\implies a+b \equiv c+d \pmod{m}$ and $a \cdot b = c \cdot d \pmod{m}$"

Proof: If $a \equiv c \pmod{m}$, then $a = c + km$ for some integer k.
If $b \equiv d \pmod{m}$, then $b = d + jm$ for some integer j.
Therefore, $a+b = c+d+(k+j)m$

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Proof: If $a \equiv c \pmod{m}$, then $a = c + km$ for some integer k.
If $b \equiv d \pmod{m}$, then $b = d + jm$ for some integer j.
Therefore, $a+b = c+d+(k+j)m$ and since $k+j$ is integer.

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Therefore, $a+b = c+d+(k+j)m$ and since $k+j$ is integer.
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Proof: If $a \equiv c \pmod{m}$, then $a = c + km$ for some integer k.
If $b \equiv d \pmod{m}$, then $b = d + jm$ for some integer j.
Therefore, $a + b = c + d + (k+j)m$ and since $k + j$ is integer.
$\implies a + b \equiv c + d \pmod{m}$. □

Modular Arithmetic: refresher.

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if and only if $(x-y)$ is divisible by m.
...or x and y have the same remainder w.r.t. m.
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or " $a \equiv c \pmod{m}$ and $b \equiv d \pmod{m}$
$\implies a+b \equiv c+d \pmod{m}$ and $a \cdot b = c \cdot d \pmod{m}$"

Proof: If $a \equiv c \pmod{m}$, then $a = c + km$ for some integer k.
If $b \equiv d \pmod{m}$, then $b = d + jm$ for some integer j.
Therefore, $a+b = c+d+(k+j)m$ and since $k+j$ is integer.
$\implies a+b \equiv c+d \pmod{m}$. □

Can calculate with representative in $\{0,\ldots,m-1\}$.

Notation

$x \pmod{m}$ or $\operatorname{mod}(x, m)$

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Work in this system.

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$a \equiv b \pmod{m}$.

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Says two integers a and b are equivalent modulo m.

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$6 \equiv$

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$6 \equiv 3 + 3 \equiv 3 + 10 \pmod{7}$.

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Inverses and Factors.

Division: multiply by multiplicative inverse.

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Check! $4(3) = 12 = 5 \pmod{7}$.

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"Common factor of 4" $\implies$
$8k - 12\ell$ is a multiple of four for any ℓ and k $\implies$
$8k \not\equiv 1 \pmod{12}$ for any k.

Greatest Common Divisor and Inverses.

Thm:
If greatest common divisor of x and m, $\gcd(x, m)$, is 1, then x has a multiplicative inverse modulo m.

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$gcd(x, m) = 1$

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Claim: The set $S = \{0x, 1x, \ldots, (m-1)x\}$ contains $y \equiv 1 \mod m$ if all distinct modulo m.

Each of m numbers in S correspond to different one of m equivalence classes modulo m.

$\implies$ One must correspond to 1 modulo m. Inverse Exists!

Proof of Claim: If not distinct, then $\exists a, b \in \{0, \ldots, m-1\}$, $a \neq b$, where
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Not distinct.

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Not distinct. Common factor 2.

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For $x=5$ and $m=6$.

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All distinct, contains 1!

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$$5x=3 \pmod 6$$

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$5x = 3 \pmod 6$ What is x?

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$5x = 3 \pmod 6$ What is x? Multiply both sides by 5.

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$5x = 3 \pmod 6$ What is x? Multiply both sides by 5.
$x = 15$

Proof review. Consequence.

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$\square$

...

For $x = 4$ and $m = 6$. All products of 4...

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Very different for elements with inverses.

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Fact: If $d|x$ and $d|y$ then $d|(x+y)$ and $d|(x-y)$.

Is it a fact? Yes? No?

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Before discussing running time of gcd procedure...

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Check 2, check 3, check 4, check 5 . . . , check $y/2$.

If $y \approx x$ roughly y uses n bits ...
2^{n-1} divisions! Exponential dependence on size!

101 bit number. $2^{100} \approx 10^{30} =$ "million, trillion, trillion" divisions!

Euclid procedure is fast.

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Algorithms at work.

Trying everything

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"(gcd x y)" at work.

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```
euclid(700,568)
```

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Notice: The first argument decreases rapidly.

Algorithms at work.

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Notice: The first argument decreases rapidly.
At least a factor of 2 in two recursive calls.

Algorithms at work.

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Notice: The first argument decreases rapidly.
At least a factor of 2 in two recursive calls.

(The second is less than the first.)

Maybe Break.

Runtime Proof.

