Motivating Puzzle

Suppose we have two shopping centers.

Shopping center 1:

3 Indian restaurants
2 Moroccan restaurants

Shopping center 2:

1 Indian restaurant
1 Moroccan restaurant

Suppose we pick a center randomly with probability 50%, and then pick a restaurant at that center with equal probability.

- If we eat at a Moroccan restaurant, what is the chance we picked center 1?

CC BY NC SA

Motivating Puzzle

Suppose we have two shopping centers.

Shopping center 1:

3 Indian restaurants
2 Moroccan restaurants

Shopping center 2:

1 Indian restaurant
1 Moroccan restaurant

Suppose we pick a center randomly with probability 50%, and then pick a restaurant from that center with equal probability.

- If we eat at a Moroccan restaurant, what's the chance we picked center 1?

Might be tempting to say that the chance is $2/3$ since 2 out of 3 Moroccan restaurants are in center 1, but this is wrong. The real answer is 44.4% (???) We'll come back to this problem later...

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Conditional Probability of an Outcome

Lecture 17, CS70 Fall 2026

Conditional Probabilities

- **Conditional Probability of an Outcome**
- Conditional Probability of an Event
- The Monty Hall Problem

Bayesian Inference

- Bayes Rule and the Total Probability Rule (TPR)
- Applications of Bayes and TPR
- Generalized Bayes and TPR

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Gem Dispenser

Suppose we have a gem dispenser that gives us:

- A diamond with probability $1/100$.
- A sapphire with probability $2/100$.
- An emerald with probability $3/100$.
- A fragment of concrete with probability $94/100$.

Let B be the event that the gem is valuable. What is the probability of getting a sapphire if we know B is true (i.e. that the gem is valuable)?

$$\frac{2/100}{6/100} = \frac{1}{3}$$

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New Idea: Conditional Probability

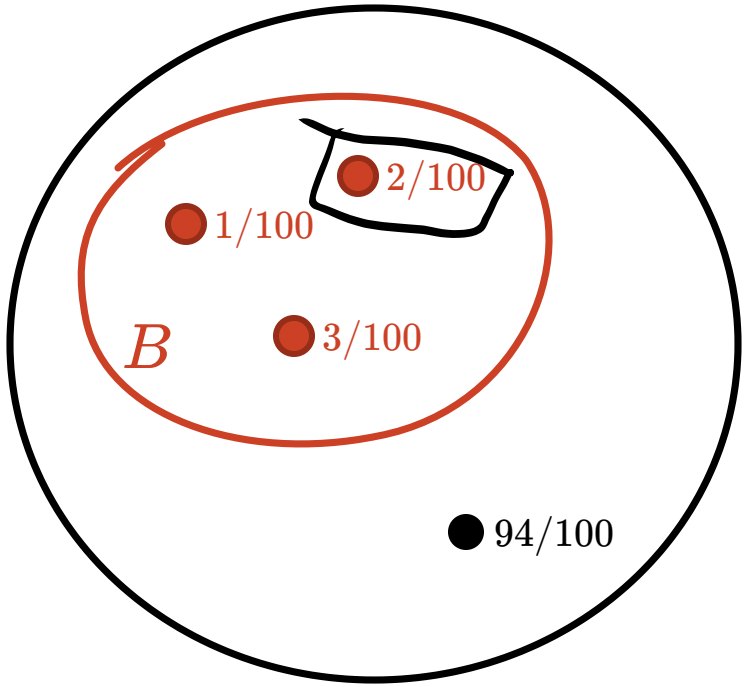
This is known as "**conditional probability**".

- Tells us the probability of an outcome given that we know that some event happened.

$P(\omega = \text{sapphire} \mid B)$

The standard notation looks like $\boldsymbol{P}(\omega|B)$.

- Can read the vertical bar as "given that".
- It is defined by the formula $\boldsymbol{P}(\omega|B) = \boldsymbol{P}(\omega)/\boldsymbol{P}(B)$
- Example: $\boldsymbol{P}(\omega = \text{sapphire}\,|B) = \frac{2/100}{6/100} = 1/3$



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Fair Dice Example

Suppose we have four fair 3-sided dice.

- Define B as the event that we get no 2s.
- What is $\boldsymbol{P}(\omega = 1131 | B)$?

1111
1112
1113

$|\Omega| = 3^4$

$P(\omega = 1131) = \frac{1}{3^4} = 1/81$

$P(B) = 16 \cdot \frac{1}{81} \quad |B| = 2^4$

$= \frac{16}{81}$

1111
1113

$\uparrow$
2

$P(\omega | B) = \frac{P(\omega)}{P(B)} = \frac{1/81}{16/81} = \frac{1}{16}$

https://joshh.ug/A

Image: Bescon 3-sided dice

Note: The notes give this example as a balls-and-bins problem, but I think it's easier to follow as dice.

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Fair Dice Solution

Suppose we have four fair 3-sided dice.

- Define B as the event that we get no 2s.
- What is $\boldsymbol{P}(\omega = 1131|B) = \boldsymbol{P}(\omega)/\boldsymbol{P}(B)$?
  - $\boldsymbol{P}(\omega) = 1/3^4 = 1/81$
  - $\boldsymbol{P}(B) = 16 \times 1/3^4 = 16/81$
  - $\boldsymbol{P}(\omega)/\boldsymbol{P}(B) = \frac{1/81}{16/81} = 1/16$

Image: Bescon 3-sided dice

Note: The notes give this example as a balls-and-bins problem, but I think it's easier to follow as dice.

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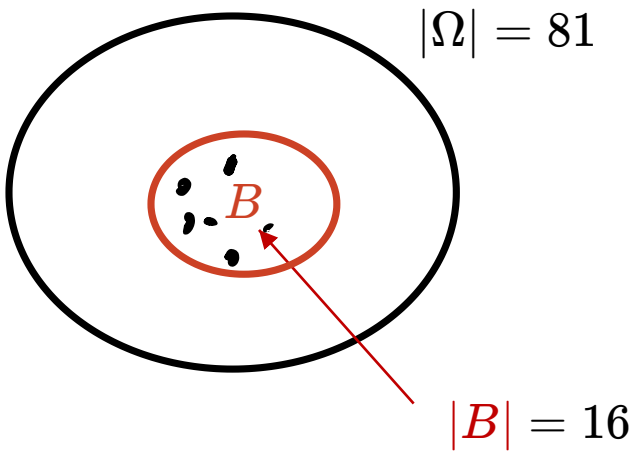
Alternate View on Fair Dice

Suppose we already know there are no 2s, i.e. the event B happened.

- What is the chance we have $\omega = 1131$? 1/16
- We're not picking from Ω, we're picking from B, so only 16 possible choices.



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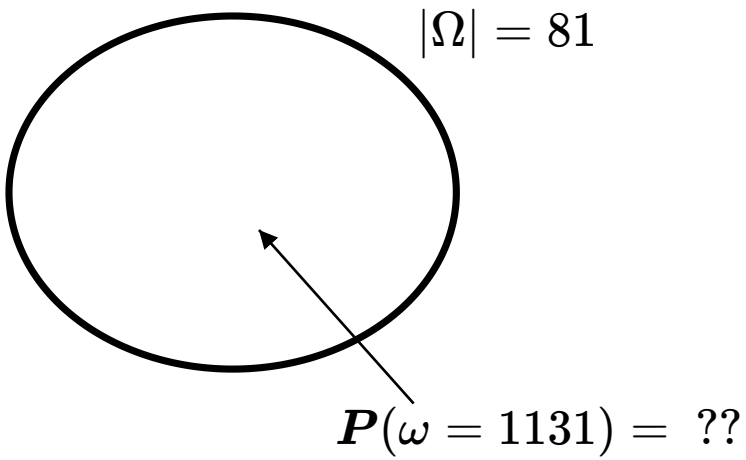
Example: Loaded Dice

Now let's suppose that the dice are loaded so that 1 occurs half the time, and 2 and 3 appear 25% of the time.

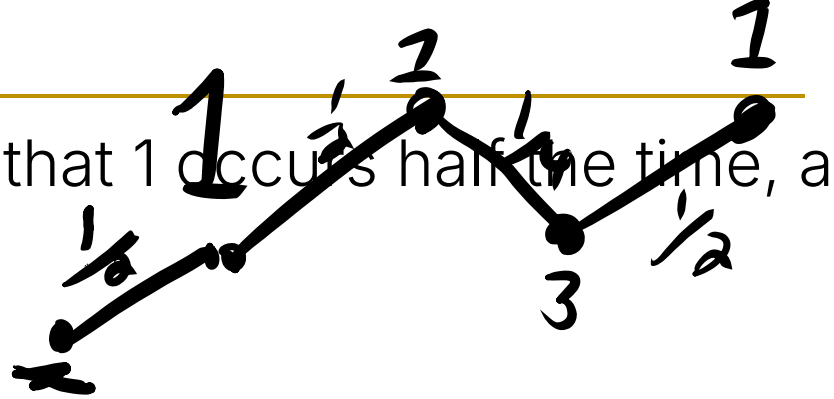
- What is $\boldsymbol{P}(\omega = 1131)$?



