Markov Chains

Definition (Markov Property) A process $X_0, X_1, \ldots$ obeys the Markov Property if

Example: Consider the process of flipping a coin that flips heads with probability $p$ until we see two consecutive heads.

Claim: For $X_0, X_1, \ldots$ a Markov chain on $S$ and $i_0 i_1 \ldots i_n$ in a sequence of states visited,

Example: Consider the process of flipping a coin that flips heads with probability $p$ until we see two consecutive heads.

$P(X_0 = H) = \ldots$

$P(X_0 = T) = \ldots$

$P(HTTHHT) = \ldots$
**Definition (Transition Matrix)** The one-step transition matrix of a chain is

**Example** Consider the process of flipping a coin that flips heads with probability $p$ until we see two consecutive heads.

**Claim** The $n$-step transition matrix $P_n$ is

**Example** Consider the process of flipping a coin that flips heads with probability $p$ until we see two consecutive heads. The transition matrix $P$ is

$$
P = \begin{bmatrix}
T & R & M \\
q & p & 0 \\
q & 0 & 1 \\
0 & 0 & 1
\end{bmatrix}
$$
Distribution over Time

Let \( \pi_0 \) be the initial distribution over the state space written as a row vector.

\[ \pi_0 \]

Note: To specify a Markov chain, you need
Hitting Time

Q. Suppose you repeatedly flip a coin with probability $p$ of heads until you see two consecutive heads. What is the expected number of flips it will take?

Note: Let $X_0, X_1, \ldots$ be a finite Markov chain on state space $S$ with transition matrix $P$. 
A Before B

Q. We repeatedly roll a six-sided die and sum the rolls modulo 3 as we go. What is the chance our sum hits 1 before it hits 2?

Q. Consider a sequence of iid trials, each of which results in n mutually exclusive categories outcomes. On each trial, let the chance of category i be $p_i > 0$. What is the chance category i appears before category j?

Note: Let $X_0, X_1, ...$ be a finite Markov chain on state space S with transition matrix $P$. 
Examples

1. An ant is sitting at the corner of a cube. At each timestep, she traverses an edge uniformly at random. What is the expected time until she reaches the other end of the cube?