# Today.

Types of graphs.

# Today.

Types of graphs.

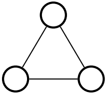
- Complete Graphs.
- Trees.
- Planar Graphs.

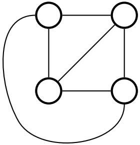
# Today.

Types of graphs.

- Complete Graphs.
- Trees.
- Planar Graphs.

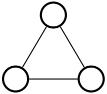
# Complete Graph.

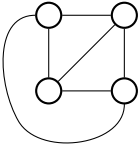




$K_n$ complete graph on $n$ vertices.

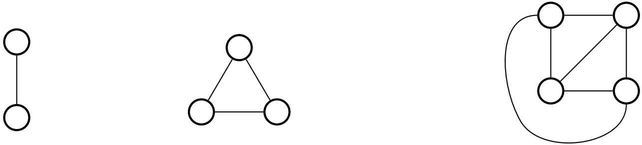
# Complete Graph.





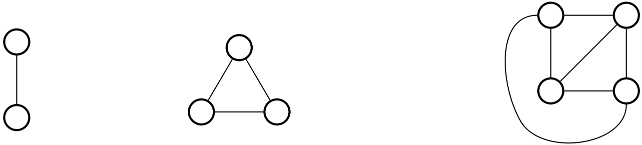
$K_n$ complete graph on $n$ vertices.
All edges are present.

# Complete Graph.



$K_n$ complete graph on $n$ vertices.
All edges are present.
Everyone is my neighbor.

# Complete Graph.



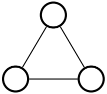
$K_n$ complete graph on $n$ vertices.

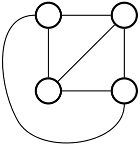
All edges are present.

Everyone is my neighbor.

Each vertex is adjacent to every other vertex.

# Complete Graph.





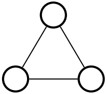
$K_n$ complete graph on $n$ vertices.

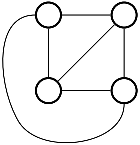
All edges are present.

Everyone is my neighbor.

Each vertex is adjacent to every other vertex.

# Complete Graph.





$K_n$ complete graph on $n$ vertices.

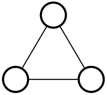
All edges are present.

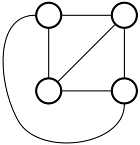
Everyone is my neighbor.

Each vertex is adjacent to every other vertex.

How many edges?

# Complete Graph.





$K_n$ complete graph on $n$ vertices.

All edges are present.

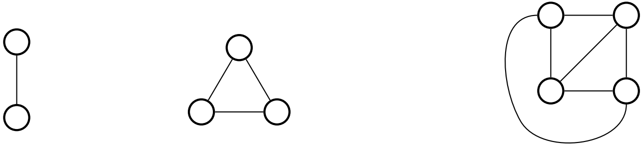
Everyone is my neighbor.

Each vertex is adjacent to every other vertex.

How many edges?

Each vertex is incident to $n-1$ edges.

# Complete Graph.



$K_n$ complete graph on $n$ vertices.

All edges are present.

Everyone is my neighbor.

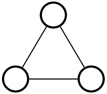
Each vertex is adjacent to every other vertex.

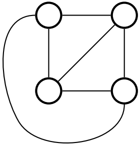
How many edges?

Each vertex is incident to $n-1$ edges.

Sum of degrees is $n(n-1)$.

# Complete Graph.





$K_n$ complete graph on $n$ vertices.

All edges are present.

Everyone is my neighbor.

Each vertex is adjacent to every other vertex.

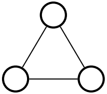
How many edges?

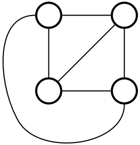
Each vertex is incident to $n-1$ edges.

Sum of degrees is $n(n-1)$.

$\implies$ Number of edges is $n(n-1)/2$.

# Complete Graph.

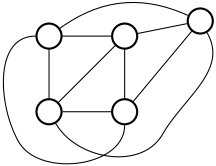




$K_n$ complete graph on $n$ vertices.
All edges are present.
Everyone is my neighbor.
Each vertex is adjacent to every other vertex.

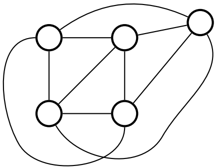
How many edges?
Each vertex is incident to $n-1$ edges.
Sum of degrees is $n(n-1)$.
$\implies$ Number of edges is $n(n-1)/2$.
Remember sum of degree is $2|E|$.

# $K_5$



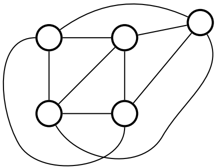
$K_5$ is not planar.

# $K_5$



$K_5$ is not planar.
Cannot be drawn in the plane without an edge crossing!

# $K_5$

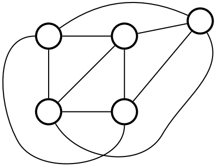


$K_5$ is not planar.

Cannot be drawn in the plane without an edge crossing!

Prove it!

# $K_5$



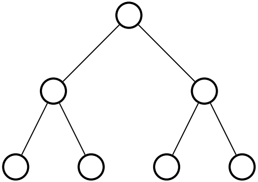
$K_5$ is not planar.

Cannot be drawn in the plane without an edge crossing!

Prove it! We will!

# A Tree, a tree.

Graph $G = (V, E)$.
Binary Tree!



More generally.

# Trees.

Definitions:

# Trees.

Definitions:

A connected graph without a cycle.

# Trees.

Definitions:

A connected graph without a cycle.
A connected graph with $|V| - 1$ edges.

# Trees.

Definitions:

A connected graph without a cycle.
A connected graph with $|V| - 1$ edges.
A connected graph where any edge removal disconnects it.

# Trees.

Definitions:

A connected graph without a cycle.
A connected graph with $|V| - 1$ edges.
A connected graph where any edge removal disconnects it.
A connected graph where any edge addition creates a cycle.

# Trees.

Definitions:

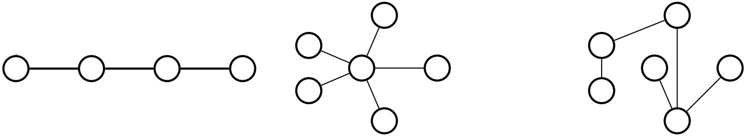
A connected graph without a cycle.
A connected graph with $|V|-1$ edges.
A connected graph where any edge removal disconnects it.
A connected graph where any edge addition creates a cycle.

# Trees.

Definitions:

A connected graph without a cycle.
A connected graph with $|V| - 1$ edges.
A connected graph where any edge removal disconnects it.
A connected graph where any edge addition creates a cycle.

Some trees.



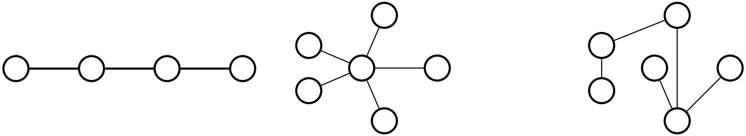
no cycle and connected?

# Trees.

Definitions:

- A connected graph without a cycle.
- A connected graph with $|V| - 1$ edges.
- A connected graph where any edge removal disconnects it.
- A connected graph where any edge addition creates a cycle.

Some trees.



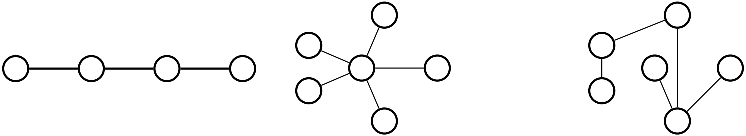
no cycle and connected? Yes.

# Trees.

Definitions:

A connected graph without a cycle.
A connected graph with $|V|-1$ edges.
A connected graph where any edge removal disconnects it.
A connected graph where any edge addition creates a cycle.

Some trees.



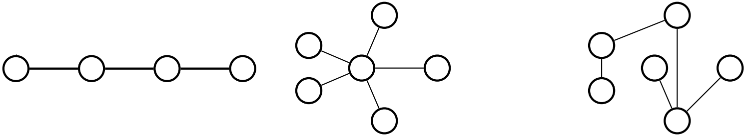
no cycle and connected? Yes.
$|V|-1$ edges and connected?

# Trees.

Definitions:

A connected graph without a cycle.
A connected graph with $|V|-1$ edges.
A connected graph where any edge removal disconnects it.
A connected graph where any edge addition creates a cycle.

Some trees.



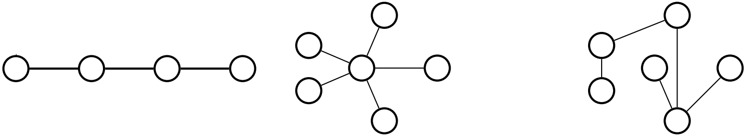
no cycle and connected? Yes.
$|V|-1$ edges and connected? Yes.

# Trees.

Definitions:

A connected graph without a cycle.
A connected graph with $|V| - 1$ edges.
A connected graph where any edge removal disconnects it.
A connected graph where any edge addition creates a cycle.

Some trees.



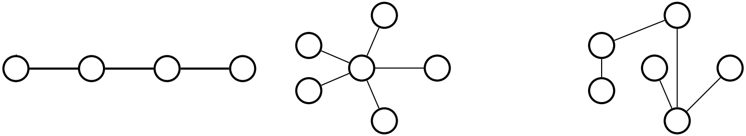
no cycle and connected? Yes.
$|V| - 1$ edges and connected? Yes.
removing any edge disconnects it.

# Trees.

Definitions:

A connected graph without a cycle.
A connected graph with $|V|-1$ edges.
A connected graph where any edge removal disconnects it.
A connected graph where any edge addition creates a cycle.

Some trees.



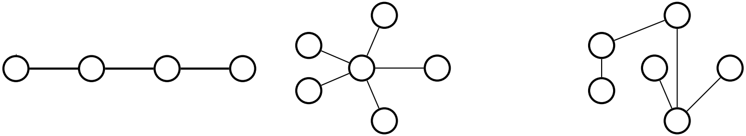
no cycle and connected? Yes.
$|V|-1$ edges and connected? Yes.
removing any edge disconnects it. Harder to check.

# Trees.

Definitions:

A connected graph without a cycle.
A connected graph with $|V|-1$ edges.
A connected graph where any edge removal disconnects it.
A connected graph where any edge addition creates a cycle.

Some trees.



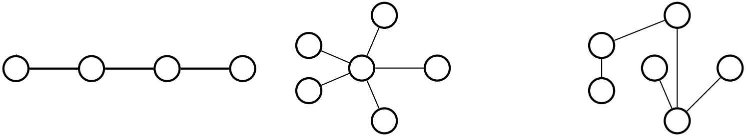
no cycle and connected? Yes.
$|V|-1$ edges and connected? Yes.
removing any edge disconnects it. Harder to check. but yes.

# Trees.

Definitions:

A connected graph without a cycle.
A connected graph with $|V|-1$ edges.
A connected graph where any edge removal disconnects it.
A connected graph where any edge addition creates a cycle.

Some trees.



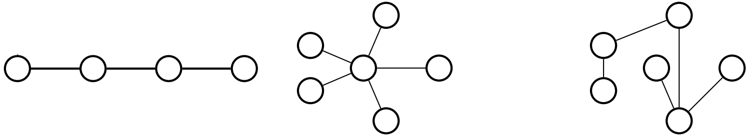
no cycle and connected? Yes.
$|V|-1$ edges and connected? Yes.
removing any edge disconnects it. Harder to check. but yes.
Adding any edge creates cycle.

# Trees.

Definitions:

A connected graph without a cycle.
A connected graph with $|V|-1$ edges.
A connected graph where any edge removal disconnects it.
A connected graph where any edge addition creates a cycle.

Some trees.



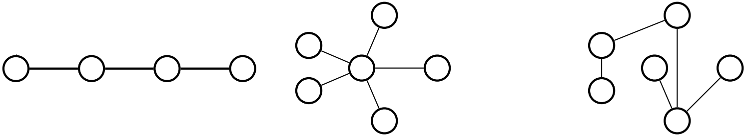
no cycle and connected? Yes.
$|V|-1$ edges and connected? Yes.
removing any edge disconnects it. Harder to check. but yes.
Adding any edge creates cycle. Harder to check.

# Trees.

Definitions:

A connected graph without a cycle.
A connected graph with $|V|-1$ edges.
A connected graph where any edge removal disconnects it.
A connected graph where any edge addition creates a cycle.

Some trees.



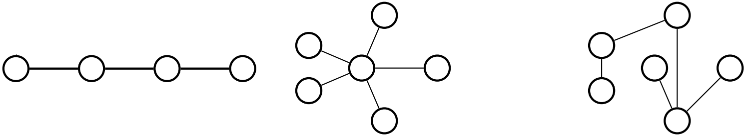
no cycle and connected? Yes.
$|V|-1$ edges and connected? Yes.
removing any edge disconnects it. Harder to check. but yes.
Adding any edge creates cycle. Harder to check. but yes.

# Trees.

Definitions:

A connected graph without a cycle.
A connected graph with $|V|-1$ edges.
A connected graph where any edge removal disconnects it.
A connected graph where any edge addition creates a cycle.

Some trees.



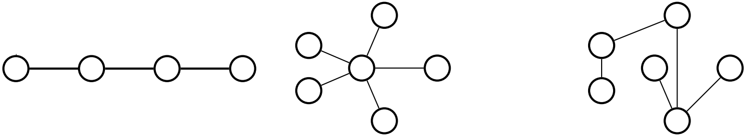
no cycle and connected? Yes.
$|V|-1$ edges and connected? Yes.
removing any edge disconnects it. Harder to check. but yes.
Adding any edge creates cycle. Harder to check. but yes.

# Trees.

Definitions:

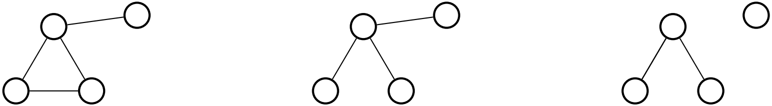
A connected graph without a cycle.
A connected graph with $|V|-1$ edges.
A connected graph where any edge removal disconnects it.
A connected graph where any edge addition creates a cycle.

Some trees.



no cycle and connected? Yes.
$|V|-1$ edges and connected? Yes.
removing any edge disconnects it. Harder to check. but yes.
Adding any edge creates cycle. Harder to check. but yes.

To tree or not to tree!