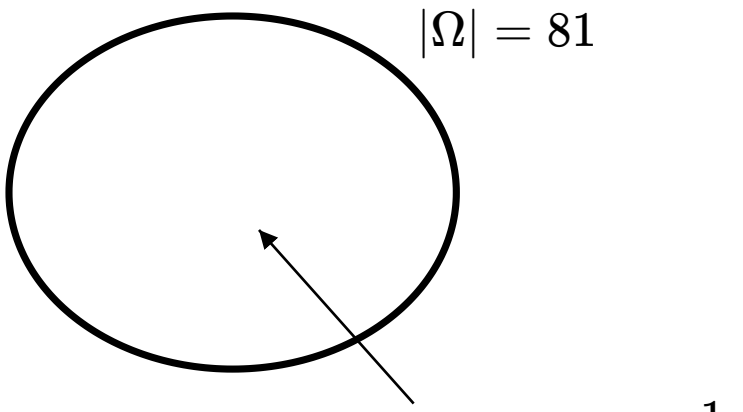
CC BY NC SA

Example: Loaded Dice

Now let's suppose that the dice are loaded so that 1 occurs half the time, and 2 and 3 appear 25% of the time.



- What is $\boldsymbol{P}(\omega = 1131)$?
  - $\left(\frac{1}{2}\right)^3 \left(\frac{1}{4}\right) = \frac{1}{32}$

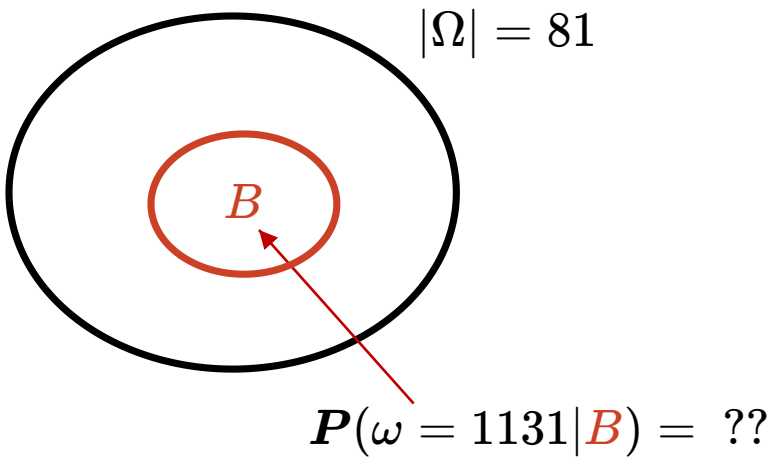


CC BY NC SA

Example: Loaded Dice

Now let's suppose that the dice are loaded so that 1 occurs half the time, and 2 and 3 appear 25% of the time. Let B be the event that no 2 occurred.

- What is $\boldsymbol{P}(\omega = 1131 | B)$?



CC BY NC SA

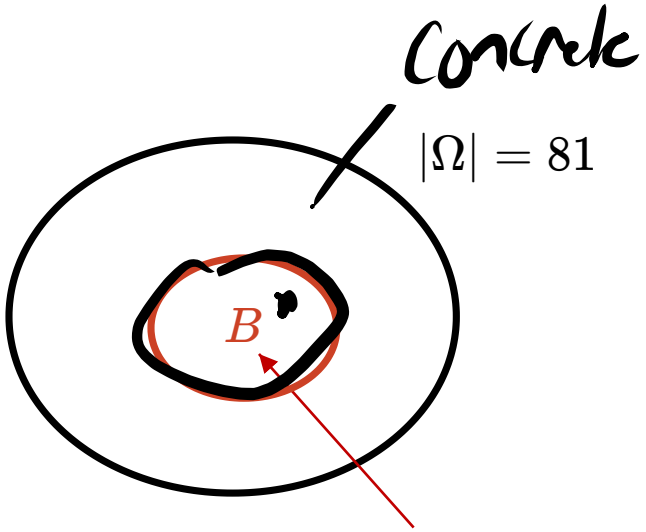
Example: Loaded Dice

Now let's suppose that the dice are loaded so that 1 occurs half the time, and 2 and 3 appear 25% of the time. Let B be the event that no 2 occurred.

$P(\omega = 1131 | B) \neq P(\omega = 1131)$

$= \frac{P(\omega = 1131)}{P(B)}$

- What is $\boldsymbol{P}(\omega = 1131 | B)$?
  - Incorrect to say: $\left(\frac{1}{2}\right)^3 \left(\frac{1}{4}\right) = \frac{1}{32}$
  - That was the probability if selected from Ω.



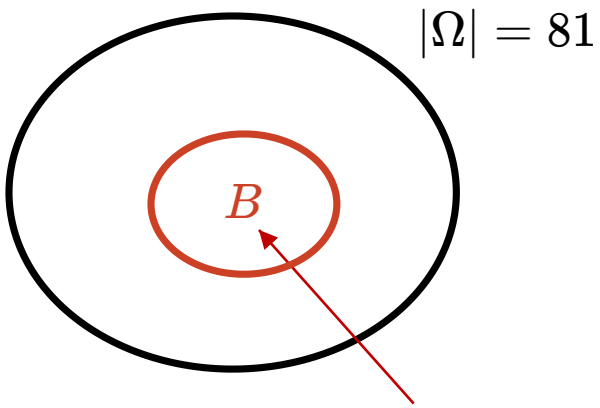
$$\boldsymbol{P}(\omega = 1131 | B) = \ ?? \neq \frac{1}{32}$$

CC BY NC SA

Example: Loaded Dice

Now let's suppose that the dice are loaded so that 1 occurs half the time, and 2 and 3 appear 25% of the time. Let B be the event that no 2 occurred.

- What is $\boldsymbol{P}(\omega = 1131 | B)$?
  - Incorrect to say: $\left(\frac{1}{2}\right)^3 \left(\frac{1}{4}\right) = \frac{1}{32}$
  - That was the probability if selected from Ω.
  - So what should we do? Divide by $\boldsymbol{P}(B)$



$$\boldsymbol{P}(\omega = 1131 | B) = \ ?? \neq \frac{1}{32}$$

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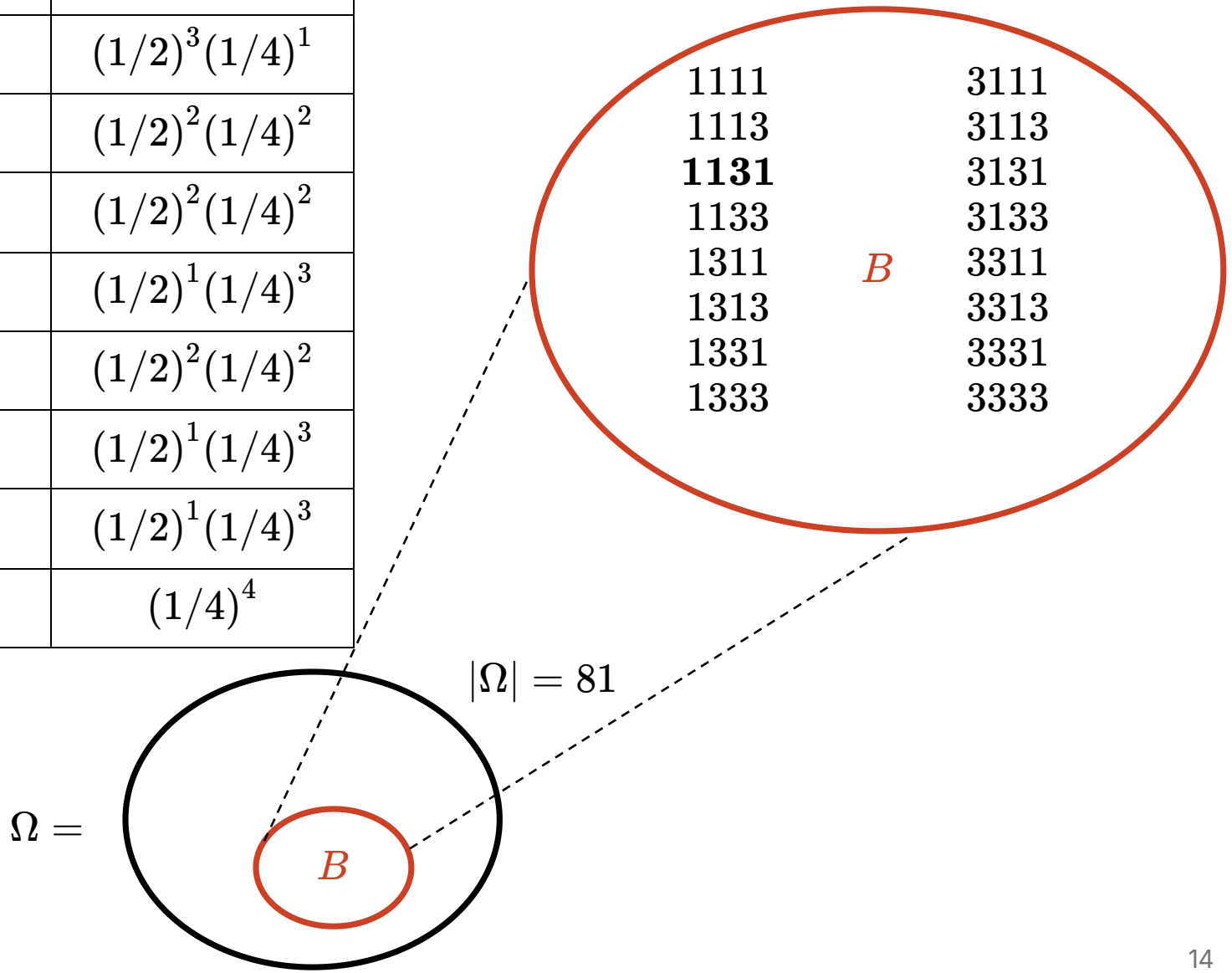
All of the Outcomes in B