```
(define (euclid x y)
 (if (= y 0)
     x
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```

Theorem: (euclid x y) uses $O(n)$ ”divisions” where $n = b(x)$.

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1 division per recursive call.
$O(n)$ divisions.

□

Runtime Proof (continued.)

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□

Finding an inverse?

We showed how to efficiently tell if there is an inverse.

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Extend euclid to find inverse.

Euclid's GCD algorithm.

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Computes the $\gcd(x, y)$ in $O(n)$ divisions.

For x and m, if $\gcd(x, m) = 1$ then x has an inverse modulo m.

Multiplicative Inverse.

GCD algorithm used to tell **if** there is a multiplicative inverse.

Multiplicative Inverse.

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How do we **find** a multiplicative inverse?

Extended GCD

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Example: For $x = 12$ and $y = 35$, $\gcd(12, 35) = 1$.

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$$(3)12 + (-1)35 = 1.$$

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$$(3)12 + (-1)35 = 1.$$

$$a = 3 \text{ and } b = -1.$$

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Example: For $x = 12$ and $y = 35$, $\gcd(12, 35) = 1$.

$$(3)12 + (-1)35 = 1.$$

$a = 3$ and $b = -1$.
The multiplicative inverse of $12 \pmod{35}$ is 3.

Make *d* out of *x* and *y*..?

```
gcd(35,12)
```

Make *d* out of *x* and *y*..?

```
gcd(35,12)
  gcd(12, 11)  ;;  gcd(12, 35%12)
```

Make *d* out of *x* and *y*..?

```
gcd(35,12)
  gcd(12, 11)  ;;  gcd(12, 35%12)
    gcd(11, 1)  ;;  gcd(11, 12%11)
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Make *d* out of *x* and *y*..?

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gcd(35,12)
  gcd(12, 11)  ;;  gcd(12, 35%12)
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        1
```

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gcd(35,12)
  gcd(12, 11)  ;;  gcd(12, 35%12)
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How did gcd get 11 from 35 and 12?

Make *d* out of *x* and *y*..?

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gcd(35,12)
  gcd(12, 11)  ;;  gcd(12, 35%12)
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```

How did gcd get 11 from 35 and 12?
$35 - \lfloor \frac{35}{12} \rfloor 12 = 35 - (2)12 = 11$

Make d out of x and y..?

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gcd(35,12)
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How did gcd get 11 from 35 and 12?
$35 - \lfloor \frac{35}{12} \rfloor 12 = 35 - (2)12 = 11$

How does gcd get 1 from 12 and 11?

Make *d* out of *x* and *y*..?

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gcd(35,12)
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$35 - \lfloor \frac{35}{12} \rfloor 12 = 35 - (2)12 = 11$

How does gcd get 1 from 12 and 11?
$12 - \lfloor \frac{12}{11} \rfloor 11 = 12 - (1)11 = 1$

Make d out of x and y..?

```
gcd(35,12)
  gcd(12, 11)  ;;  gcd(12, 35%12)
    gcd(11, 1)  ;;  gcd(11, 12%11)
      gcd(1,0)
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How did gcd get 11 from 35 and 12?
$35 - \lfloor \frac{35}{12} \rfloor 12 = 35 - (2)12 = 11$

How does gcd get 1 from 12 and 11?
$12 - \lfloor \frac{12}{11} \rfloor 11 = 12 - (1)11 = 1$

Algorithm finally returns 1.

Make d out of x and y..?

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  gcd(12, 11)  ;;  gcd(12, 35%12)
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Algorithm finally returns 1.

But we want 1 from sum of multiples of 35 and 12?

Make *d* out of *x* and *y*..?

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Get 1 from 12 and 11.

Make *d* out of *x* and *y*..?

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Algorithm finally returns 1.

But we want 1 from sum of multiples of 35 and 12?

Get 1 from 12 and 11.
$1 = 12 - (1)11$

Make d out of x and y..?

```
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$35 - \lfloor \frac{35}{12} \rfloor 12 = 35 - (2)12 = 11$

How does gcd get 1 from 12 and 11?
$12 - \lfloor \frac{12}{11} \rfloor 11 = 12 - (1)11 = 1$

Algorithm finally returns 1.

But we want 1 from sum of multiples of 35 and 12?