# Equivalence of Definitions.

**Theorem:**
"G connected and has $|V| - 1$ edges" $\equiv$
"G is connected and has no cycles."

# Equivalence of Definitions.

**Theorem:**
"G connected and has $|V|-1$ edges" $\equiv$
"G is connected and has no cycles."

**Lemma:** If $v$ is degree 1 in connected $G$, then $G-v$ is connected.
**Proof:**
For $x \neq v, y \neq v \in V$,

# Equivalence of Definitions.

**Theorem:**
"G connected and has $|V|-1$ edges" $\equiv$
"G is connected and has no cycles."

**Lemma:** If $v$ is degree 1 in connected $G$, then $G-v$ is connected.
**Proof:**
For $x \neq v, y \neq v \in V$,
there is path between $x$ and $y$ in $G$ since connected.

# Equivalence of Definitions.

**Theorem:**
"G connected and has $|V|-1$ edges" $\equiv$
"G is connected and has no cycles."

**Lemma:** If $v$ is degree 1 in connected $G$, then $G-v$ is connected.
**Proof:**
For $x \neq v, y \neq v \in V$,
there is path between $x$ and $y$ in $G$ since connected.
and does not use $v$ (degree 1)

# Equivalence of Definitions.

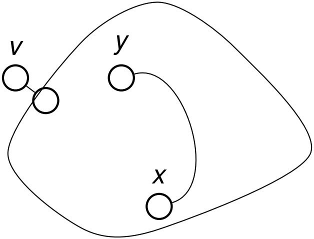
**Theorem:**
"G connected and has $|V|-1$ edges" $\equiv$
"G is connected and has no cycles."

**Lemma:** If $v$ is degree 1 in connected $G$, then $G-v$ is connected.
**Proof:**
For $x \neq v, y \neq v \in V$,
there is path between $x$ and $y$ in $G$ since connected.
and does not use $v$ (degree 1)
$\implies G-v$ is connected.

# Equivalence of Definitions.

**Theorem:**
"G connected and has $|V|-1$ edges" $\equiv$
"G is connected and has no cycles."

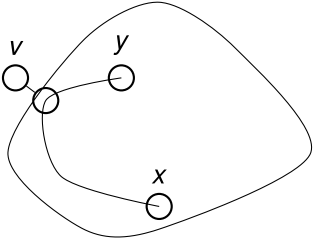
**Lemma:** If $v$ is degree 1 in connected $G$, then $G-v$ is connected.
**Proof:**
For $x \neq v, y \neq v \in V$,
there is path between $x$ and $y$ in $G$ since connected.
and does not use $v$ (degree 1)
$\implies G-v$ is connected. □



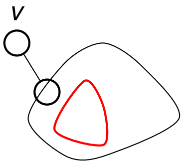
# Equivalence of Definitions.

**Theorem:**
"G connected and has $|V| - 1$ edges" $\equiv$
"G is connected and has no cycles."

**Lemma:** If $v$ is degree 1 in connected $G$, then $G - v$ is connected.
**Proof:**
For $x \neq v, y \neq v \in V$,
there is path between $x$ and $y$ in $G$ since connected.
and does not use $v$ (degree 1)
$\implies G - v$ is connected. □



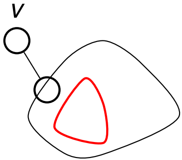
# Proof of only if.




**Thm:**
"G connected and has $|V|-1$ edges" $\equiv$
"G is connected and has no cycles."

**Proof of $\implies$ :**

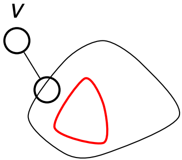
# Proof of only if.




**Thm:**
"G connected and has $|V|-1$ edges" $\equiv$
"G is connected and has no cycles."

**Proof of $\implies$ :** By induction on $|V|$.

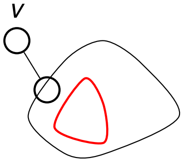
# Proof of only if.




**Thm:**
"G connected and has $|V|-1$ edges" $\equiv$
"G is connected and has no cycles."

**Proof of $\implies$ :** By induction on $|V|$.
Base Case: $|V|=1$. $0=|V|-1$ edges and has no cycles.

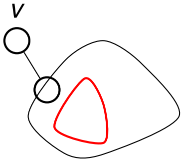
# Proof of only if.




**Thm:**
"G connected and has $|V| - 1$ edges" $\equiv$
"G is connected and has no cycles."

**Proof of $\implies$ :** By induction on $|V|$.
Base Case: $|V| = 1$. $0 = |V| - 1$ edges and has no cycles.

# Proof of only if.

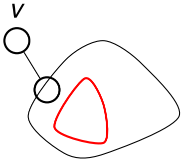



**Thm:**
"G connected and has $|V|-1$ edges" $\equiv$
"G is connected and has no cycles."

**Proof of $\implies$ :** By induction on $|V|$.
Base Case: $|V| = 1$. $0 = |V| - 1$ edges and has no cycles.

Induction Step:

# Proof of only if.

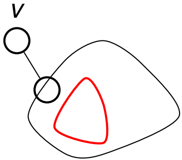



**Thm:**
"G connected and has $|V|-1$ edges" $\equiv$
"G is connected and has no cycles."

**Proof of $\implies$ :** By induction on $|V|$.
Base Case: $|V|=1$. $0=|V|-1$ edges and has no cycles.

Induction Step:
**Claim:** There is a degree 1 node.

# Proof of only if.

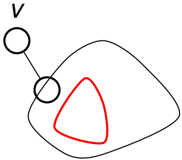



**Thm:**
"G connected and has $|V|-1$ edges" $\equiv$
"G is connected and has no cycles."

**Proof of $\implies$ :** By induction on $|V|$.
Base Case: $|V|=1$. $0=|V|-1$ edges and has no cycles.

Induction Step:
**Claim:** There is a degree 1 node.
**Proof:** First, connected $\implies$ every vertex degree $\geq 1$.

# Proof of only if.

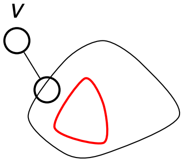



**Thm:**
"G connected and has $|V|-1$ edges" $\equiv$
"G is connected and has no cycles."

**Proof of $\implies$:** By induction on $|V|$.
Base Case: $|V|=1$. $0=|V|-1$ edges and has no cycles.

Induction Step:
**Claim:** There is a degree 1 node.
**Proof:** First, connected $\implies$ every vertex degree $\geq 1$.
Sum of degrees is $2|V|-2$

# Proof of only if.

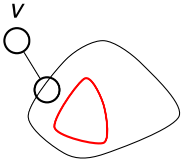



**Thm:**
"G connected and has $|V|-1$ edges" $\equiv$
"G is connected and has no cycles."

**Proof of $\implies$ :** By induction on $|V|$.
Base Case: $|V|=1$. $0=|V|-1$ edges and has no cycles.

Induction Step:
**Claim:** There is a degree 1 node.
**Proof:** First, connected $\implies$ every vertex degree $\geq 1$.
Sum of degrees is $2|V|-2$
Average degree $2-2/|V|$

# Proof of only if.

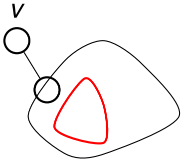



**Thm:**
"G connected and has $|V|-1$ edges" $\equiv$
"G is connected and has no cycles."

**Proof of $\implies$ :** By induction on $|V|$.
Base Case: $|V|=1$. $0=|V|-1$ edges and has no cycles.

Induction Step:
**Claim:** There is a degree 1 node.
**Proof:** First, connected $\implies$ every vertex degree $\geq 1$.
Sum of degrees is $2|V|-2$
Average degree $2-2/|V|$
Not everyone is bigger than average!

# Proof of only if.

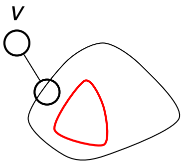



**Thm:**
"G connected and has $|V|-1$ edges" $\equiv$
"G is connected and has no cycles."

**Proof of $\implies$ :** By induction on $|V|$.
Base Case: $|V| = 1$. $0 = |V| - 1$ edges and has no cycles.

Induction Step:
**Claim:** There is a degree 1 node.
**Proof:** First, connected $\implies$ every vertex degree $\geq 1$.
Sum of degrees is $2|V| - 2$
Average degree $2 - 2/|V|$
Not everyone is bigger than average! $\square$

# Proof of only if.

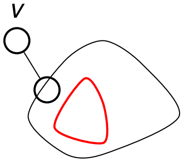



**Thm:**
"G connected and has $|V|-1$ edges" $\equiv$
"G is connected and has no cycles."

**Proof of $\Longrightarrow$ :** By induction on $|V|$.
Base Case: $|V|=1$. $0=|V|-1$ edges and has no cycles.

Induction Step:
**Claim:** There is a degree 1 node.
**Proof:** First, connected $\Longrightarrow$ every vertex degree $\geq 1$.
Sum of degrees is $2|V|-2$
Average degree $2-2/|V|$
Not everyone is bigger than average! $\square$
By degree 1 removal lemma, $G-v$ is connected.

# Proof of only if.

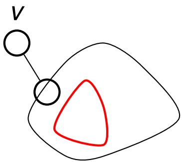



**Thm:**
"G connected and has $|V|-1$ edges" $\equiv$
"G is connected and has no cycles."

**Proof of $\implies$ :** By induction on $|V|$.
Base Case: $|V|=1$. $0=|V|-1$ edges and has no cycles.

Induction Step:
**Claim:** There is a degree 1 node.
**Proof:** First, connected $\implies$ every vertex degree $\geq 1$.
Sum of degrees is $2|V|-2$
Average degree $2-2/|V|$
Not everyone is bigger than average! □
By degree 1 removal lemma, $G-v$ is connected.
$G-v$ has $|V|-1$ vertices and $|V|-2$ edges so by induction

# Proof of only if.



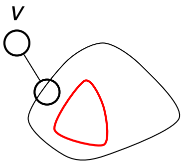

**Thm:**
"G connected and has $|V|-1$ edges" $\equiv$
"G is connected and has no cycles."

**Proof of $\implies$ :** By induction on $|V|$.
Base Case: $|V|=1$. $0=|V|-1$ edges and has no cycles.

Induction Step:
**Claim:** There is a degree 1 node.
**Proof:** First, connected $\implies$ every vertex degree $\geq 1$.
Sum of degrees is $2|V|-2$
Average degree $2-2/|V|$
Not everyone is bigger than average! $\square$

By degree 1 removal lemma, $G-v$ is connected.
$G-v$ has $|V|-1$ vertices and $|V|-2$ edges so by induction
$\implies$ no cycle in $G-v$.

# Proof of only if.



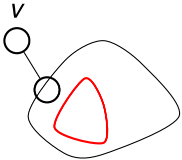

**Thm:**
"G connected and has $|V|-1$ edges" $\equiv$
"G is connected and has no cycles."

**Proof of $\implies$:** By induction on $|V|$.
Base Case: $|V|=1$. $0=|V|-1$ edges and has no cycles.

Induction Step:
**Claim:** There is a degree 1 node.
**Proof:** First, connected $\implies$ every vertex degree $\geq 1$.
Sum of degrees is $2|V|-2$
Average degree $2-2/|V|$
Not everyone is bigger than average! □

By degree 1 removal lemma, $G-v$ is connected.
$G-v$ has $|V|-1$ vertices and $|V|-2$ edges so by induction
$\implies$ no cycle in $G-v$.
And no cycle in $G$ since degree 1 cannot participate in cycle.

# Proof of only if.




**Thm:**
"G connected and has $|V| - 1$ edges" $\equiv$
"G is connected and has no cycles."

**Proof of $\implies$ :** By induction on $|V|$.
Base Case: $|V| = 1$. $0 = |V| - 1$ edges and has no cycles.

Induction Step:
**Claim:** There is a degree 1 node.
**Proof:** First, connected $\implies$ every vertex degree $\geq 1$.
Sum of degrees is $2|V| - 2$
Average degree $2 - 2/|V|$
Not everyone is bigger than average! □
By degree 1 removal lemma, $G - v$ is connected.
$G - v$ has $|V| - 1$ vertices and $|V| - 2$ edges so by induction
$\implies$ no cycle in $G - v$.
And no cycle in $G$ since degree 1 cannot participate in cycle. □

# Proof of if

**Thm:**
"G is connected and has no cycles"
$\Longrightarrow$ "G connected and has $|V|-1$ edges"
**Proof:**

# Proof of if

**Thm:**
"G is connected and has no cycles"
$\implies$ "G connected and has $|V| - 1$ edges"
**Proof:**
Walk from a vertex using untraversed edges.

# Proof of if

**Thm:**
"G is connected and has no cycles"
$\implies$ "G connected and has $|V| - 1$ edges"
**Proof:**
Walk from a vertex using untraversed edges.
Until get stuck.

# Proof of if

**Thm:**
"G is connected and has no cycles"
$\implies$ "G connected and has $|V| - 1$ edges"
**Proof:**
Walk from a vertex using untraversed edges.
Until get stuck.
**Claim:** Degree 1 vertex.

# Proof of if

**Thm:**
"G is connected and has no cycles"
$\implies$ "G connected and has $|V| - 1$ edges"
**Proof:**
Walk from a vertex using untraversed edges.
Until get stuck.
**Claim:** Degree 1 vertex.
**Proof of Claim:**
Can't visit more than once since no cycle.

# Proof of if

**Thm:**
"G is connected and has no cycles"
$\implies$ "G connected and has $|V| - 1$ edges"
**Proof:**
Walk from a vertex using untraversed edges.
Until get stuck.
**Claim:** Degree 1 vertex.
**Proof of Claim:**
Can't visit more than once since no cycle.
Entered.

# Proof of if

**Thm:**
"G is connected and has no cycles"
$\implies$ "G connected and has $|V| - 1$ edges"
**Proof:**
Walk from a vertex using untraversed edges.
Until get stuck.
**Claim:** Degree 1 vertex.
**Proof of Claim:**
Can't visit more than once since no cycle.
Entered. Didn't leave.

# Proof of if

**Thm:**
"G is connected and has no cycles"
$\implies$ "G connected and has $|V| - 1$ edges"
**Proof:**
Walk from a vertex using untraversed edges.
Until get stuck.
**Claim:** Degree 1 vertex.
**Proof of Claim:**
Can't visit more than once since no cycle.
Entered. Didn't leave. Only one incident edge.