1/16

ω	$P(\omega)$	ω	$P(\omega)$
1111	$(1/2)^4$	3111	$(1/2)^3(1/4)^1$
1113	$(1/2)^3(1/4)^1$	3113	$(1/2)^2(1/4)^2$
1131	$(1/2)^3(1/4)^1$	3131	$(1/2)^2(1/4)^2$
1133	$(1/2)^2(1/4)^2$	3133	$(1/2)^1(1/4)^3$
1311	$(1/2)^3(1/4)^1$	3311	$(1/2)^2(1/4)^2$
1313	$(1/2)^2(1/4)^2$	3313	$(1/2)^1(1/4)^3$
1331	$(1/2)^2(1/4)^2$	3331	$(1/2)^1(1/4)^3$
1333	$(1/2)^1(1/4)^3$	3333	$(1/4)^4$

Above are all the probabilities of the outcomes in B.



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Probability of All of the Outcomes in B

$P(B) = \sum$

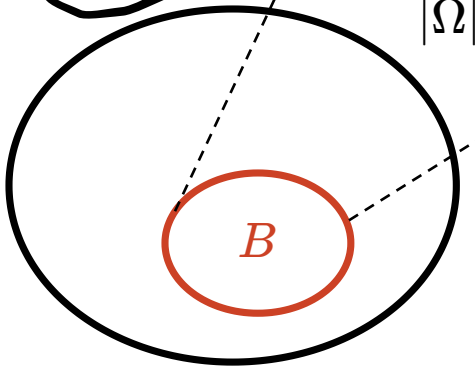
ω	$P(\omega)$	ω	$P(\omega)$
1111	1/16	3111	1/32
1113	1/32	3113	1/64
1131	1/32	3131	1/64
1133	1/64	3133	1/128
1311	1/32	3311	1/64
1313	1/64	3313	1/128
1331	1/64	3331	1/128
1333	1/128	3333	1/256

B

1111	3111
1113	3113
1131	3131
1133	3133
1311	3311
1313	3313
1331	3331
1333	3333

$|\Omega| = 81$

$\Omega =$



Above are all the probabilities of the outcomes in B simplified.

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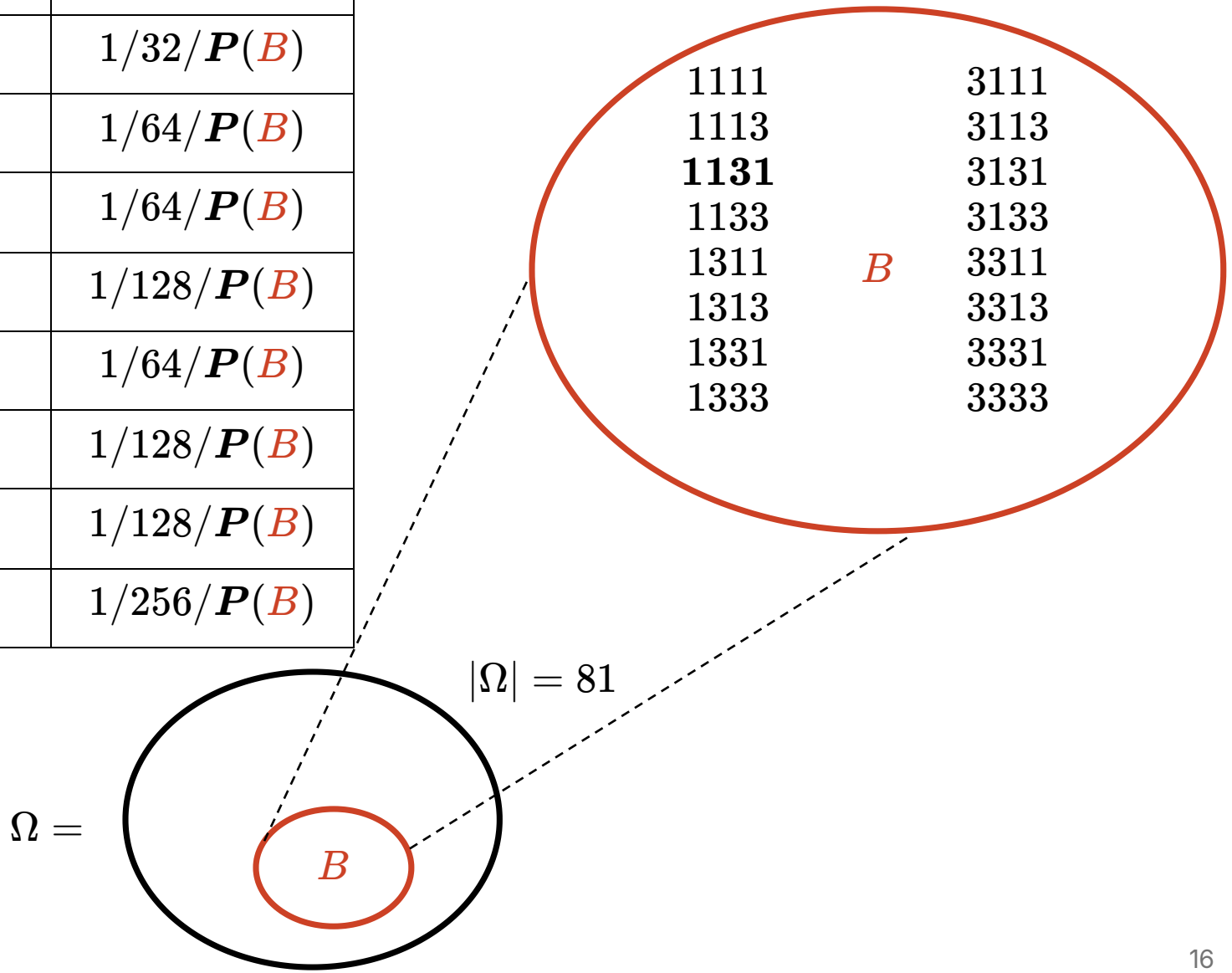
Conditional Probability of All of the Outcomes in B

ω	$P(\omega \mid B)$	ω	$P(\omega \mid B)$
1111	$1/16/P(B)$	3111	$1/32/P(B)$
1113	$1/32/P(B)$	3113	$1/64/P(B)$
1131	$1/32/P(B)$	3131	$1/64/P(B)$
1133	$1/64/P(B)$	3133	$1/128/P(B)$
1311	$1/32/P(B)$	3311	$1/64/P(B)$
1313	$1/64/P(B)$	3313	$1/128/P(B)$
1331	$1/64/P(B)$	3331	$1/128/P(B)$
1333	$1/128/P(B)$	3333	$1/256/P(B)$

We're effectively rescaling all of the probabilities.