Get 1 from 12 and 11.
$1 = 12 - (1)11 = 12 - (1)(35 - (2)12)$
Get 11 from 35 and 12 and plugin....

Make d out of x and y..?

```
gcd(35,12)
  gcd(12, 11)  ;;  gcd(12, 35%12)
    gcd(11, 1)  ;;  gcd(11, 12%11)
      gcd(1,0)
        1
```

How did gcd get 11 from 35 and 12?
$35 - \lfloor \frac{35}{12} \rfloor 12 = 35 - (2)12 = 11$

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But we want 1 from sum of multiples of 35 and 12?

Get 1 from 12 and 11.
$1 = 12 - (1)11 = 12 - (1)(35 - (2)12) = (3)12 + (-1)35$
Get 11 from 35 and 12 and plugin.... Simplify.

Make d out of x and y..?

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  gcd(12, 11)  ;;  gcd(12, 35%12)
    gcd(11, 1)  ;;  gcd(11, 12%11)
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```

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But we want 1 from sum of multiples of 35 and 12?

Get 1 from 12 and 11.
$1 = 12 - (1)11 = 12 - (1)(35 - (2)12) = (3)12 + (-1)35$
Get 11 from 35 and 12 and plugin.... Simplify. $a = 3$ and $b = -1$.

Extended GCD Algorithm.

```
ext-gcd(x,y)
  if y = 0 then return(x, 1, 0)
     else
         (d, a, b) := ext-gcd(y, mod(x,y))
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Claim: Returns (d, a, b): $d = gcd(a, b)$ and $d = ax + by$.

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Example:

```
ext-gcd(35,12)
  ext-gcd(12, 11)
```

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Claim: Returns (d, a, b): $d = gcd(a, b)$ and $d = ax + by$.
Example: $a - \lfloor x/y \rfloor \cdot b =$

```
    ext-gcd(35,12)
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        ext-gcd(11, 1)
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          return (1,1,0) ;; 1 = (1)1 + (0) 0
```

Extended GCD Algorithm.

```
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  if y = 0 then return(x, 1, 0)
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Claim: Returns (d,a,b): $d = gcd(a,b)$ and $d = ax + by$.
Example: $a - \lfloor x/y \rfloor \cdot b = 1 - \lfloor 11/1 \rfloor \cdot 0 = 1$

```
ext-gcd(35,12)
  ext-gcd(12, 11)
    ext-gcd(11, 1)
      ext-gcd(1,0)
      return (1,1,0) ;; 1 = (1)1 + (0) 0
    return (1,0,1)   ;; 1 = (0)11 + (1)1
```

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  if y = 0 then return(x, 1, 0)
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```

Claim: Returns (d,a,b): $d = gcd(a,b)$ and $d = ax + by$.
Example: $a - \lfloor x/y \rfloor \cdot b = 0 - \lfloor 12/11 \rfloor \cdot 1 = -1$

```
ext-gcd(35,12)
  ext-gcd(12, 11)
    ext-gcd(11, 1)
      ext-gcd(1,0)
      return (1,1,0) ;; 1 = (1)1 + (0) 0
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  return (1,1,-1)    ;; 1 = (1)12 + (-1)11
```

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```

Claim: Returns (d, a, b): $d = gcd(a, b)$ and $d = ax + by$.
Example: $a - \lfloor x/y \rfloor \cdot b = \ 1 - \lfloor 35/12 \rfloor \cdot (-1) = 3$

```
    ext-gcd(35,12)
      ext-gcd(12, 11)
        ext-gcd(11, 1)
          ext-gcd(1,0)
          return (1,1,0) ;; 1 = (1)1 + (0) 0
        return (1,0,1)   ;; 1 = (0)11 + (1)1
      return (1,1,-1)    ;; 1 = (1)12 + (-1)11
   return (1,-1, 3)      ;; 1 = (-1)35 +(3)12
```

Extended GCD Algorithm.

```
ext-gcd(x,y)
  if y = 0 then return(x, 1, 0)
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      return (1,1,-1)    ;; 1 = (1)12 + (-1)11
   return (1,-1, 3)      ;; 1 = (-1)35 +(3)12
```

Extended GCD Algorithm.

```
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         return (d, b, a - floor(x/y)  * b)
```

Theorem: Returns (d, a, b), where $d = gcd(a, b)$ and

$$d = ax + by.$$

Correctness.