# Proof of if

**Thm:**
"G is connected and has no cycles"
$\implies$ "G connected and has $|V| - 1$ edges"
**Proof:**
Walk from a vertex using untraversed edges.
Until get stuck.
**Claim:** Degree 1 vertex.
**Proof of Claim:**
Can't visit more than once since no cycle.
Entered. Didn't leave. Only one incident edge. $\square$

# Proof of if

**Thm:**
"G is connected and has no cycles"
$\implies$ "G connected and has $|V| - 1$ edges"
**Proof:**
Walk from a vertex using untraversed edges.
Until get stuck.
**Claim:** Degree 1 vertex.
**Proof of Claim:**
Can't visit more than once since no cycle.
Entered. Didn't leave. Only one incident edge. $\square$
Removing node doesn't create cycle.

# Proof of if

**Thm:**
"G is connected and has no cycles"
$\implies$ "G connected and has $|V| - 1$ edges"
**Proof:**
Walk from a vertex using untraversed edges.
Until get stuck.
**Claim:** Degree 1 vertex.
**Proof of Claim:**
Can't visit more than once since no cycle.
Entered. Didn't leave. Only one incident edge. □
Removing node doesn't create cycle.
New graph is connected.

# Proof of if

**Thm:**
"G is connected and has no cycles"
$\implies$ "G connected and has $|V| - 1$ edges"
**Proof:**
Walk from a vertex using untraversed edges.
Until get stuck.
**Claim:** Degree 1 vertex.
**Proof of Claim:**
Can't visit more than once since no cycle.
Entered. Didn't leave. Only one incident edge. □
Removing node doesn't create cycle.
New graph is connected.
Removing degree 1 node doesn't disconnect from Degree 1 lemma.

# Proof of if

**Thm:**
"G is connected and has no cycles"
$\implies$ "G connected and has $|V| - 1$ edges"
**Proof:**
Walk from a vertex using untraversed edges.
Until get stuck.
**Claim:** Degree 1 vertex.
**Proof of Claim:**
Can't visit more than once since no cycle.
Entered. Didn't leave. Only one incident edge. $\square$
Removing node doesn't create cycle.
New graph is connected.
Removing degree 1 node doesn't disconnect from Degree 1 lemma.
By induction $G - v$ has $|V| - 2$ edges.

# Proof of if

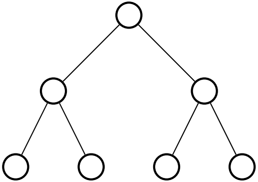
**Thm:**
"G is connected and has no cycles"
$\implies$ "G connected and has $|V| - 1$ edges"
**Proof:**
Walk from a vertex using untraversed edges.
Until get stuck.
**Claim:** Degree 1 vertex.
**Proof of Claim:**
Can't visit more than once since no cycle.
Entered. Didn't leave. Only one incident edge. □
Removing node doesn't create cycle.
New graph is connected.
Removing degree 1 node doesn't disconnect from Degree 1 lemma.
By induction $G - v$ has $|V| - 2$ edges.
$G$ has one more or $|V| - 1$ edges.

# Proof of if

**Thm:**
"G is connected and has no cycles"
$\implies$ "G connected and has $|V| - 1$ edges"
**Proof:**
Walk from a vertex using untraversed edges.
Until get stuck.
**Claim:** Degree 1 vertex.
**Proof of Claim:**
Can't visit more than once since no cycle.
Entered. Didn't leave. Only one incident edge. □
Removing node doesn't create cycle.
New graph is connected.
Removing degree 1 node doesn't disconnect from Degree 1 lemma.
By induction $G - v$ has $|V| - 2$ edges.
$G$ has one more or $|V| - 1$ edges. □

# Tree's fall apart.

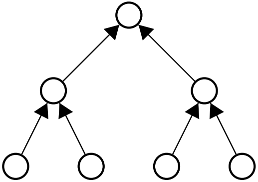
**Thm:** There is one vertex whose removal disconnects $|V|/2$ nodes from each other.



Idea of proof.

# Tree's fall apart.

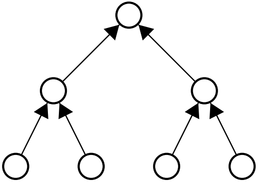
**Thm:** There is one vertex whose removal disconnects $|V|/2$ nodes from each other.



Idea of proof.
Point edge toward bigger side.

# Tree's fall apart.

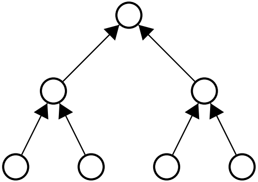
**Thm:** There is one vertex whose removal disconnects $|V|/2$ nodes from each other.



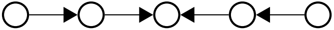
Idea of proof.
Point edge toward bigger side.
    Remove center node.

# Tree's fall apart.

**Thm:** There is one vertex whose removal disconnects $|V|/2$ nodes from each other.

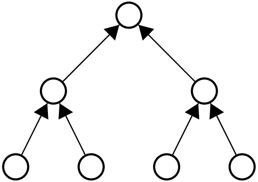


Idea of proof.
Point edge toward bigger side.
    Remove center node.

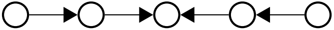


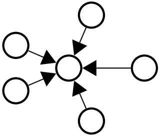
# Tree's fall apart.

**Thm:** There is one vertex whose removal disconnects $|V|/2$ nodes from each other.



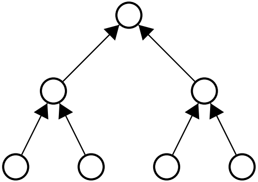
Idea of proof.
Point edge toward bigger side.
Remove center node.



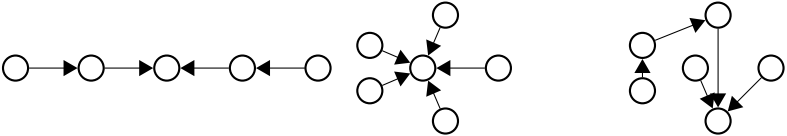


# Tree's fall apart.

**Thm:** There is one vertex whose removal disconnects $|V|/2$ nodes from each other.

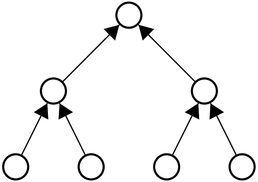


Idea of proof.
Point edge toward bigger side.
Remove center node.

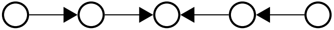


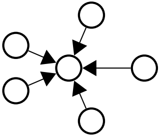
# Tree's fall apart.

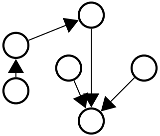
**Thm:** There is one vertex whose removal disconnects $|V|/2$ nodes from each other.



Idea of proof.
Point edge toward bigger side.
Remove center node.





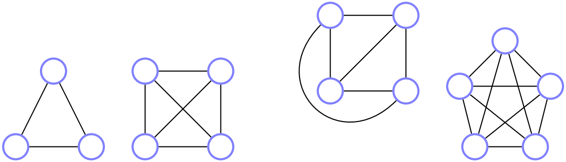


# Planar graphs.

A graph that can be drawn in the plane without edge crossings.

# Planar graphs.

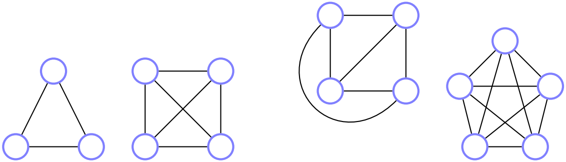
A graph that can be drawn in the plane without edge crossings.



Planar?

# Planar graphs.

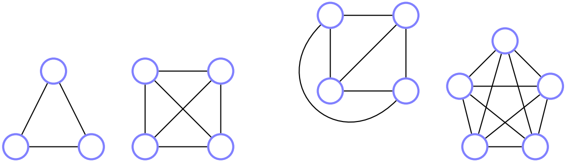
A graph that can be drawn in the plane without edge crossings.



Planar? Yes for Triangle.

# Planar graphs.

A graph that can be drawn in the plane without edge crossings.

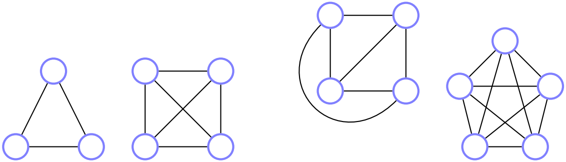


Planar? Yes for Triangle.

Four node complete?

# Planar graphs.

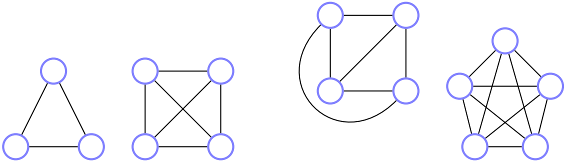
A graph that can be drawn in the plane without edge crossings.



Planar? Yes for Triangle.
Four node complete? Yes.

# Planar graphs.

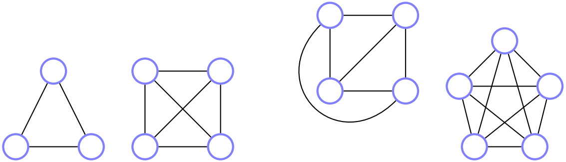
A graph that can be drawn in the plane without edge crossings.



Planar? Yes for Triangle.
Four node complete? Yes.
Five node complete or $K_5$?

# Planar graphs.

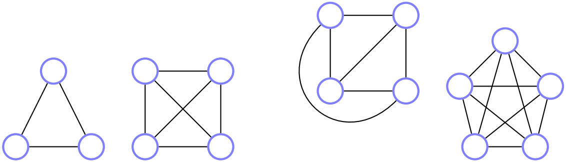
A graph that can be drawn in the plane without edge crossings.



Planar? Yes for Triangle.
Four node complete? Yes.
Five node complete or $K_5$? No!

# Planar graphs.

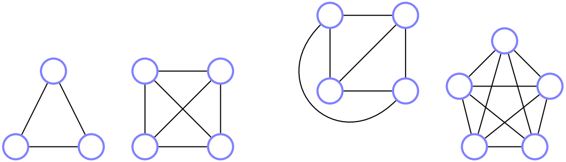
A graph that can be drawn in the plane without edge crossings.



Planar? Yes for Triangle.
Four node complete? Yes.
Five node complete or $K_5$? No! Why?

# Planar graphs.

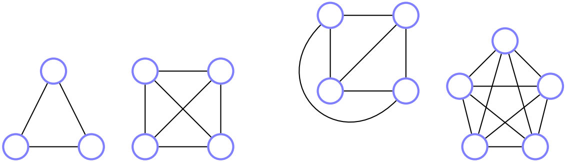
A graph that can be drawn in the plane without edge crossings.



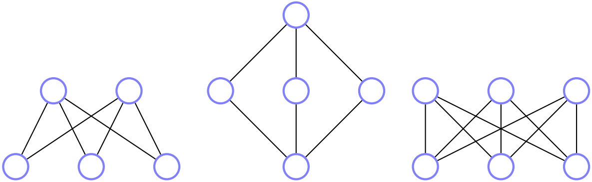
Planar? Yes for Triangle.
Four node complete? Yes.
Five node complete or $K_5$? No! Why? Later.

# Planar graphs.

A graph that can be drawn in the plane without edge crossings.

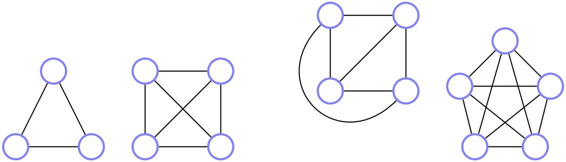


Planar? Yes for Triangle.
Four node complete? Yes.
Five node complete or $K_5$? No! Why? Later.

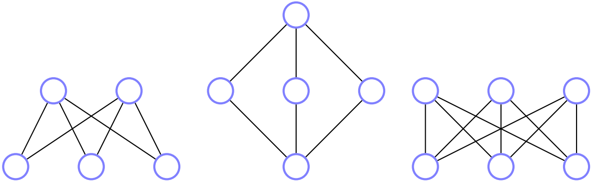


# Planar graphs.

A graph that can be drawn in the plane without edge crossings.



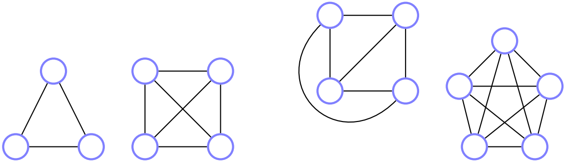
Planar? Yes for Triangle.
Four node complete? Yes.
Five node complete or $K_5$? No! Why? Later.



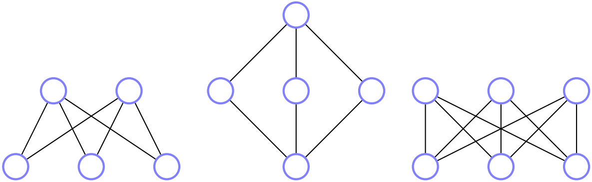
Two to three nodes, bipartite?

# Planar graphs.

A graph that can be drawn in the plane without edge crossings.



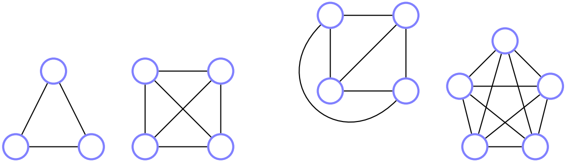
Planar? Yes for Triangle.
Four node complete? Yes.
Five node complete or $K_5$? No! Why? Later.



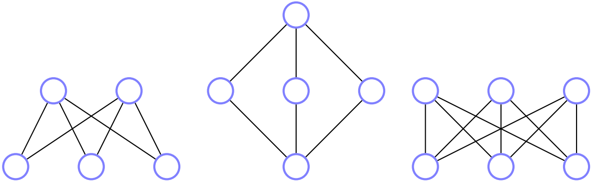
Two to three nodes, bipartite? Yes.

# Planar graphs.

A graph that can be drawn in the plane without edge crossings.



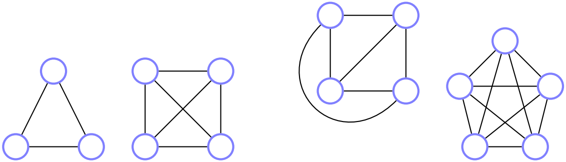
Planar? Yes for Triangle.
Four node complete? Yes.
Five node complete or $K_5$? No! Why? Later.



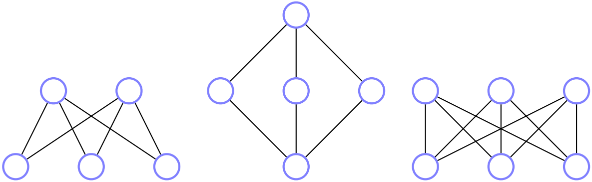
Two to three nodes, bipartite? Yes.
Three to three nodes, complete/bipartite or $K_{3,3}$.