Conditional Probability of All of the Outcomes in B

$(3/4)^4$

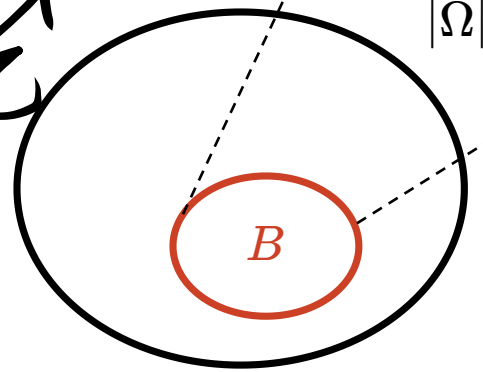
ω	$P(\omega \mid B)$	ω	$P(\omega \mid B)$
1111	16/81	3111	8/81
1113	8/81	3113	4/81
1131	8/81	3131	4/81
1133	4/81	3133	2/81
1311	8/81	3311	4/81
1313	4/81	3313	2/81
1331	4/81	3331	2/81
1333	2/81	3333	1/81

Above we have $P(\omega \mid B)$.

- Observe: $\sum_{\omega \in B} P(\omega \mid B) = 1$

1

$\Omega =$



$|\Omega| = 81$

1111 3111
1113 3113
1131 3131
1133 3133
1311 B 3311
1313 3313
1331 3331
1333 3333

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Another Viewpoint (Bonus Slide)

Another way of thinking about this: We know that the conditional probabilities must sum to 1.

- If we're picking only from B, then the sum of the chance of all possible choices from B must be 100%.

$$\sum_{\omega \in B} \boldsymbol{P}(\omega \mid B) = 1$$

If we didn't rescale, i.e. $\boldsymbol{P}(\omega \mid B) = \boldsymbol{P}(\omega)$, then we'd end up with a contradiction:

$$\sum_{\omega \in B} \boldsymbol{P}(\omega \mid B) = \sum_{\omega \in B} \boldsymbol{P}(\omega) = \boldsymbol{P}(B)$$

To avoid this problem, we must divide by $\boldsymbol{P}(B)$, i.e. $\boldsymbol{P}(\omega|B) = \frac{\boldsymbol{P}(\omega)}{\boldsymbol{P}(B)}$

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$$P(w \mid B) = \frac{P(w)}{P(B)}$$

Conditional Probability of an Event

Lecture 17, CS70 Fall 2026

Conditional Probabilities

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- **Conditional Probability of an Event**
- The Monty Hall Problem

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- Bayes Rule and the Total Probability Rule (TPR)
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Another Dice Rolling Problem

Suppose we are back to fair 3-sided dice. Let A be the event where we roll at least three 1s. What is $\boldsymbol{P}(A)$?

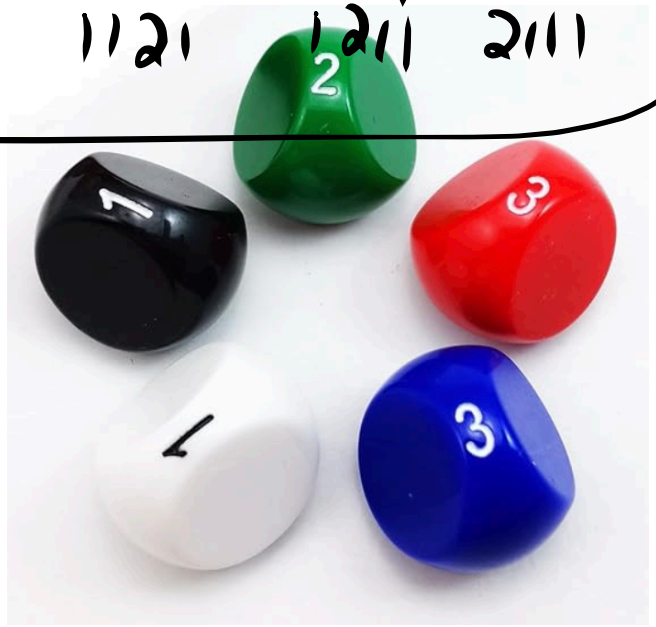
81

A

1111 1113 1131 1311 3111

1112 1121 1211 2111

$\frac{9}{81}$



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Another Dice Rolling Problem

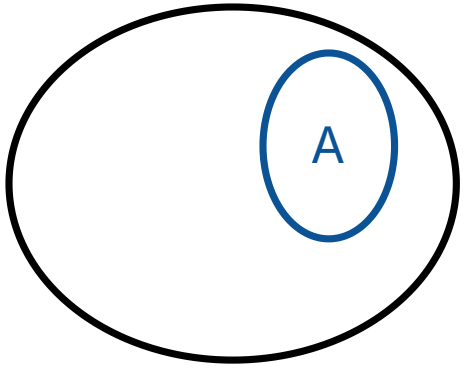
Suppose we are back to fair 3-sided dice. Let A be the event where we roll at least three 1s. What is $\boldsymbol{P}(A)$?

- This is just $\underbrace{8\times 1/81}_{\text{Chance of three 1s}} + \underbrace{1/81}_{\text{Chance of four 1s.}} = 9/81.$

2111	**3111**	**1111**
1211	**1311**	
1121	**1131**	
1112	**1113**	

That is, since the probability space is uniform: $\boldsymbol{P}(\mathrm{A}) = \frac{|A|}{|\Omega|}$

$\Omega =$



$|A| = 9$

CC BY NC SA

Conditional Probability of an Event

Suppose we are back to fair 3-sided dice. Let A be the event where we roll at least three 1s. Let B be the event from before where we roll no 2s.

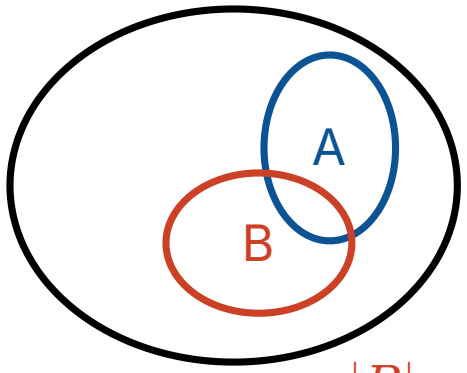
$$P(\omega|B) = \frac{P(\omega)}{P(B)}$$

prob of any outcome given B

What is $\boldsymbol{P}(A|B)$?

2111	**3111**	**1111**
1211	**1311**	
1121	**1131**	
1112	**1113**	

$\Omega =$



$|A| = 9$

$|B| = 16$

CC BY NC SA

Conditional Probability of an Event

Suppose we are back to fair 3-sided dice. Let A be the event where we roll at least three 1s. Let B be the event from before where we roll no 2s.

What is $\boldsymbol{P}(A|B)$?

2111	**3111**	**1111**
1211	**1311**	
1121	**1131**	
1112	**1113**	

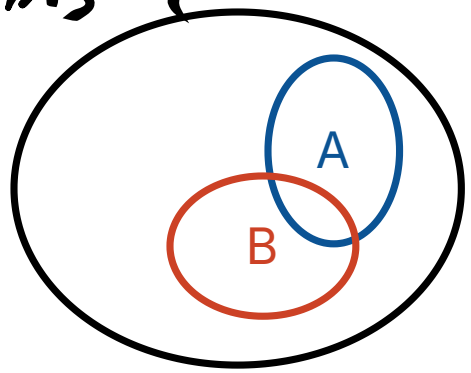
We know that the probability of an event is just the sum of probabilities of individual outcomes.

$$\boldsymbol{P}(A|B) = \sum_{\omega \in A} P(\omega \mid B)$$

terms 9

So: $\boldsymbol{P}(A|B) = \boldsymbol{P}(\omega = 2111 \mid B) + \cdots + \boldsymbol{P}(\omega = 1111 \mid B)$

$\Omega =$



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Conditional Probability of an Event

Suppose we are back to fair 3-sided dice. Let A be the event where we roll at least three 1s. Let B be the event from before where we roll no 2s.

https://joshh.ug/B

What is $\boldsymbol{P}(A|B)$?

2111	**3111**	**1111**
1211	**1311**	
1121	**1131**	
1112	**1113**	

We know that the probability of an event is just the sum of probabilities of individual outcomes.

$$\boldsymbol{P}(A|B) = \sum_{\omega \in A} P(\omega \mid B)$$

1/16

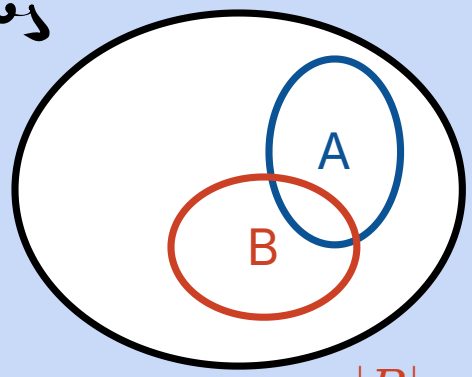
16 outcomes

So: $\boldsymbol{P}(A|B) = \boldsymbol{P}(\omega = 2111 \mid B) + \cdots + \boldsymbol{P}(\omega = 1111 \mid B)$

0

What are these values?