Proof: Strong Induction.[1]

[1] Assume d is $gcd(x,y)$ by previous proof.

Correctness.

Proof: Strong Induction.[1]
Base: ext-gcd$(x, 0)$ returns $(d = x, 1, 0)$ with $x = (1)x + (0)y$.

[1] Assume d is $gcd(x, y)$ by previous proof.

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Proof: Strong Induction.[1]
Base: ext-gcd($x,0$) returns ($d=x,1,0$) with $x=(1)x+(0)y$.

Induction Step: Returns (d,A,B) with $d=Ax+By$
Ind hyp: **ext-gcd**($y, \mod(x,y)$) returns (d,a,b) with
$d=ay+b(\mod(x,y))$

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ext-gcd(x, y) calls **ext-gcd**(y, $\mod(x, y)$) so

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Ind hyp: **ext-gcd**(y, mod (x,y)) returns (d, a, b) with
$d = ay + b(\bmod (x,y))$

ext-gcd(x,y) calls **ext-gcd**(y, mod (x,y)) so

$$d = ay + b \cdot (\bmod (x,y))$$

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$$\begin{aligned} d &= ay + b \cdot (\mod(x, y)) \\ &= ay + b \cdot (x - \lfloor \frac{x}{y} \rfloor y) \end{aligned}$$

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$$\begin{aligned} d &= ay + b \cdot (\mod(x,y)) \\ &= ay + b \cdot (x - \lfloor \frac{x}{y} \rfloor y) \\ &= bx + (a - \lfloor \frac{x}{y} \rfloor \cdot b)y \end{aligned}$$

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And ext-gcd returns $(d, b, (a - \lfloor \frac{x}{y} \rfloor \cdot b))$ so theorem holds!

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Ind hyp: **ext-gcd**$(y, \mod(x,y))$ returns (d,a,b) with
$d=ay+b(\mod(x,y))$

ext-gcd(x,y) calls **ext-gcd**$(y, \mod(x,y))$ so

$$\begin{aligned} d &= ay+b\cdot(\mod(x,y)) \\ &= ay+b\cdot(x-\lfloor\frac{x}{y}\rfloor y) \\ &= bx+(a-\lfloor\frac{x}{y}\rfloor\cdot b)y \end{aligned}$$

And ext-gcd returns $(d,b,(a-\lfloor\frac{x}{y}\rfloor\cdot b))$ so theorem holds! □

[1] Assume d is $gcd(x,y)$ by previous proof.

Review Proof: step.

Prove: returns (d, A, B) where $d = Ax + By$.

```
ext-gcd(x,y)
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```

Recursively: $d = ay + b(x - \lfloor \frac{x}{y} \rfloor \cdot y)$

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Recursively: $d = ay + b(x - \lfloor \frac{x}{y} \rfloor \cdot y) \implies d = bx - (a - \lfloor \frac{x}{y} \rfloor b)y$

Returns $(d, b, (a - \lfloor \frac{x}{y} \rfloor \cdot b))$.

Hand Calculation Method for Inverses.

Example: $\gcd(7,60) = 1$.

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Example: $\gcd(7,60) = 1$.
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$$\begin{aligned} 7(0) + 60(1) &= 60 \\ 7(1) + 60(0) &= 7 \end{aligned}$$

Hand Calculation Method for Inverses.

Example: $\gcd(7,60) = 1$.
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$$\begin{aligned} 7(0) + 60(1) &= 60 \\ 7(1) + 60(0) &= 7 \\ 7(-8) + 60(1) &= 4 \end{aligned}$$

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$$\begin{aligned} 7(0)+60(1) &= 60 \\ 7(1)+60(0) &= 7 \\ 7(-8)+60(1) &= 4 \\ 7(9)+60(-1) &= 3 \end{aligned}$$

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$$
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7(0) + 60(1) &= 60 \\
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7(-17) + 60(2) &= 1
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Confirm:

Hand Calculation Method for Inverses.

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$$

Confirm: $-119+120 = 1$

Wrap-up

Conclusion: Can find multiplicative inverses in $O(n)$ time!

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Very different from elementary school: try 1, try 2, try 3...

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Inverse of $500,000,357$ modulo $1,000,000,000,000$?

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Inverse of $500,000,357$ modulo $1,000,000,000,000$?
≤ 80 divisions.

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Internet Security.

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Public Key Cryptography: 512 digits.

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512 divisions vs.

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