# Planar graphs.

A graph that can be drawn in the plane without edge crossings.



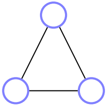
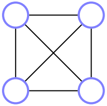
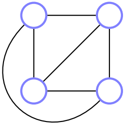
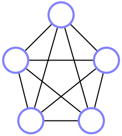
Planar? Yes for Triangle.
Four node complete? Yes.
Five node complete or $K_5$? No! Why? Later.



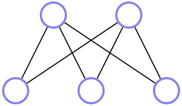
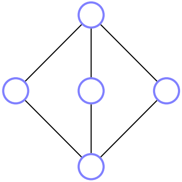
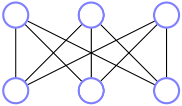
Two to three nodes, bipartite? Yes.
Three to three nodes, complete/bipartite or $K_{3,3}$. No.

# Planar graphs.

A graph that can be drawn in the plane without edge crossings.

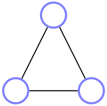
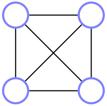
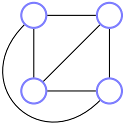
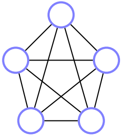
Planar? Yes for Triangle.
Four node complete? Yes.
Five node complete or $K_5$? No! Why? Later.

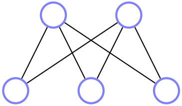
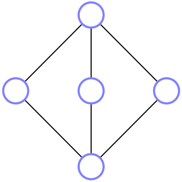
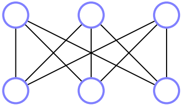
Two to three nodes, bipartite? Yes.
Three to three nodes, complete/bipartite or $K_{3,3}$. No. Why?

# Planar graphs.

A graph that can be drawn in the plane without edge crossings.

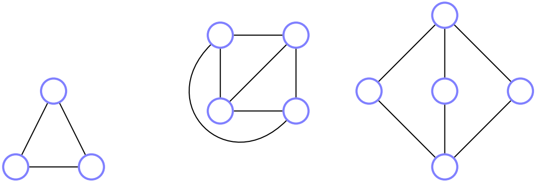
   

Planar? Yes for Triangle.
Four node complete? Yes.
Five node complete or $K_5$? No! Why? Later.

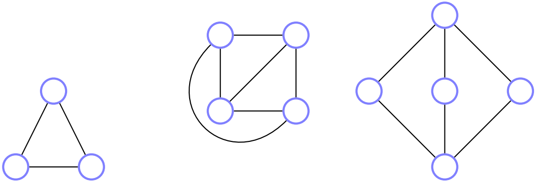
  

Two to three nodes, bipartite? Yes.
Three to three nodes, complete/bipartite or $K_{3,3}$. No. Why? Later.

# Euler's Formula.

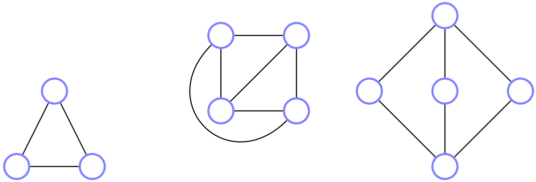


# Euler's Formula.



Faces: connected regions of the plane.

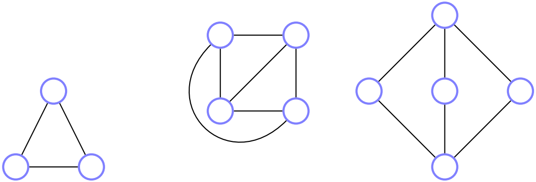
# Euler's Formula.



Faces: connected regions of the plane.

How many faces for

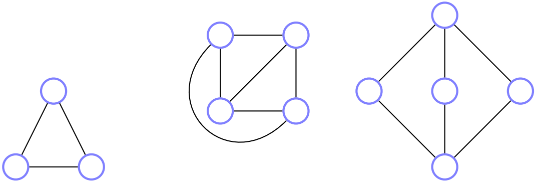
# Euler's Formula.



Faces: connected regions of the plane.

How many faces for
triangle?

# Euler's Formula.

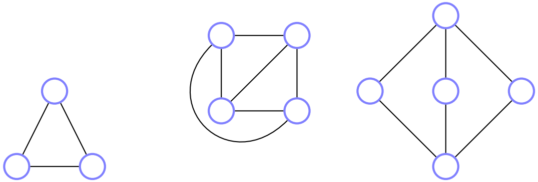


Faces: connected regions of the plane.

How many faces for

triangle? 2

# Euler’s Formula.



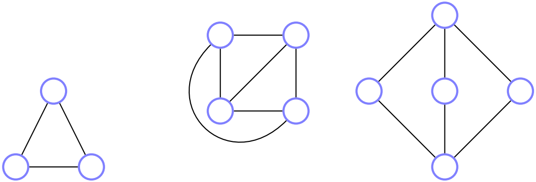
Faces: connected regions of the plane.

How many faces for

triangle? 2

complete on four vertices or $K_4$?

# Euler's Formula.



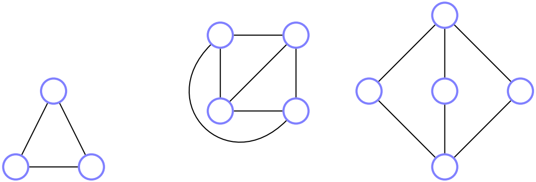
Faces: connected regions of the plane.

How many faces for

triangle? 2

complete on four vertices or $K_4$? 4

# Euler's Formula.



Faces: connected regions of the plane.

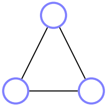
How many faces for

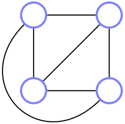
triangle? 2

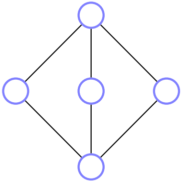
complete on four vertices or $K_4$? 4

bipartite, complete two/three or $K_{2,3}$?

# Euler's Formula.







Faces: connected regions of the plane.

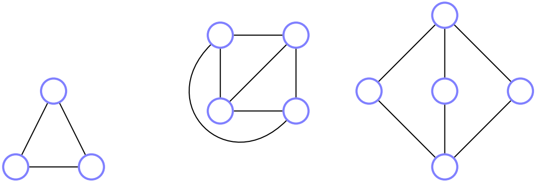
How many faces for

triangle? 2

complete on four vertices or $K_4$? 4

bipartite, complete two/three or $K_{2,3}$? 3

# Euler's Formula.



Faces: connected regions of the plane.

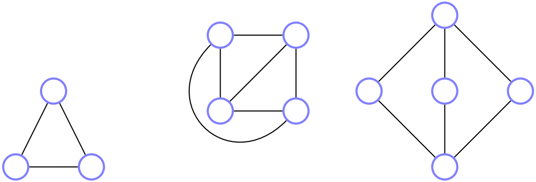
How many faces for

triangle? 2

complete on four vertices or $K_4$? 4

bipartite, complete two/three or $K_{2,3}$? 3

# Euler's Formula.



Faces: connected regions of the plane.

How many faces for

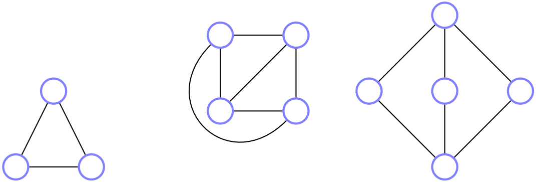
triangle? 2

complete on four vertices or $K_4$? 4

bipartite, complete two/three or $K_{2,3}$? 3

$v$ is number of vertices, $e$ is number of edges, $f$ is number of faces.

# Euler's Formula.



Faces: connected regions of the plane.

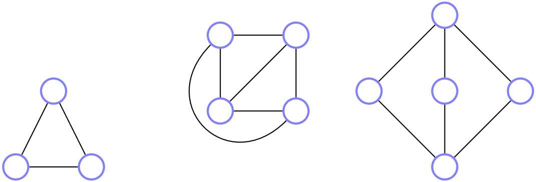
How many faces for

triangle? 2
complete on four vertices or $K_4$? 4
bipartite, complete two/three or $K_{2,3}$? 3

$v$ is number of vertices, $e$ is number of edges, $f$ is number of faces.

Euler's Formula: Connected planar graph has $v + f = e + 2$.

# Euler's Formula.



Faces: connected regions of the plane.

How many faces for

triangle? 2

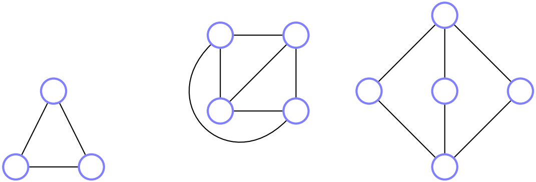
complete on four vertices or $K_4$? 4

bipartite, complete two/three or $K_{2,3}$? 3

$v$ is number of vertices, $e$ is number of edges, $f$ is number of faces.

Euler's Formula: Connected planar graph has $v + f = e + 2$.

# Euler's Formula.



Faces: connected regions of the plane.

How many faces for

triangle? 2

complete on four vertices or $K_4$? 4

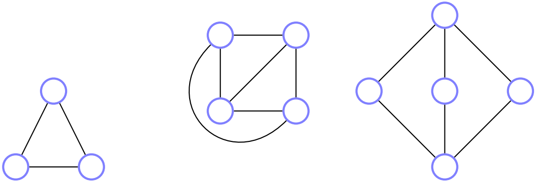
bipartite, complete two/three or $K_{2,3}$? 3

$v$ is number of vertices, $e$ is number of edges, $f$ is number of faces.

Euler's Formula: Connected planar graph has $v + f = e + 2$.

Triangle:

# Euler's Formula.



Faces: connected regions of the plane.

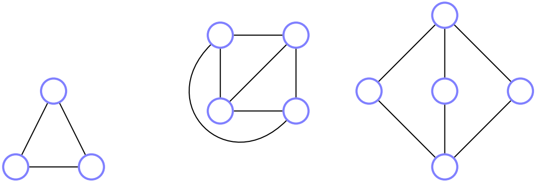
How many faces for
triangle? 2
complete on four vertices or $K_4$? 4
bipartite, complete two/three or $K_{2,3}$? 3

$v$ is number of vertices, $e$ is number of edges, $f$ is number of faces.

Euler's Formula: Connected planar graph has $v + f = e + 2$.

Triangle: $3 + 2 = 3 + 2$!

# Euler's Formula.



Faces: connected regions of the plane.

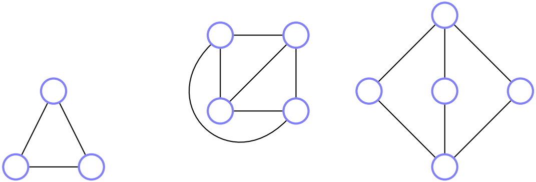
How many faces for
triangle? 2
complete on four vertices or $K_4$? 4
bipartite, complete two/three or $K_{2,3}$? 3

$v$ is number of vertices, $e$ is number of edges, $f$ is number of faces.

Euler's Formula: Connected planar graph has $v + f = e + 2$.

Triangle: $3 + 2 = 3 + 2$!
$K_4$:

# Euler's Formula.



Faces: connected regions of the plane.

How many faces for

triangle? 2

complete on four vertices or $K_4$? 4

bipartite, complete two/three or $K_{2,3}$? 3

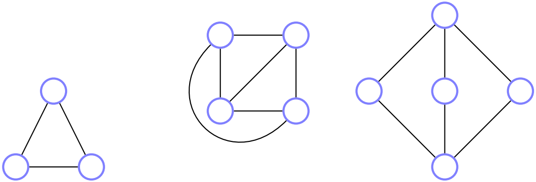
$v$ is number of vertices, $e$ is number of edges, $f$ is number of faces.

Euler's Formula: Connected planar graph has $v + f = e + 2$.

Triangle: $3 + 2 = 3 + 2$!

$K_4$: $4 + 4 = 6 + 2$!

# Euler's Formula.



Faces: connected regions of the plane.

How many faces for

triangle? 2

complete on four vertices or $K_4$? 4

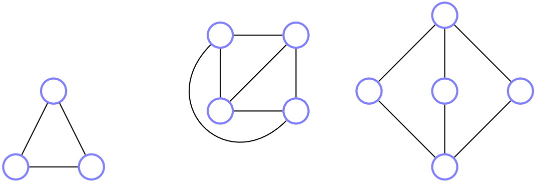
bipartite, complete two/three or $K_{2,3}$? 3

$v$ is number of vertices, $e$ is number of edges, $f$ is number of faces.

Euler's Formula: Connected planar graph has $v + f = e + 2$.

Triangle: $3 + 2 = 3 + 2$!
$K_4$: $4 + 4 = 6 + 2$!
$K_{2,3}$:

# Euler's Formula.



Faces: connected regions of the plane.

How many faces for

triangle? 2

complete on four vertices or $K_4$? 4

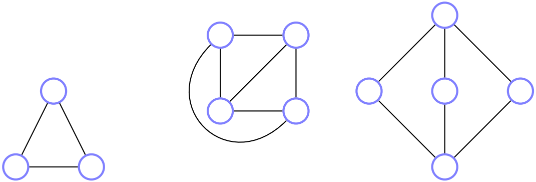
bipartite, complete two/three or $K_{2,3}$? 3

$v$ is number of vertices, $e$ is number of edges, $f$ is number of faces.

Euler's Formula: Connected planar graph has $v + f = e + 2$.

Triangle: $3 + 2 = 3 + 2$!
$K_4$: $4 + 4 = 6 + 2$!
$K_{2,3}$: $5 + 3 = 6 + 2$!

# Euler's Formula.



Faces: connected regions of the plane.

How many faces for

triangle? 2

complete on four vertices or $K_4$? 4

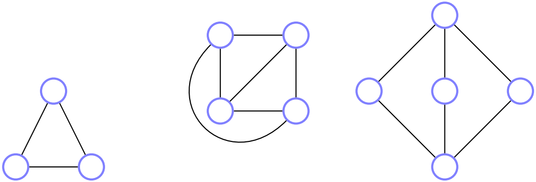
bipartite, complete two/three or $K_{2,3}$? 3

$v$ is number of vertices, $e$ is number of edges, $f$ is number of faces.

Euler's Formula: Connected planar graph has $v + f = e + 2$.

Triangle: $3 + 2 = 3 + 2$!
$K_4$: $4 + 4 = 6 + 2$!
$K_{2,3}$: $5 + 3 = 6 + 2$!