$\Omega =$



$|A| = 9$

$|B| = 16$

CC BY NC SA

Conditional Probability of an Event

Suppose we are back to fair 3-sided dice. Let A be the event where we roll at least three 1s. Let B be the event from before where we roll no 2s.

What is $\boldsymbol{P}(A|B)$?

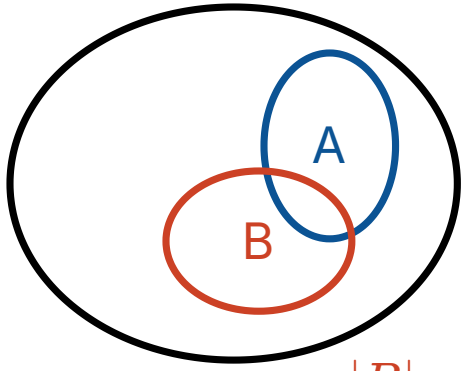
2111	**3111**	**1111**
1211	**1311**	
1121	**1131**	
1112	**1113**	

We know that the probability of an event is just the sum of probabilities of individual outcomes.

$$\boldsymbol{P}(A|B) = \sum_{\omega \in A} P(\omega \mid B)$$

So: $\boldsymbol{P}(A|B) = \underbrace{\boldsymbol{P}(\omega = 2111 \mid B)}_{0} + \cdots + \underbrace{\boldsymbol{P}(\omega = 1111 \mid B)}_{1/16}$

$\Omega =$



$|A| = 9$

$|B| = 16$

CC BY NC SA

Conditional Probability of an Event

Suppose we are back to fair 3-sided dice. Let A be the event where we roll at least three 1s. Let B be the event from before where we roll no 2s.

$P(\overset{w=}{2111}|B) + P(w=1211|B) + \ldots + P(w=1111|B)$

0 0 $\frac{1}{16}$

What is $\boldsymbol{P}(A|B)$?

2111	3111	1111
1211	1311	
1121	1131	
1112	1113	

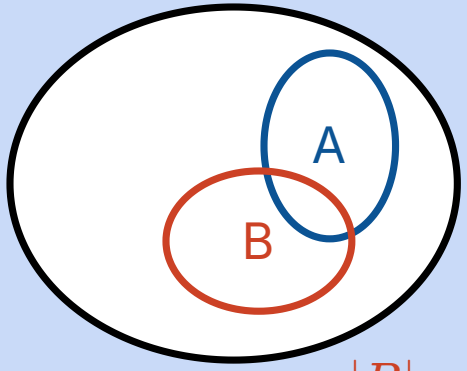
0 $4 \cdot \frac{1}{16}$ $= 5/16$

We know that the probability of an event is just the sum of probabilities of individual outcomes.

$$\boldsymbol{P}(A|B) = \sum_{\omega \in A} P(\omega \mid B)$$

So: $\boldsymbol{P}(A|B) =$??

$\Omega =$



$|A| = 9$

$|B| = 16$

Conditional Probability of an Event

Suppose we are back to fair 3-sided dice. Let A be the event where we roll at least three 1s. Let B be the event from before where we roll no 2s.

What is $\boldsymbol{P}(A|B)$?

$$\underbrace{\cancel{\begin{matrix}2111\\1211\\1121\\1112\end{matrix}}}_{\text{blue}} \qquad \underbrace{\begin{matrix}3111\\1311\\1131\\1113\end{matrix}}_{\text{purple}} \qquad \left.\begin{matrix}1111\end{matrix}\right] A$$

We know that the probability of an event is just the sum of probabilities of individual outcomes.

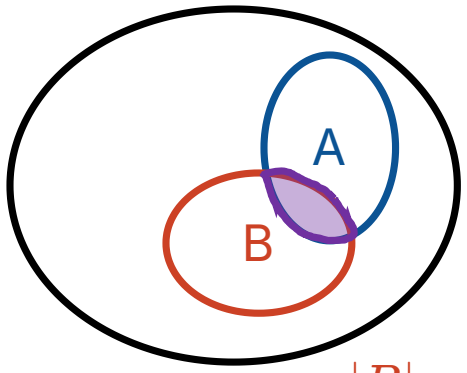
$|\Omega| = 81$

$$\boldsymbol{P}(A|B) = \sum_{\omega \in A} P(\omega \mid B)$$

So: $\boldsymbol{P}(A|B) = 5/16$

- This is because 5 elements from A are also in B.

$\Omega =$



$|A| = 9$

$|B| = 16$

CC BY NC SA

Conditional Probability of an Event

We just saw that:

$$\boldsymbol{P}(A|B) = \sum_{\omega \in A} P(\omega \mid B)$$

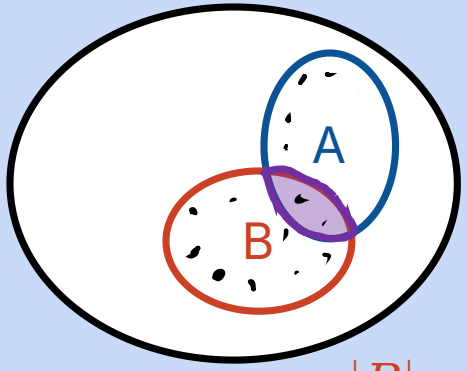
we basically checked if w was part of B before adding it

Which of the following is also correct? Select all that apply.

- $\boldsymbol{P}(A \mid B) = \sum_{\omega \in B} \boldsymbol{P}(\omega \mid B)$ → 1
- $\boldsymbol{P}(A \mid B) = \sum_{\omega \in A \cup B} \boldsymbol{P}(\omega \mid B)$
- $\boldsymbol{P}(A \mid B) = \sum_{\omega \in A \cap B} \boldsymbol{P}(\omega \mid B)$

https://joshh.ug/C

$\Omega =$



$|A| = 9$

$|B| = 16$

CC BY NC SA

Conditional Probability of an Event

We just saw that:

$$\boldsymbol{P}(A|B) = \sum_{\omega \in A} P(\omega \mid B)$$

Which of the following is also correct? Select all that apply.

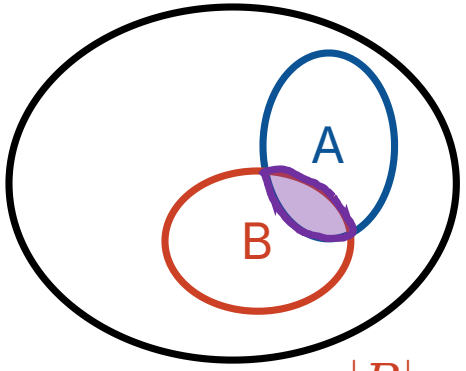
- $\boldsymbol{P}(A \mid B) = \sum_{\omega \in B} \boldsymbol{P}(\omega \mid B)$. No, this is just 1.
- $\boldsymbol{P}(A \mid B) = \sum_{\omega \in A \cup B} \boldsymbol{P}(\omega \mid B)$. No, this is also just 1.
- $\boldsymbol{P}(A \mid B) = \sum_{\omega \in A \cap B} \boldsymbol{P}(\omega \mid B)$. Yes! This is equivalent to only summing over the purple items. Items only in A have probability $\boldsymbol{P}(\omega \in (A \backslash B)|B) = 0$

$\cancel{2111}$ $\cancel{1211}$ $\cancel{1121}$ $\cancel{1112}$

3111 1311 1131 1113

1111

$\Omega =$



$|A| = 9$

$|B| = 16$

Conditional Probability of an Event

We just saw that:

$$\boldsymbol{P}(A|B) = \sum_{\omega \in A} P(\omega \mid B)$$

Or we can write this as:

$$\boldsymbol{P}(A|B) = \sum_{\omega \in A \cap B} P(\omega \mid B)$$

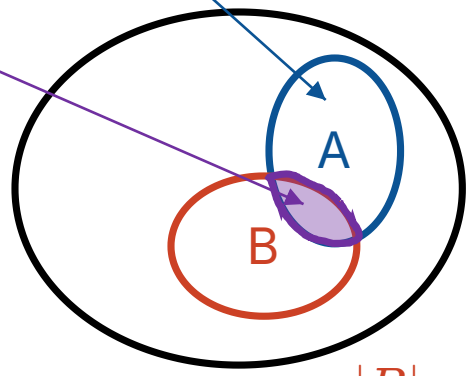
Optional to add these since $\boldsymbol{P}(\omega \in A \backslash B | B) = 0$