# Euler's Formula.



Faces: connected regions of the plane.

How many faces for
 triangle? 2
 complete on four vertices or $K_4$? 4
 bipartite, complete two/three or $K_{2,3}$? 3

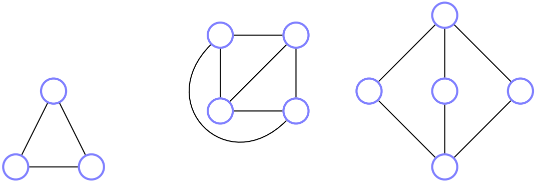
$v$ is number of vertices, $e$ is number of edges, $f$ is number of faces.

Euler's Formula: Connected planar graph has $v + f = e + 2$.

Triangle: $3 + 2 = 3 + 2$!
$K_4$: $4 + 4 = 6 + 2$!
$K_{2,3}$: $5 + 3 = 6 + 2$!

Examples = 3!

# Euler's Formula.



Faces: connected regions of the plane.

How many faces for

triangle? 2
complete on four vertices or $K_4$? 4
bipartite, complete two/three or $K_{2,3}$? 3

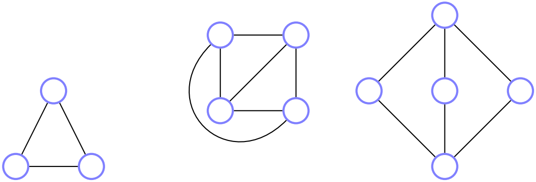
$v$ is number of vertices, $e$ is number of edges, $f$ is number of faces.

Euler's Formula: Connected planar graph has $v+f=e+2$.

Triangle: $3+2=3+2$!
$K_4$: $4+4=6+2$!
$K_{2,3}$: $5+3=6+2$!

Examples = 3! Proven!

# Euler's Formula.



Faces: connected regions of the plane.

How many faces for

triangle? 2

complete on four vertices or $K_4$? 4

bipartite, complete two/three or $K_{2,3}$? 3

$v$ is number of vertices, $e$ is number of edges, $f$ is number of faces.

Euler's Formula: Connected planar graph has $v + f = e + 2$.

Triangle: $3 + 2 = 3 + 2$!
$K_4$: $4 + 4 = 6 + 2$!
$K_{2,3}$: $5 + 3 = 6 + 2$!

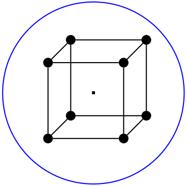
Examples = 3! Proven! Not!!!!

# Euler and Polyhedron.

Greeks knew formula for polyhedron.

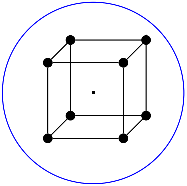
# Euler and Polyhedron.

Greeks knew formula for polyhedron.



# Euler and Polyhedron.

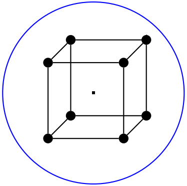
Greeks knew formula for polyhedron.



Faces?

# Euler and Polyhedron.

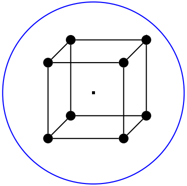
Greeks knew formula for polyhedron.



Faces? 6. Edges?

# Euler and Polyhedron.

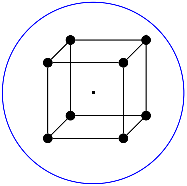
Greeks knew formula for polyhedron.



Faces? 6.     Edges? 12.

# Euler and Polyhedron.

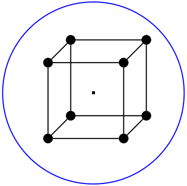
Greeks knew formula for polyhedron.



Faces? 6. Edges? 12. Vertices?

# Euler and Polyhedron.

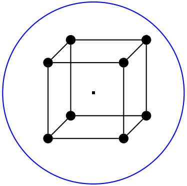
Greeks knew formula for polyhedron.



Faces? 6. Edges? 12. Vertices? 8.

# Euler and Polyhedron.

Greeks knew formula for polyhedron.

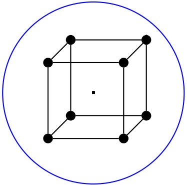


Faces? 6. Edges? 12. Vertices? 8.

Euler: Connected planar graph: $v + f = e + 2$.

# Euler and Polyhedron.

Greeks knew formula for polyhedron.

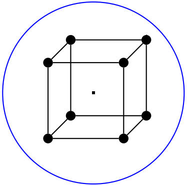


Faces? 6. Edges? 12. Vertices? 8.

Euler: Connected planar graph: $v + f = e + 2$.

# Euler and Polyhedron.

Greeks knew formula for polyhedron.



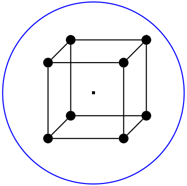
Faces? 6. Edges? 12. Vertices? 8.

Euler: Connected planar graph: $v + f = e + 2$.

$8 + 6 = 12 + 2$.

# Euler and Polyhedron.

Greeks knew formula for polyhedron.



Faces? 6. Edges? 12. Vertices? 8.

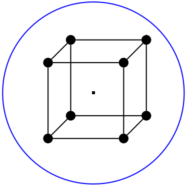
Euler: Connected planar graph: $v + f = e + 2$.

$8 + 6 = 12 + 2$.

Greeks couldn't prove it.

# Euler and Polyhedron.

Greeks knew formula for polyhedron.



Faces? 6. Edges? 12. Vertices? 8.

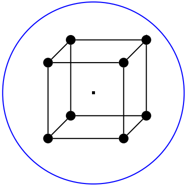
Euler: Connected planar graph: $v + f = e + 2$.

$8 + 6 = 12 + 2$.

Greeks couldn't prove it. Induction?

# Euler and Polyhedron.

Greeks knew formula for polyhedron.



Faces? 6. Edges? 12. Vertices? 8.

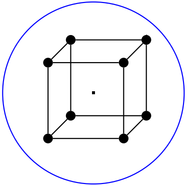
Euler: Connected planar graph: $v + f = e + 2$.

$8 + 6 = 12 + 2$.

Greeks couldn't prove it. Induction? Remove vertice for polyhedron?

# Euler and Polyhedron.

Greeks knew formula for polyhedron.



Faces? 6.  Edges? 12.  Vertices? 8.

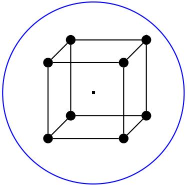
Euler: Connected planar graph: $v + f = e + 2$.

$8 + 6 = 12 + 2$.

Greeks couldn't prove it. Induction? Remove vertice for polyhedron?

# Euler and Polyhedron.

Greeks knew formula for polyhedron.



Faces? 6. Edges? 12. Vertices? 8.

Euler: Connected planar graph: $v + f = e + 2$.

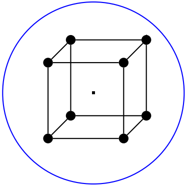
$8 + 6 = 12 + 2$.

Greeks couldn't prove it. Induction? Remove vertice for polyhedron?

Polyhedron without holes

# Euler and Polyhedron.

Greeks knew formula for polyhedron.



Faces? 6. Edges? 12. Vertices? 8.

Euler: Connected planar graph: $v + f = e + 2$.

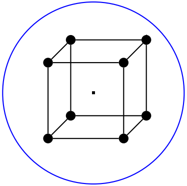
$8 + 6 = 12 + 2$.

Greeks couldn't prove it. Induction? Remove vertice for polyhedron?

Polyhedron without holes $\equiv$

# Euler and Polyhedron.

Greeks knew formula for polyhedron.



Faces? 6. Edges? 12. Vertices? 8.

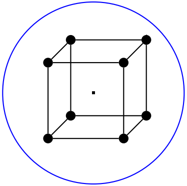
Euler: Connected planar graph: $v + f = e + 2$.
$8 + 6 = 12 + 2$.

Greeks couldn't prove it. Induction? Remove vertice for polyhedron?

Polyhedron without holes $\equiv$ Planar graphs.

# Euler and Polyhedron.

Greeks knew formula for polyhedron.



Faces? 6. Edges? 12. Vertices? 8.

Euler: Connected planar graph: $v + f = e + 2$.
$8 + 6 = 12 + 2$.

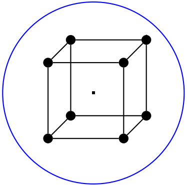
Greeks couldn't prove it. Induction? Remove vertice for polyhedron?

Polyhedron without holes $\equiv$ Planar graphs.

Surround by sphere.

# Euler and Polyhedron.

Greeks knew formula for polyhedron.



Faces? 6.    Edges? 12.    Vertices? 8.

Euler: Connected planar graph: $v+f=e+2$.
$8+6=12+2$.

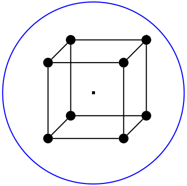
Greeks couldn't prove it. Induction? Remove vertice for polyhedron?

Polyhedron without holes $\equiv$ Planar graphs.

Surround by sphere.
Project from point inside polytope onto sphere.

# Euler and Polyhedron.

Greeks knew formula for polyhedron.



Faces? 6. Edges? 12. Vertices? 8.

Euler: Connected planar graph: $v + f = e + 2$.
$8 + 6 = 12 + 2$.

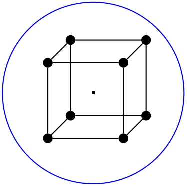
Greeks couldn't prove it. Induction? Remove vertice for polyhedron?

Polyhedron without holes $\equiv$ Planar graphs.

Surround by sphere.
Project from point inside polytope onto sphere.
Sphere

# Euler and Polyhedron.

Greeks knew formula for polyhedron.



Faces? 6. Edges? 12. Vertices? 8.

Euler: Connected planar graph: $v + f = e + 2$.

$8 + 6 = 12 + 2$.

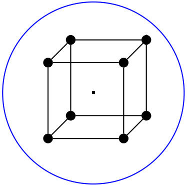
Greeks couldn't prove it. Induction? Remove vertice for polyhedron?

Polyhedron without holes $\equiv$ Planar graphs.

Surround by sphere.
Project from point inside polytope onto sphere.
Sphere $\equiv$ Plane!

# Euler and Polyhedron.

Greeks knew formula for polyhedron.



Faces? 6. Edges? 12. Vertices? 8.

Euler: Connected planar graph: $v + f = e + 2$.

$8 + 6 = 12 + 2$.

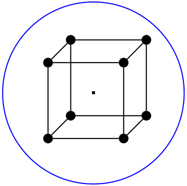
Greeks couldn't prove it. Induction? Remove vertice for polyhedron?

Polyhedron without holes $\equiv$ Planar graphs.

Surround by sphere.
Project from point inside polytope onto sphere.
Sphere $\equiv$ Plane! Topologically.

# Euler and Polyhedron.

Greeks knew formula for polyhedron.



Faces? 6. Edges? 12. Vertices? 8.

Euler: Connected planar graph: $v + f = e + 2$.
$8 + 6 = 12 + 2$.

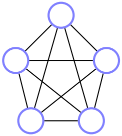
Greeks couldn't prove it. Induction? Remove vertice for polyhedron?

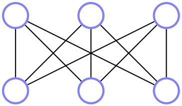
Polyhedron without holes $\equiv$ Planar graphs.

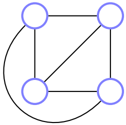
Surround by sphere.
Project from point inside polytope onto sphere.
Sphere $\equiv$ Plane! Topologically.

Euler proved formula thousands of years later!

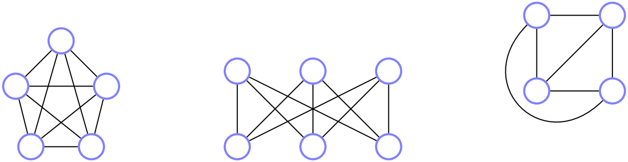
# Euler and planarity of $K_5$ and $K_{3,3}$





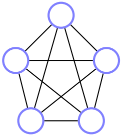


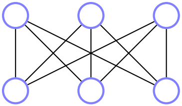
# Euler and planarity of $K_5$ and $K_{3,3}$

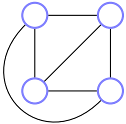


Euler: $v + f = e + 2$ for connected planar graph.

# Euler and planarity of $K_5$ and $K_{3,3}$

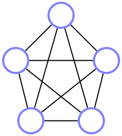


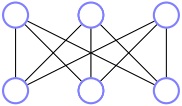


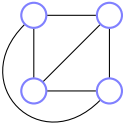


Euler: $v + f = e + 2$ for connected planar graph.
We consider graphs where $v \geq 3$.

# Euler and planarity of $K_5$ and $K_{3,3}$





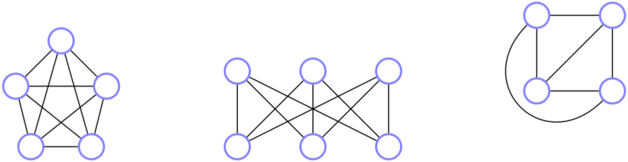


Euler: $v + f = e + 2$ for connected planar graph.

We consider graphs where $v \geq 3$.

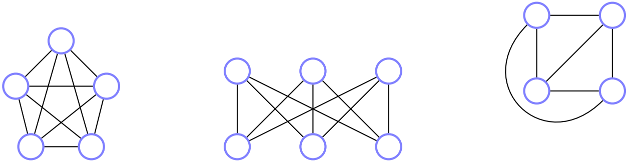
Each face is adjacent to edge at least 3 times for simple graph.

# Euler and planarity of $K_5$ and $K_{3,3}$



Euler: $v + f = e + 2$ for connected planar graph.
We consider graphs where $v \geq 3$.
Each face is adjacent to edge at least 3 times for simple graph.
$\geq 3f$ face-edge adjacencies.

# Euler and planarity of $K_5$ and $K_{3,3}$



Euler: $v + f = e + 2$ for connected planar graph.

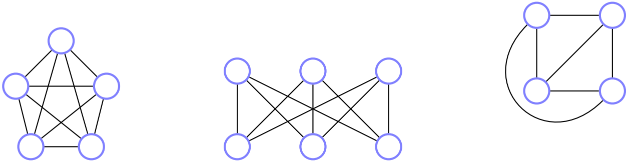
We consider graphs where $v \geq 3$.

Each face is adjacent to edge at least 3 times for simple graph.

$\geq 3f$ face-edge adjacencies.

Each edge is adjacent to (at most) two faces.

# Euler and planarity of $K_5$ and $K_{3,3}$



Euler: $v + f = e + 2$ for connected planar graph.

We consider graphs where $v \geq 3$.

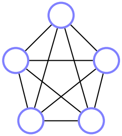
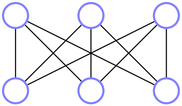
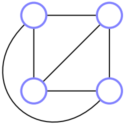
Each face is adjacent to edge at least 3 times for simple graph.

$\geq 3f$ face-edge adjacencies.

Each edge is adjacent to (at most) two faces.

$\leq 2e$ face-edge adjacencies.

# Euler and planarity of $K_5$ and $K_{3,3}$

Euler: $v + f = e + 2$ for connected planar graph.

We consider graphs where $v \geq 3$.

Each face is adjacent to edge at least 3 times for simple graph.

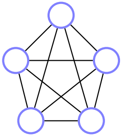
$\geq 3f$ face-edge adjacencies.

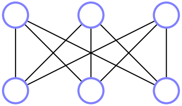
Each edge is adjacent to (at most) two faces.

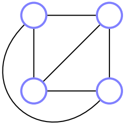
$\leq 2e$ face-edge adjacencies.

$\implies 3f \leq 2e$

# Euler and planarity of $K_5$ and $K_{3,3}$







Euler: $v + f = e + 2$ for connected planar graph.

We consider graphs where $v \geq 3$.

Each face is adjacent to edge at least 3 times for simple graph.

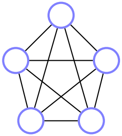
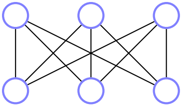
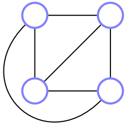
$\geq 3f$ face-edge adjacencies.

Each edge is adjacent to (at most) two faces.

$\leq 2e$ face-edge adjacencies.

$\implies 3f \leq 2e$ *for*

# Euler and planarity of $K_5$ and $K_{3,3}$

Euler: $v + f = e + 2$ for connected planar graph.

We consider graphs where $v \geq 3$.

Each face is adjacent to edge at least 3 times for simple graph.

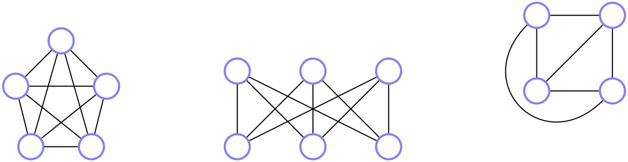
$\geq 3f$ face-edge adjacencies.

Each edge is adjacent to (at most) two faces.

$\leq 2e$ face-edge adjacencies.

$\implies 3f \leq 2e$ *for any*

# Euler and planarity of $K_5$ and $K_{3,3}$



Euler: $v + f = e + 2$ for connected planar graph.

We consider graphs where $v \geq 3$.

Each face is adjacent to edge at least 3 times for simple graph.

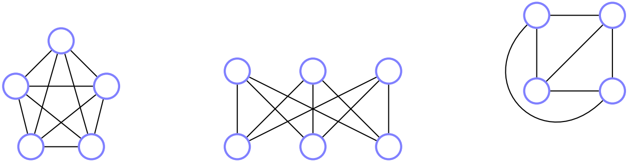
$\geq 3f$ face-edge adjacencies.

Each edge is adjacent to (at most) two faces.

$\leq 2e$ face-edge adjacencies.

$\implies 3f \leq 2e$ *for any planar*

# Euler and planarity of $K_5$ and $K_{3,3}$



Euler: $v + f = e + 2$ for connected planar graph.

We consider graphs where $v \geq 3$.

Each face is adjacent to edge at least 3 times for simple graph.

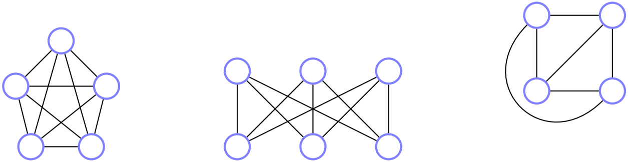
$\geq 3f$ face-edge adjacencies.

Each edge is adjacent to (at most) two faces.

$\leq 2e$ face-edge adjacencies.

$\implies$ $3f \leq 2e$ *for any planar graph with more than* 2 *vertices*

# Euler and planarity of $K_5$ and $K_{3,3}$



Euler: $v + f = e + 2$ for connected planar graph.

We consider graphs where $v \geq 3$.

Each face is adjacent to edge at least 3 times for simple graph.

$\geq 3f$ face-edge adjacencies.

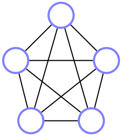
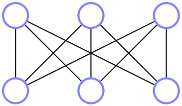
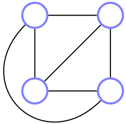
Each edge is adjacent to (at most) two faces.

$\leq 2e$ face-edge adjacencies.

$\implies 3f \leq 2e$ *for any planar graph with more than* 2 *vertices*

... or $\frac{2}{3}e \geq f$.

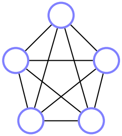
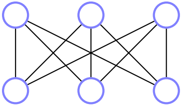
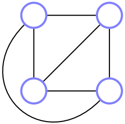
# Euler and planarity of $K_5$ and $K_{3,3}$

Euler: $v + f = e + 2$ for connected planar graph.
  We consider graphs where $v \geq 3$.
  Each face is adjacent to edge at least 3 times for simple graph.
$\geq 3f$ face-edge adjacencies.
  Each edge is adjacent to (at most) two faces.
    $\leq 2e$ face-edge adjacencies.
$\Longrightarrow$ $3f \leq 2e$ *for any planar graph with more than* 2 *vertices*
... or $\frac{2}{3}e \geq f$.

+ Euler:

# Euler and planarity of $K_5$ and $K_{3,3}$

Euler: $v + f = e + 2$ for connected planar graph.

We consider graphs where $v \geq 3$.

Each face is adjacent to edge at least 3 times for simple graph.

$\geq 3f$ face-edge adjacencies.

Each edge is adjacent to (at most) two faces.

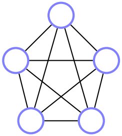
$\leq 2e$ face-edge adjacencies.

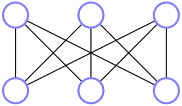
$\Longrightarrow$ $3f \leq 2e$ *for any planar graph with more than* 2 *vertices*

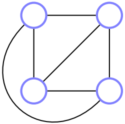
... or $\frac{2}{3}e \geq f$.

+ Euler: $v + \frac{2}{3}e \geq e + 2$

# Euler and planarity of $K_5$ and $K_{3,3}$







Euler: $v + f = e + 2$ for connected planar graph.

We consider graphs where $v \geq 3$.

Each face is adjacent to edge at least 3 times for simple graph.

$\geq 3f$ face-edge adjacencies.

Each edge is adjacent to (at most) two faces.

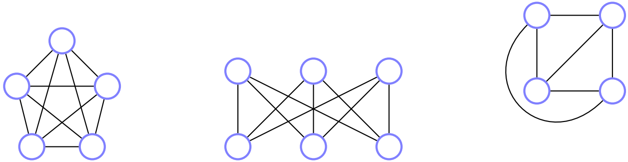
$\leq 2e$ face-edge adjacencies.

$\implies 3f \leq 2e$ *for any planar graph with more than* 2 *vertices*

... or $\frac{2}{3}e \geq f$.

+ Euler: $v + \frac{2}{3}e \geq e + 2 \implies e \leq 3v - 6$

# Euler and planarity of $K_5$ and $K_{3,3}$



Euler: $v + f = e + 2$ for connected planar graph.

We consider graphs where $v \geq 3$.

Each face is adjacent to edge at least 3 times for simple graph.

$\geq 3f$ face-edge adjacencies.

Each edge is adjacent to (at most) two faces.

$\leq 2e$ face-edge adjacencies.

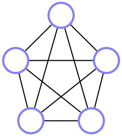
$\implies 3f \leq 2e$ *for any planar graph with more than* 2 *vertices*

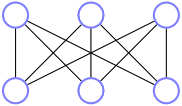
... or $\frac{2}{3}e \geq f$.

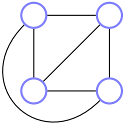
+ Euler: $v + \frac{2}{3}e \geq e + 2 \implies e \leq 3v - 6$

$K_5$

# Euler and planarity of $K_5$ and $K_{3,3}$







Euler: $v + f = e + 2$ for connected planar graph.

We consider graphs where $v \geq 3$.

Each face is adjacent to edge at least 3 times for simple graph.

$\geq 3f$ face-edge adjacencies.

Each edge is adjacent to (at most) two faces.

$\leq 2e$ face-edge adjacencies.

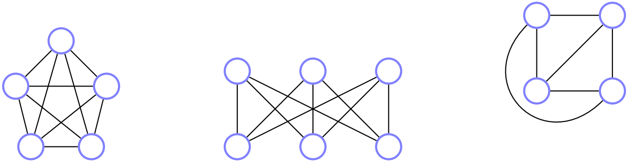
$\implies 3f \leq 2e$ *for any planar graph with more than* 2 *vertices*

... or $\frac{2}{3}e \geq f$.

+ Euler: $v + \frac{2}{3}e \geq e + 2 \implies e \leq 3v - 6$

$K_5$ Edges?

# Euler and planarity of $K_5$ and $K_{3,3}$



Euler: $v + f = e + 2$ for connected planar graph.

We consider graphs where $v \geq 3$.

Each face is adjacent to edge at least 3 times for simple graph.

$\geq 3f$ face-edge adjacencies.

Each edge is adjacent to (at most) two faces.

$\leq 2e$ face-edge adjacencies.

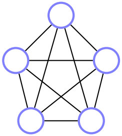
$\Longrightarrow$ $3f \leq 2e$ *for any planar graph with more than* 2 *vertices*

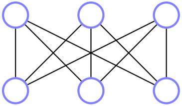
... or $\frac{2}{3}e \geq f$.

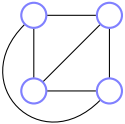
+ Euler: $v + \frac{2}{3}e \geq e + 2 \implies e \leq 3v - 6$

$K_5$ Edges? $4 + 3 + 2 + 1$

# Euler and planarity of $K_5$ and $K_{3,3}$







Euler: $v + f = e + 2$ for connected planar graph.

We consider graphs where $v \geq 3$.

Each face is adjacent to edge at least 3 times for simple graph.

$\geq 3f$ face-edge adjacencies.

Each edge is adjacent to (at most) two faces.

$\leq 2e$ face-edge adjacencies.

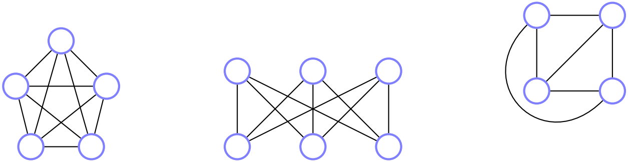
$\Longrightarrow$ $3f \leq 2e$ *for any planar graph with more than* 2 *vertices*

... or $\frac{2}{3}e \geq f$.

+ Euler: $v + \frac{2}{3}e \geq e + 2 \implies e \leq 3v - 6$

$K_5$ Edges? $4 + 3 + 2 + 1 = 10$.

# Euler and planarity of $K_5$ and $K_{3,3}$

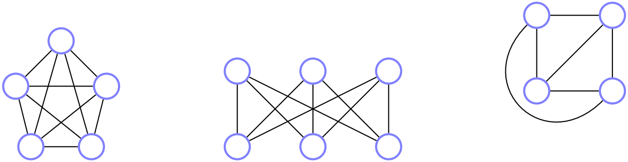


Euler: $v + f = e + 2$ for connected planar graph.
We consider graphs where $v \geq 3$.
Each face is adjacent to edge at least 3 times for simple graph.
$\geq 3f$ face-edge adjacencies.
Each edge is adjacent to (at most) two faces.
$\leq 2e$ face-edge adjacencies.
$\implies 3f \leq 2e$ *for any planar graph with more than* 2 *vertices*
... or $\frac{2}{3}e \geq f$.

+ Euler: $v + \frac{2}{3}e \geq e + 2 \implies e \leq 3v - 6$

$K_5$ Edges? $4+3+2+1 = 10$. Vertices?

# Euler and planarity of $K_5$ and $K_{3,3}$

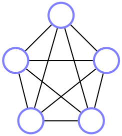
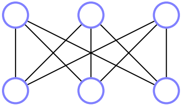
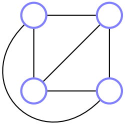


Euler: $v + f = e + 2$ for connected planar graph.
We consider graphs where $v \geq 3$.
Each face is adjacent to edge at least 3 times for simple graph.
$\geq 3f$ face-edge adjacencies.
Each edge is adjacent to (at most) two faces.
$\leq 2e$ face-edge adjacencies.
$\implies 3f \leq 2e$ *for any planar graph with more than* 2 *vertices*
... or $\frac{2}{3}e \geq f$.

+ Euler: $v + \frac{2}{3}e \geq e + 2 \implies e \leq 3v - 6$

$K_5$ Edges? $4 + 3 + 2 + 1 = 10$. Vertices? 5.

# Euler and planarity of $K_5$ and $K_{3,3}$

Euler: $v + f = e + 2$ for connected planar graph.

We consider graphs where $v \geq 3$.

Each face is adjacent to edge at least 3 times for simple graph.

$\geq 3f$ face-edge adjacencies.

Each edge is adjacent to (at most) two faces.

$\leq 2e$ face-edge adjacencies.

$\implies 3f \leq 2e$ *for any planar graph with more than* 2 *vertices*

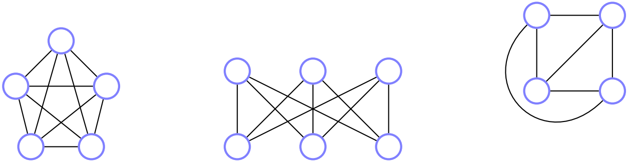
... or $\frac{2}{3}e \geq f$.

+ Euler: $v + \frac{2}{3}e \geq e + 2 \implies e \leq 3v - 6$

$K_5$ Edges? $4+3+2+1 = 10$. Vertices? 5.

$10 \not\leq 3(5) - 6 = 9$.

# Euler and planarity of $K_5$ and $K_{3,3}$

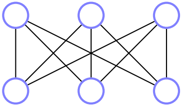


Euler: $v + f = e + 2$ for connected planar graph.
We consider graphs where $v \geq 3$.
Each face is adjacent to edge at least 3 times for simple graph.
$\geq 3f$ face-edge adjacencies.
Each edge is adjacent to (at most) two faces.
$\leq 2e$ face-edge adjacencies.
$\implies 3f \leq 2e$ *for any planar graph with more than* 2 *vertices*
... or $\frac{2}{3}e \geq f$.