$\cancel{2111}$ $\cancel{1211}$ $\cancel{1121}$ $\cancel{1112}$

3111 1311 1131 1113

1111

$\Omega =$



$|A| = 9$

$|B| = 16$

CC BY NC SA

Conditional Probability of an Event

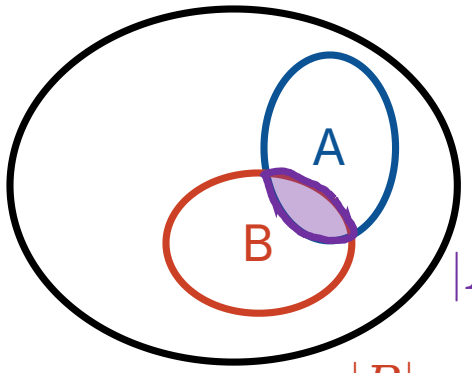
We've now derived that:

$$\boldsymbol{P}(A|B) = \sum_{\omega \in A \cap B} \boldsymbol{P}(\omega \mid B)$$

We're still not quite to the standard notation. To get there, let's do a bit more algebra:

$$\boldsymbol{P}(A|B) = \sum_{\omega \in A \cap B} \boldsymbol{P}(\omega \mid B)$$

$\Omega =$



$|A| = 9$

$|A \cap B| = 5$

$|B| = 16$

CC BY NC SA

Conditional Probability of an Event

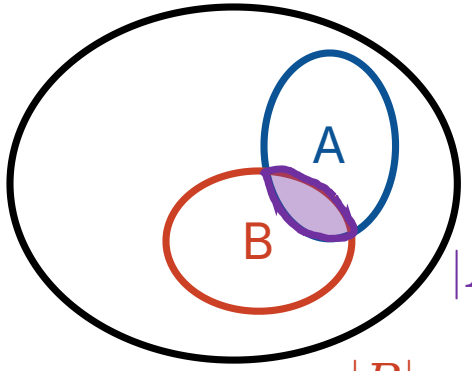
We've now derived that:

$$\boldsymbol{P}(A|B) = \sum_{\omega \in A \cap B} \boldsymbol{P}(\omega \mid B)$$

We're still not quite to the standard notation. To get there, let's do a bit more algebra:

$$\boldsymbol{P}(A|B) = \sum_{\omega \in A \cap B} \boldsymbol{P}(\omega \mid B) = \sum_{\omega \in A \cap B} \frac{\boldsymbol{P}(\omega)}{\boldsymbol{P}(B)} = \frac{\sum_{\omega \in A \cap B} \boldsymbol{P}(\omega)}{\boldsymbol{P}(B)} = \frac{\boldsymbol{P}(A \cap B)}{\boldsymbol{P}(B)}$$

$\Omega =$



$|A| = 9$

$|A \cap B| = 5$

$|B| = 16$

CC BY NC SA

Conditional Probability of an Event

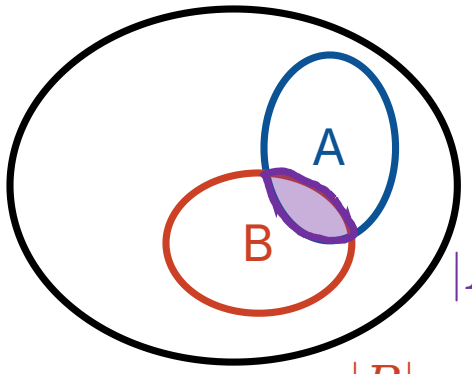
This is the standard form for the conditional probability of an event.

$$\boldsymbol{P}(A|B) = \frac{\boldsymbol{P}(A \cap B)}{\boldsymbol{P}(B)}$$

Interpretation:

- The numerator is the sum of the unconditional probabilities of the members of $A \cap B$.
- Then we rescale that sum by the probability of the event $\boldsymbol{P}(B)$ (which is the sum of the unconditional probabilities of the members of B).

$\Omega =$



$|A| = 9$

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$|B| = 16$

CC BY NC SA

Definition of Conditional Probability

For events $A,\ B \subseteq \Omega$ in the same probability space such that $P(B) > 0$, the conditional probability of A given B is:

$$\boldsymbol{P}(A|B) = \frac{\boldsymbol{P}(A \cap B)}{\boldsymbol{P}(B)}$$

This expression is very common in the real world! Know it well.

- But don't lose sight of what it really means: You're adding up all the probabilities of the purple items, each of which is scaled by dividing by the sum of the probabilities of the red items.

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Example Using Conditional Probability Formula: Dealing Cards

Suppose the first card we're dealt is an ace, and we want to know the chance our next card will also be an ace. Here:

https://joshh.ug/D

- B: Event that first card was an ace.
- A: Event that second card was an ace.

Let's verify that $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$.

CC BY NC SA

Example Using Conditional Probability Formula: Dealing Cards

Suppose the first card we're dealt is an ace, and we want to know the chance our next card will also be an ace. Here:

- B: Event that first card was an ace.
- A: Event that second card was an ace.

Can also reason directly: One ace is gone, so 3 out of 51 cards are aces, so $P(A \mid B) = 3/51$

Let's verify that $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$.

- $P(B) = 4/52$ since 4 out of 52 cards are aces.
- $P(A \cap B) = \frac{12}{52 \times 51}$ since there are 52×51 ways of drawing two cards, and $4 \times 3 = 12$ ways of getting two aces.
- Thus chance that next card will be an ace is $\frac{12}{52 \times 51} / \frac{4}{52} = \frac{3}{51}$

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Back to the Shopping Center Puzzle

Suppose we have two shopping centers.

Shopping center 1:

3 Indian restaurants
2 Moroccan restaurants

Shopping center 2:

1 Indian restaurant
1 Moroccan restaurant

Suppose we pick a center randomly with probability 50%, and then pick a restaurant from that center with equal probability.

- If we eat at a Moroccan restaurant, what's the chance we picked center 1?

Hint, let C_1 be the event we picked center 1, and C_2 be the event we picked center 2. Then (by the definition of conditional probability):

$$P(C_1|\bullet) = \frac{P(C_1 \cap \bullet)}{P(\bullet)}$$

Video viewers: Consider pausing and calculating the answer.

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Back to the Shopping Center Puzzle

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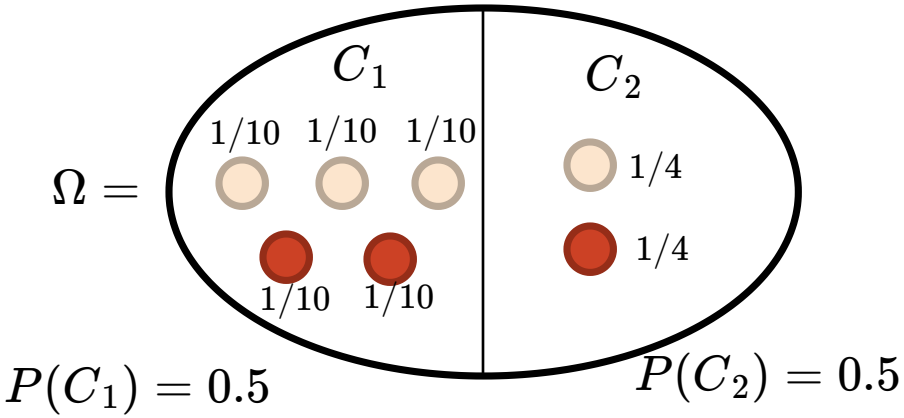

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Back to the Shopping Center Puzzle

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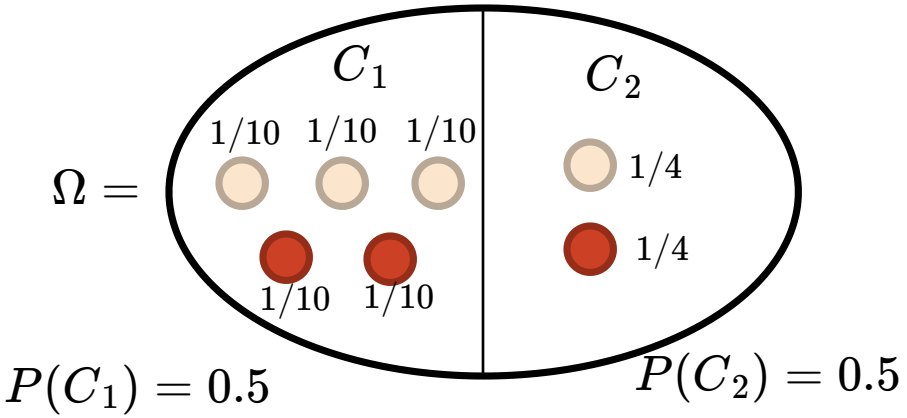

$$P(C_1|\bullet) = \frac{P(C_1 \cap \bullet)}{P(\bullet)}$$

$$= \frac{1/10 + 1/10}{1/10 + 1/10 + 1/4}$$

$$= \frac{2/10}{9/20} = \frac{4}{9}$$

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The Monty Hall Problem

Lecture 17, CS70 Fall 2026

Conditional Probabilities

- Conditional Probability of an Outcome
- Conditional Probability of an Event
- **The Monty Hall Problem**