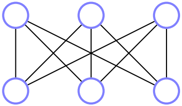
+ Euler: $v + \frac{2}{3}e \geq e + 2 \implies e \leq 3v - 6$

$K_5$ Edges? $4 + 3 + 2 + 1 = 10$. Vertices? 5.
$10 \not\leq 3(5) - 6 = 9$. $\implies K_5$ is not planar.

# $K_{3,3}$ non-planarity.

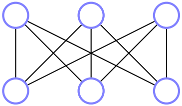


# $K_{3,3}$ non-planarity.



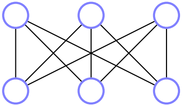
Euler: $v + \frac{2}{3}e \geq e + 2$

# $K_{3,3}$ non-planarity.



Euler: $v + \frac{2}{3}e \geq e + 2 \implies e \leq 3v - 6$

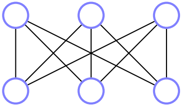
# $K_{3,3}$ non-planarity.



Euler: $v + \frac{2}{3}e \geq e + 2 \implies e \leq 3v - 6$

$K_{3,3}$?

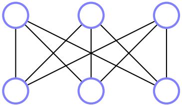
# $K_{3,3}$ non-planarity.



Euler: $v + \frac{2}{3}e \geq e + 2 \implies e \leq 3v - 6$

$K_{3,3}$? Edges?

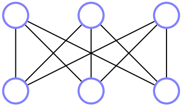
# $K_{3,3}$ non-planarity.



Euler: $v + \frac{2}{3}e \geq e + 2 \implies e \leq 3v - 6$

$K_{3,3}$? Edges? 9.

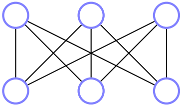
# $K_{3,3}$ non-planarity.



Euler: $v + \frac{2}{3}e \geq e + 2 \implies e \leq 3v - 6$

$K_{3,3}$? Edges? 9. Vertices. 6.

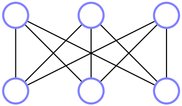
# $K_{3,3}$ non-planarity.



Euler: $v + \frac{2}{3}e \geq e + 2 \implies e \leq 3v - 6$

$K_{3,3}$? Edges? 9. Vertices. 6. $9 \leq 3(6) - 6$?

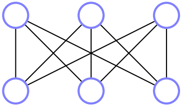
# $K_{3,3}$ non-planarity.



Euler: $v + \frac{2}{3}e \geq e + 2 \implies e \leq 3v - 6$

$K_{3,3}$? Edges? 9. Vertices. 6. $9 \leq 3(6) - 6$? Sure!

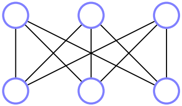
# $K_{3,3}$ non-planarity.



Euler: $v + \frac{2}{3}e \geq e + 2 \implies e \leq 3v - 6$

$K_{3,3}$? Edges? 9. Vertices. 6. $9 \leq 3(6) - 6$? Sure!

# $K_{3,3}$ non-planarity.

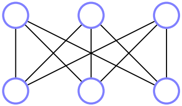


Euler: $v + \frac{2}{3}e \geq e + 2 \implies e \leq 3v - 6$

$K_{3,3}$? Edges? 9. Vertices. 6. $9 \leq 3(6) - 6$? Sure!

Planar?

# $K_{3,3}$ non-planarity.

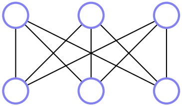


Euler: $v + \frac{2}{3}e \geq e + 2 \implies e \leq 3v - 6$

$K_{3,3}$? Edges? 9. Vertices. 6. $9 \leq 3(6) - 6$? Sure!

Planar? No.

# $K_{3,3}$ non-planarity.

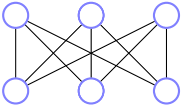


Euler: $v + \frac{2}{3}e \geq e + 2 \implies e \leq 3v - 6$

$K_{3,3}$? Edges? 9. Vertices. 6. $9 \leq 3(6) - 6$? Sure!

Planar? No.

# $K_{3,3}$ non-planarity.



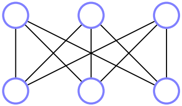
Euler: $v + \frac{2}{3}e \geq e + 2 \implies e \leq 3v - 6$

$K_{3,3}$? Edges? 9. Vertices. 6. $9 \leq 3(6) - 6$? Sure!

Planar? No.

No cycles that are triangles.

# $K_{3,3}$ non-planarity.



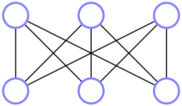
Euler: $v + \frac{2}{3}e \geq e + 2 \implies e \leq 3v - 6$

$K_{3,3}$? Edges? 9. Vertices. 6. $9 \leq 3(6) - 6$? Sure!

Planar? No.

No cycles that are triangles.
Cycles of length $\geq 4$.

# $K_{3,3}$ non-planarity.



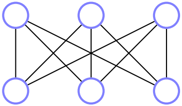
Euler: $v + \frac{2}{3}e \geq e + 2 \implies e \leq 3v - 6$

$K_{3,3}$? Edges? 9. Vertices. 6. $9 \leq 3(6) - 6$? Sure!

Planar? No.

No cycles that are triangles.
  Cycles of length $\geq 4$.
At least $4f$ face-edge adjacencies,

# $K_{3,3}$ non-planarity.



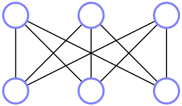
Euler: $v + \frac{2}{3}e \geq e + 2 \implies e \leq 3v - 6$

$K_{3,3}$? Edges? 9. Vertices. 6. $9 \leq 3(6) - 6$? Sure!

Planar? No.

No cycles that are triangles.
  Cycles of length $\geq 4$.
At least $4f$ face-edge adjacencies,
  and at most $2e$.

# $K_{3,3}$ non-planarity.



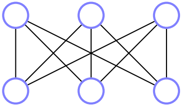
Euler: $v + \frac{2}{3}e \geq e + 2 \implies e \leq 3v - 6$

$K_{3,3}$? Edges? 9. Vertices. 6. $9 \leq 3(6) - 6$? Sure!

Planar? No.

No cycles that are triangles.
  Cycles of length $\geq 4$.
At least $4f$ face-edge adjacencies,
  and at most $2e$.
.... $4f \leq 2e$

# $K_{3,3}$ non-planarity.



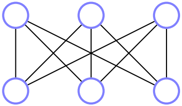
Euler: $v + \frac{2}{3}e \geq e + 2 \implies e \leq 3v - 6$

$K_{3,3}$? Edges? 9. Vertices. 6. $9 \leq 3(6) - 6$? Sure!

Planar? No.

No cycles that are triangles.
  Cycles of length $\geq 4$.
At least $4f$ face-edge adjacencies,
   and at most $2e$.
.... $4f \leq 2e$ for

# $K_{3,3}$ non-planarity.



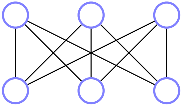
Euler: $v + \frac{2}{3}e \geq e + 2 \implies e \leq 3v - 6$

$K_{3,3}$? Edges? 9. Vertices. 6. $9 \leq 3(6) - 6$? Sure!

Planar? No.

No cycles that are triangles.
  Cycles of length $\geq 4$.
At least $4f$ face-edge adjacencies,
  and at most $2e$.
.... $4f \leq 2e$ for any

# $K_{3,3}$ non-planarity.



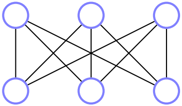
Euler: $v + \frac{2}{3}e \geq e + 2 \implies e \leq 3v - 6$

$K_{3,3}$? Edges? 9. Vertices. 6. $9 \leq 3(6) - 6$? Sure!

Planar? No.

No cycles that are triangles.
  Cycles of length $\geq 4$.
At least $4f$ face-edge adjacencies,
  and at most $2e$.
.... $4f \leq 2e$ for any bipartite

# $K_{3,3}$ non-planarity.



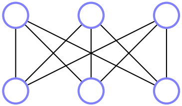
Euler: $v + \frac{2}{3}e \geq e + 2 \implies e \leq 3v - 6$

$K_{3,3}$? Edges? 9. Vertices. 6. $9 \leq 3(6) - 6$? Sure!

Planar? No.

No cycles that are triangles.
  Cycles of length $\geq 4$.
At least $4f$ face-edge adjacencies,
  and at most $2e$.
.... $4f \leq 2e$ for any bipartite planar graph.

# $K_{3,3}$ non-planarity.



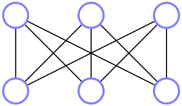
Euler: $v + \frac{2}{3}e \geq e + 2 \implies e \leq 3v - 6$

$K_{3,3}$? Edges? 9. Vertices. 6. $9 \leq 3(6) - 6$? Sure!

Planar? No.

No cycles that are triangles.
  Cycles of length $\geq 4$.
At least $4f$ face-edge adjacencies,
  and at most $2e$.
.... $4f \leq 2e$ for any bipartite planar graph.
Euler: $v + \frac{1}{2}e \geq e + 2$

# $K_{3,3}$ non-planarity.



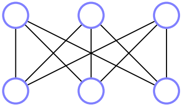
Euler: $v + \frac{2}{3}e \geq e + 2 \implies e \leq 3v - 6$

$K_{3,3}$? Edges? 9. Vertices. 6. $9 \leq 3(6) - 6$? Sure!

Planar? No.

No cycles that are triangles.
  Cycles of length $\geq 4$.
At least $4f$ face-edge adjacencies,
  and at most $2e$.
.... $4f \leq 2e$ for any bipartite planar graph.
Euler: $v + \frac{1}{2}e \geq e + 2 \implies e \leq 2v - 4$ for bipartite planar graph

# $K_{3,3}$ non-planarity.



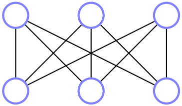
Euler: $v + \frac{2}{3}e \geq e + 2 \implies e \leq 3v - 6$

$K_{3,3}$? Edges? 9. Vertices. 6. $9 \leq 3(6) - 6$? Sure!

Planar? No.

No cycles that are triangles.
  Cycles of length $\geq 4$.
At least $4f$ face-edge adjacencies,
  and at most $2e$.
.... $4f \leq 2e$ for any bipartite planar graph.
Euler: $v + \frac{1}{2}e \geq e + 2 \implies e \leq 2v - 4$ for bipartite planar graph

# $K_{3,3}$ non-planarity.



Euler: $v + \frac{2}{3}e \geq e + 2 \implies e \leq 3v - 6$

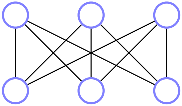
$K_{3,3}$? Edges? 9. Vertices. 6. $9 \leq 3(6) - 6$? Sure!

Planar? No.

No cycles that are triangles.
 Cycles of length $\geq 4$.
At least $4f$ face-edge adjacencies,
 and at most $2e$.
.... $4f \leq 2e$ for any bipartite planar graph.
Euler: $v + \frac{1}{2}e \geq e + 2 \implies e \leq 2v - 4$ for bipartite planar graph

$9 \not\leq 2(6) - 4$.

# $K_{3,3}$ non-planarity.



Euler: $v + \frac{2}{3}e \geq e + 2 \implies e \leq 3v - 6$

$K_{3,3}$? Edges? 9. Vertices. 6. $9 \leq 3(6) - 6$? Sure!

Planar? No.

No cycles that are triangles.
  Cycles of length $\geq 4$.
At least $4f$ face-edge adjacencies,
  and at most $2e$.
.... $4f \leq 2e$ for any bipartite planar graph.
Euler: $v + \frac{1}{2}e \geq e + 2 \implies e \leq 2v - 4$ for bipartite planar graph

$9 \not\leq 2(6) - 4$. $\implies$ $K_{3,3}$ is not planar!

# Tree.

A tree is a connected acyclic graph.

# Tree.

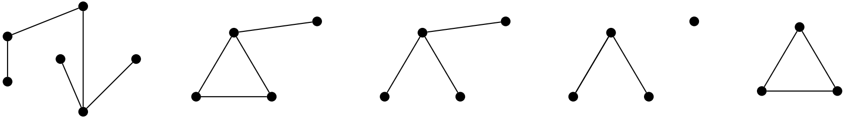
A tree is a connected acyclic graph.

To tree or not to tree!

# Tree.

A tree is a connected acyclic graph.

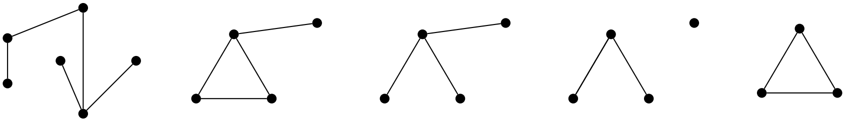
To tree or not to tree!



# Tree.

A tree is a connected acyclic graph.

To tree or not to tree!

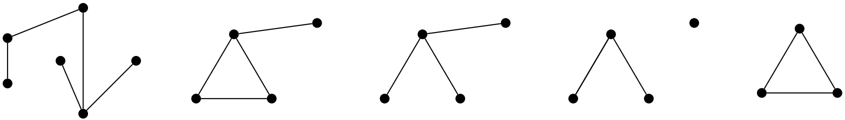


Yes.

# Tree.

A tree is a connected acyclic graph.

To tree or not to tree!

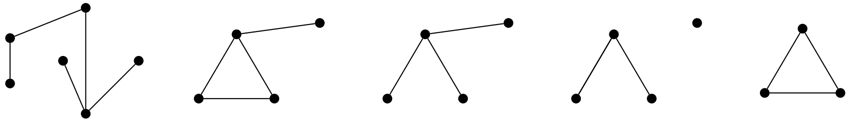


Yes. No.

# Tree.

A tree is a connected acyclic graph.

To tree or not to tree!

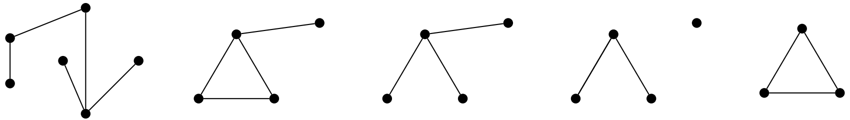


Yes. No. Yes.

# Tree.

A tree is a connected acyclic graph.

To tree or not to tree!

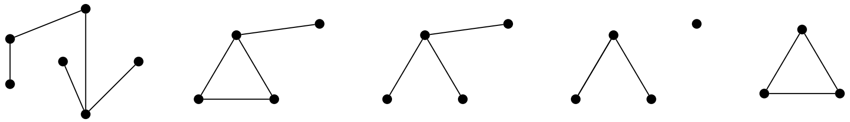


Yes. No. Yes. No.