Bayesian Inference

- Bayes Rule and the Total Probability Rule (TPR)
- Applications of Bayes and TPR
- Generalized Bayes and TPR

CC BY NC SA

The Monty Hall Problem

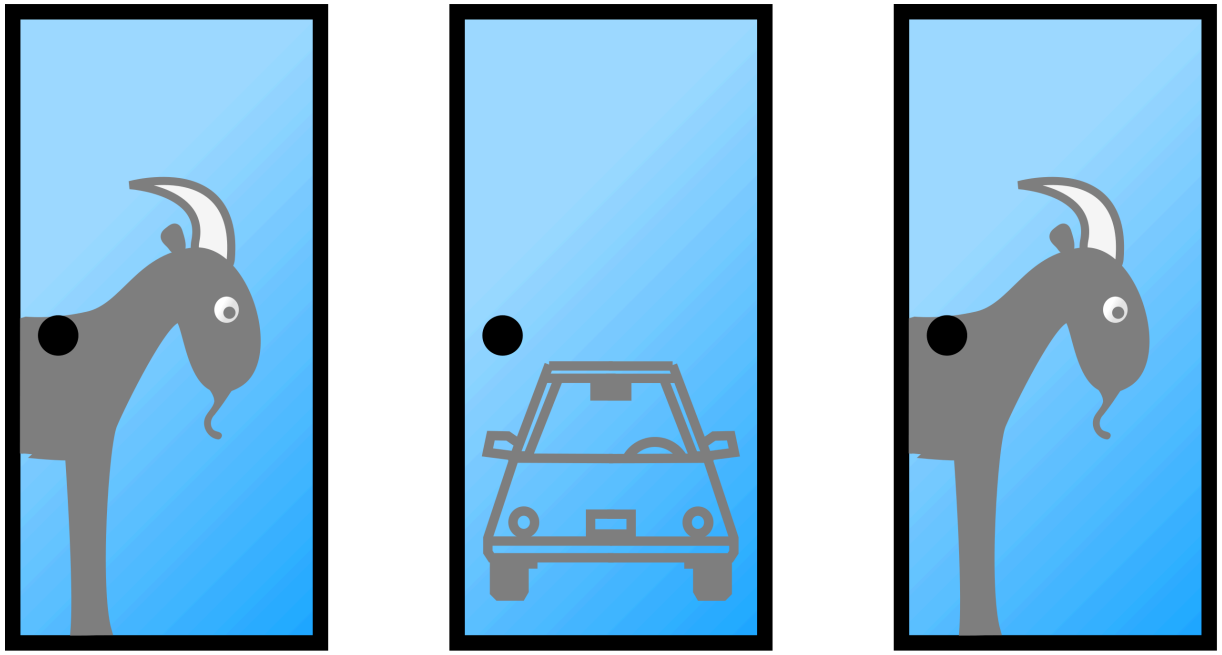
Let's consider a much trickier problem using the same framework as before.

CC BY NC SA

The Monty Hall Problem

Let's start by imagining a game. In this game there are 3 doors with prizes behind them, and you want to win the car.

- One door has a car behind it.
- Two doors have a goat behind them.



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The Monty Hall Problem

Let's start by imagining a game. In this game there are 3 doors with prizes behind them, and you want to win the car.

- One door has a car behind it.
- Two doors have a goat behind them.

The rules are as follows:

- You pick one of 3 doors.
- The host reveals what is behind one of the other doors. The answer is **ALWAYS** a goat.
- You **ALWAYS** get a chance to switch doors after the host shows you the goat.

Let's try to play a few rounds.

CC BY NC SA

Let’s Play: The Monty Hall Game

Pick a door.

1 2 3

Click a door to choose it.

Results

0 games

Stay 0 won of 0 –

Switch 0 won of 0 –

Play 100 games, always stay switch

show each game Run Reset

Adapted from the Monty Hall Game in the Rossman/Chance Applet Collection, using its door, goat and car pictures.

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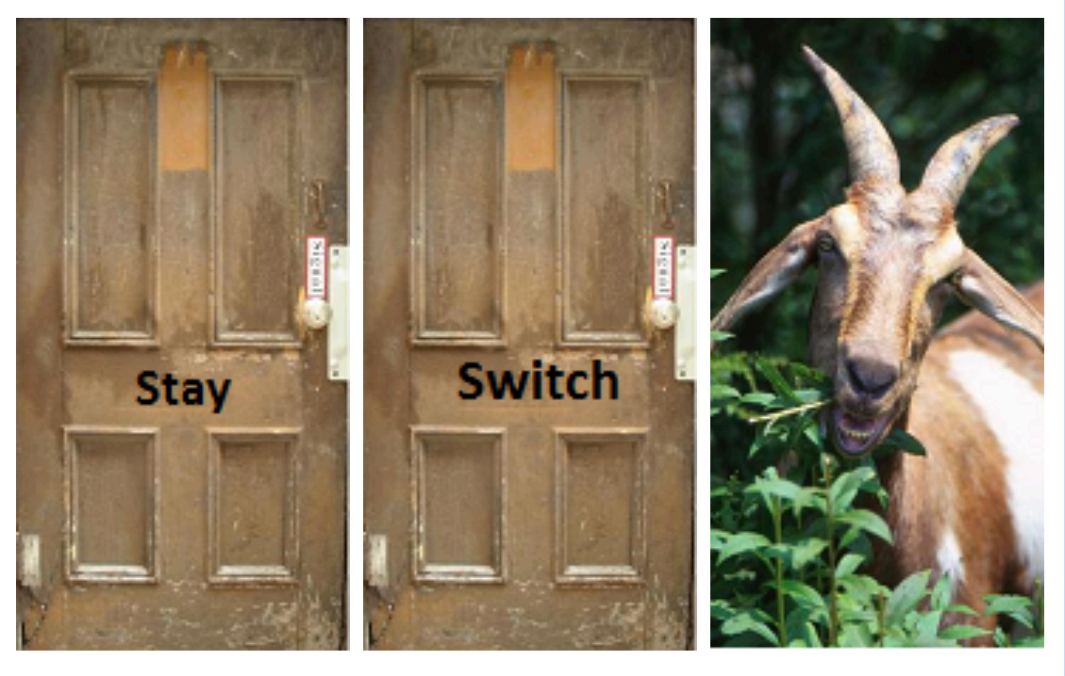
The Monty Hall Problem

Suppose we're playing, pick the left door, and we get to this state:



Should you switch doors? (Note: You are **ALWAYS** shown a goat)

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The Monty Hall Problem

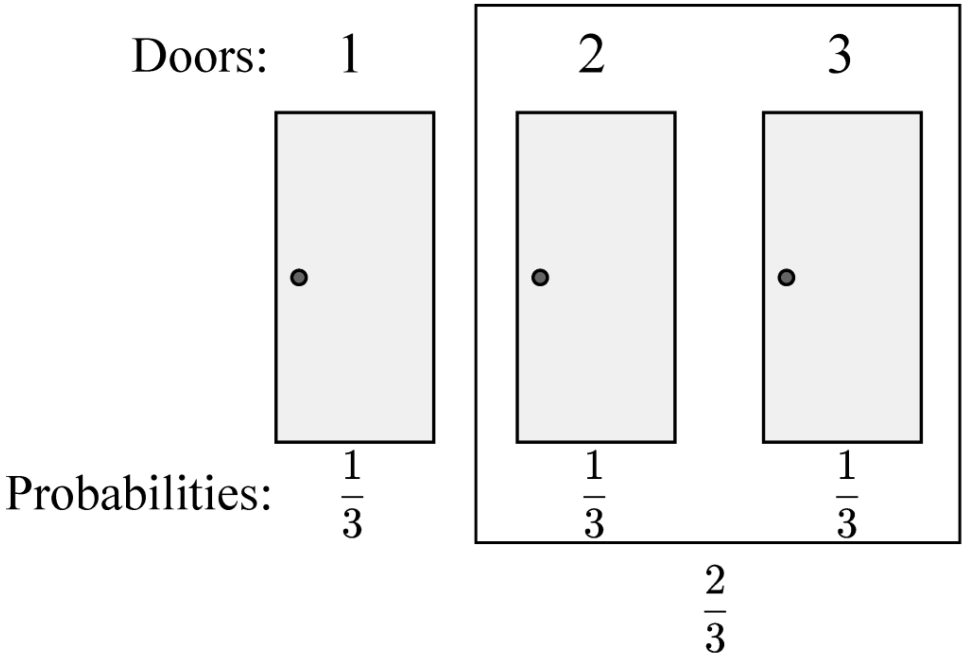
Strangely the answer is yes: your chance of winning is better if you switch!