# Tree.

A tree is a connected acyclic graph.

To tree or not to tree!

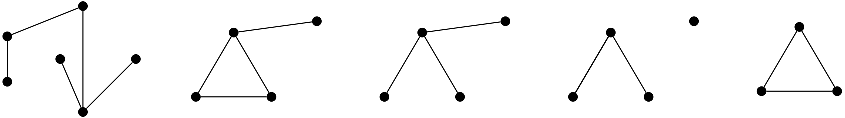


Yes. No. Yes. No. No.

# Tree.

A tree is a connected acyclic graph.

To tree or not to tree!



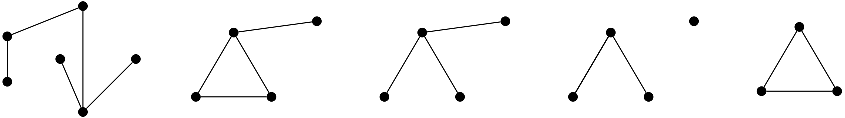
Yes. No. Yes. No. No.

Faces?

# Tree.

A tree is a connected acyclic graph.

To tree or not to tree!



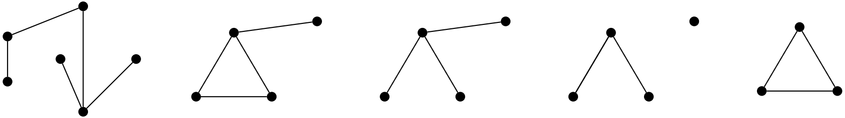
Yes. No. Yes. No. No.

Faces?   1.

# Tree.

A tree is a connected acyclic graph.

To tree or not to tree!



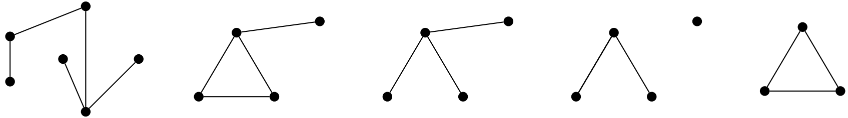
Yes. No. Yes. No. No.

Faces? 1. 2.

# Tree.

A tree is a connected acyclic graph.

To tree or not to tree!



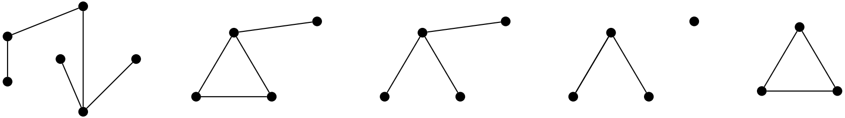
Yes. No. Yes. No. No.

Faces? 1. 2. 1.

# Tree.

A tree is a connected acyclic graph.

To tree or not to tree!



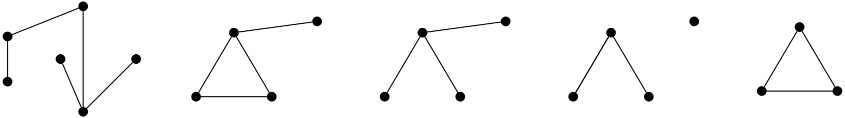
Yes. No. Yes. No. No.

Faces? 1. 2. 1. 1.

# Tree.

A tree is a connected acyclic graph.

To tree or not to tree!



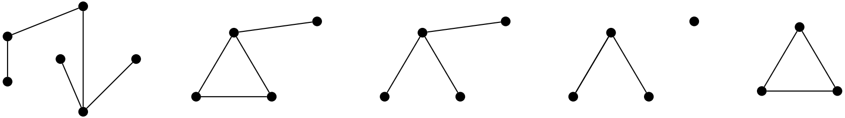
Yes. No. Yes. No. No.

Faces? 1. 2. 1. 1. 2.

# Tree.

A tree is a connected acyclic graph.

To tree or not to tree!



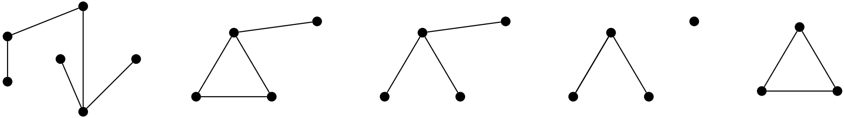
Yes. No. Yes. No. No.

Faces? 1. 2. 1. 1. 2.
Vertices/Edges.

# Tree.

A tree is a connected acyclic graph.

To tree or not to tree!



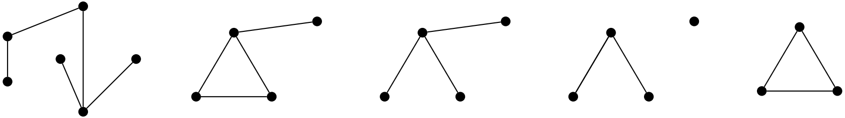
Yes. No. Yes. No. No.

Faces? 1. 2. 1. 1. 2.
Vertices/Edges. Recall: $e = v - 1$ for tree.

# Tree.

A tree is a connected acyclic graph.

To tree or not to tree!



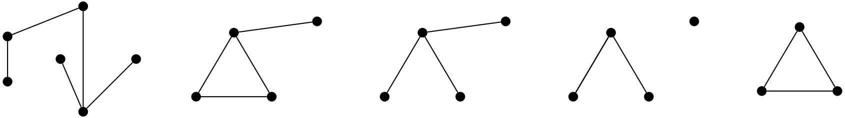
Yes. No. Yes. No. No.

Faces? 1. 2. 1. 1. 2.
Vertices/Edges. Recall: $e = v - 1$ for tree.

# Tree.

A tree is a connected acyclic graph.

To tree or not to tree!



Yes. No. Yes. No. No.

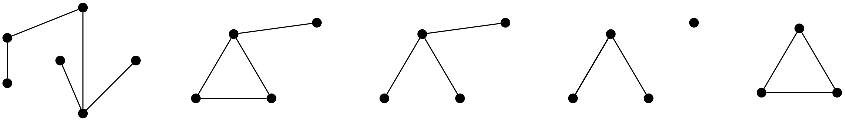
Faces? 1. 2. 1. 1. 2.
Vertices/Edges. Recall: $e = v - 1$ for tree.

One face for trees!

# Tree.

A tree is a connected acyclic graph.

To tree or not to tree!



Yes. No. Yes. No. No.

Faces?   1. 2. 1. 1. 2.
Vertices/Edges. Recall: $e = v - 1$ for tree.

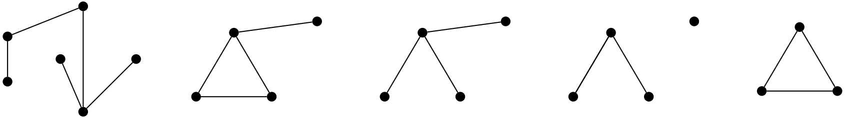
One face for trees!

Euler works for trees: $v + f = e + 2$.

# Tree.

A tree is a connected acyclic graph.

To tree or not to tree!



Yes. No. Yes. No. No.

Faces? 1. 2. 1. 1. 2.
Vertices/Edges. Recall: $e = v - 1$ for tree.

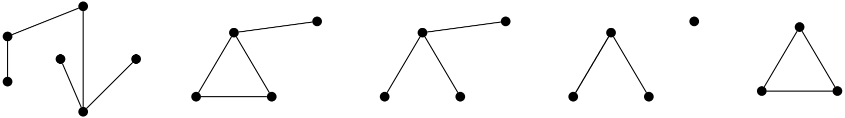
One face for trees!

Euler works for trees: $v + f = e + 2$.
$v + 1 = v - 1 + 2$

# Tree.

A tree is a connected acyclic graph.

To tree or not to tree!



Yes. No. Yes. No. No.

Faces? 1. 2. 1. 1. 2.
Vertices/Edges. Recall: $e = v - 1$ for tree.

One face for trees!

Euler works for trees: $v + f = e + 2$.
$v + 1 = v - 1 + 2$

# Euler's formula.

Euler: Connected planar graph has $v+f=e+2$.

# Euler's formula.

Euler: Connected planar graph has $v + f = e + 2$.

**Proof sketch:**

# Euler's formula.

Euler: Connected planar graph has $v + f = e + 2$.

**Proof sketch:** Induction on $e$.

# Euler's formula.

Euler: Connected planar graph has $v + f = e + 2$.

**Proof sketch:** Induction on $e$.
Base:

# Euler's formula.

Euler: Connected planar graph has $v+f=e+2$.

**Proof sketch:** Induction on $e$.
Base: $e=0$,

# Euler's formula.

Euler: Connected planar graph has $v+f=e+2$.

**Proof sketch:** Induction on $e$.
Base: $e=0$, $v=f=1$.

# Euler's formula.

Euler: Connected planar graph has $v+f=e+2$.

**Proof sketch:** Induction on $e$.
Base: $e=0$, $v=f=1$.
Induction Step:

# Euler's formula.

Euler: Connected planar graph has $v + f = e + 2$.

**Proof sketch:** Induction on $e$.
Base: $e = 0$, $v = f = 1$.
Induction Step:
If it is a tree.

# Euler's formula.

Euler: Connected planar graph has $v + f = e + 2$.

**Proof sketch:** Induction on $e$.
Base: $e = 0$, $v = f = 1$.
Induction Step:
If it is a tree. Done.

# Euler's formula.

Euler: Connected planar graph has $v + f = e + 2$.

**Proof sketch:** Induction on $e$.
Base: $e = 0$, $v = f = 1$.
Induction Step:
  If it is a tree. Done.
  If not a tree.

# Euler's formula.

Euler: Connected planar graph has $v + f = e + 2$.

**Proof sketch:** Induction on $e$.
Base: $e = 0$, $v = f = 1$.
Induction Step:
If it is a tree. Done.
If not a tree.
Find a cycle.

# Euler's formula.

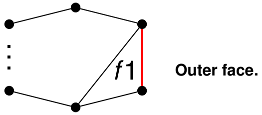
Euler: Connected planar graph has $v+f=e+2$.

**Proof sketch:** Induction on $e$.
Base: $e=0$, $v=f=1$.
Induction Step:
If it is a tree. Done.
If not a tree.
Find a cycle. Remove edge.

# Euler's formula.

Euler: Connected planar graph has $v+f=e+2$.

**Proof sketch:** Induction on $e$.
Base: $e=0$, $v=f=1$.
Induction Step:
If it is a tree. Done.
If not a tree.
Find a cycle. Remove edge.

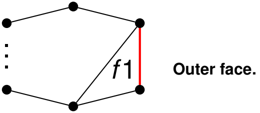



Joins two faces.

# Euler's formula.

Euler: Connected planar graph has $v+f=e+2$.

**Proof sketch:** Induction on $e$.
Base: $e=0$, $v=f=1$.
Induction Step:
If it is a tree. Done.
If not a tree.
Find a cycle. Remove edge.

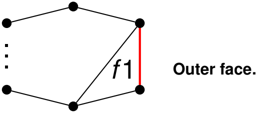



Joins two faces.
New graph: $v$-vertices.

# Euler's formula.

Euler: Connected planar graph has $v+f=e+2$.

**Proof sketch:** Induction on $e$.
Base: $e=0$, $v=f=1$.
Induction Step:
If it is a tree. Done.
If not a tree.
Find a cycle. Remove edge.

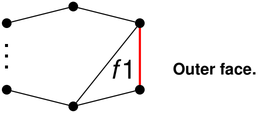



Joins two faces.
New graph: $v$-vertices. $e-1$ edges.

# Euler's formula.

Euler: Connected planar graph has $v+f=e+2$.

**Proof sketch:** Induction on $e$.
Base: $e=0$, $v=f=1$.
Induction Step:
If it is a tree. Done.
If not a tree.
Find a cycle. Remove edge.

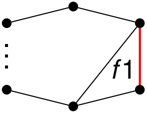



Joins two faces.
New graph: $v$-vertices. $e-1$ edges. $f-1$ faces.

# Euler's formula.

Euler: Connected planar graph has $v+f=e+2$.

**Proof sketch:** Induction on $e$.
Base: $e=0$, $v=f=1$.
Induction Step:
If it is a tree. Done.
If not a tree.
Find a cycle. Remove edge.



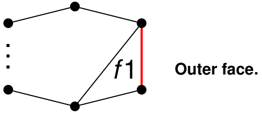

**Outer face.**

Joins two faces.
New graph: $v$-vertices. $e-1$ edges. $f-1$ faces. Planar.

# Euler's formula.

Euler: Connected planar graph has $v + f = e + 2$.

**Proof sketch:** Induction on $e$.
Base: $e = 0$, $v = f = 1$.
Induction Step:
If it is a tree. Done.
If not a tree.
Find a cycle. Remove edge.

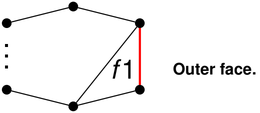



Joins two faces.
New graph: $v$-vertices. $e-1$ edges. $f-1$ faces. Planar.
$v + (f-1) = (e-1) + 2$ by induction hypothesis.

# Euler's formula.

Euler: Connected planar graph has $v+f=e+2$.

**Proof sketch:** Induction on $e$.
Base: $e=0$, $v=f=1$.
Induction Step:
If it is a tree. Done.
If not a tree.
Find a cycle. Remove edge.

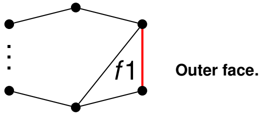



Joins two faces.
New graph: $v$-vertices. $e-1$ edges. $f-1$ faces. Planar.
$v+(f-1)=(e-1)+2$ by induction hypothesis.
Therefore $v+f=e+2$.

# Euler's formula.

Euler: Connected planar graph has $v + f = e + 2$.

**Proof sketch:** Induction on $e$.
Base: $e = 0$, $v = f = 1$.
Induction Step:
If it is a tree. Done.
If not a tree.
Find a cycle. Remove edge.




Joins two faces.
New graph: $v$-vertices. $e - 1$ edges. $f - 1$ faces. Planar.
$v + (f - 1) = (e - 1) + 2$ by induction hypothesis.
Therefore $v + f = e + 2$. □

# Summary

Graphs, trees, complete graphs, planar graphs.

# Summary

Graphs, trees, complete graphs, planar graphs.

Euler's formula.

# Summary

Graphs, trees, complete graphs, planar graphs.

Euler's formula.

Have a nice weekend!