Explanation (1/4)

The chance that you were correct with your first guess is 1/3.

Only two possibilities exist:

- You were right (with probability 1/3).
- You were wrong (with probability 2/3).



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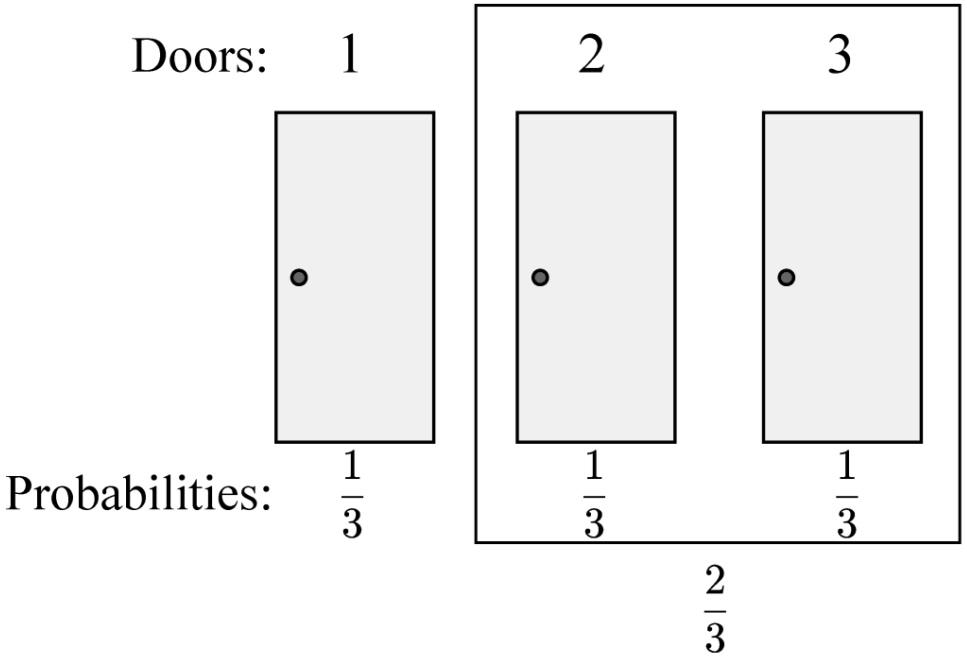
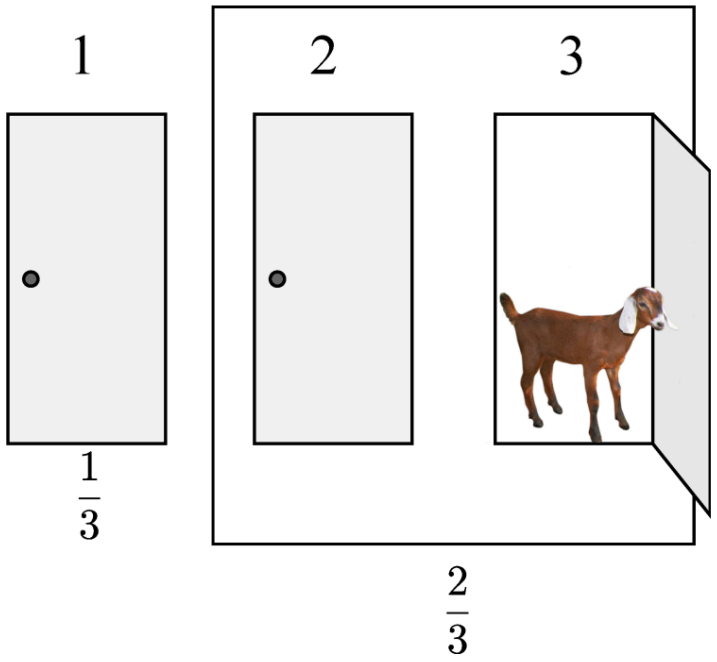
Explanation (2/4)

Suppose the host opens door 3, how do the probabilities change?

- Chance that your original choice is correct is unchanged. It's still 1/3.





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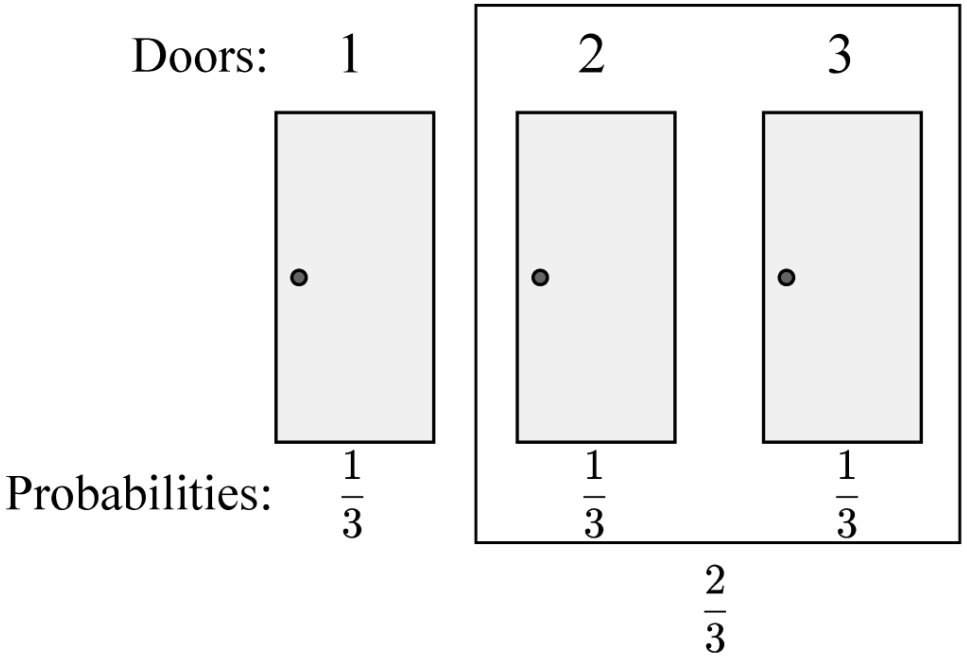
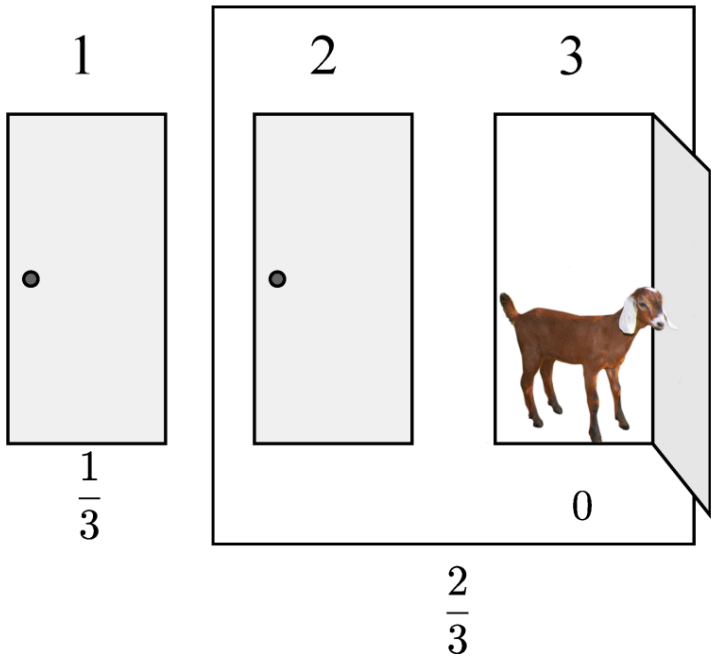
Explanation (3/4)

Suppose the host opens door 3, how do the probabilities change?

- Chance that your original choice is correct is unchanged. It's still 1/3.
- Chance that door 3 has the car is now zero.





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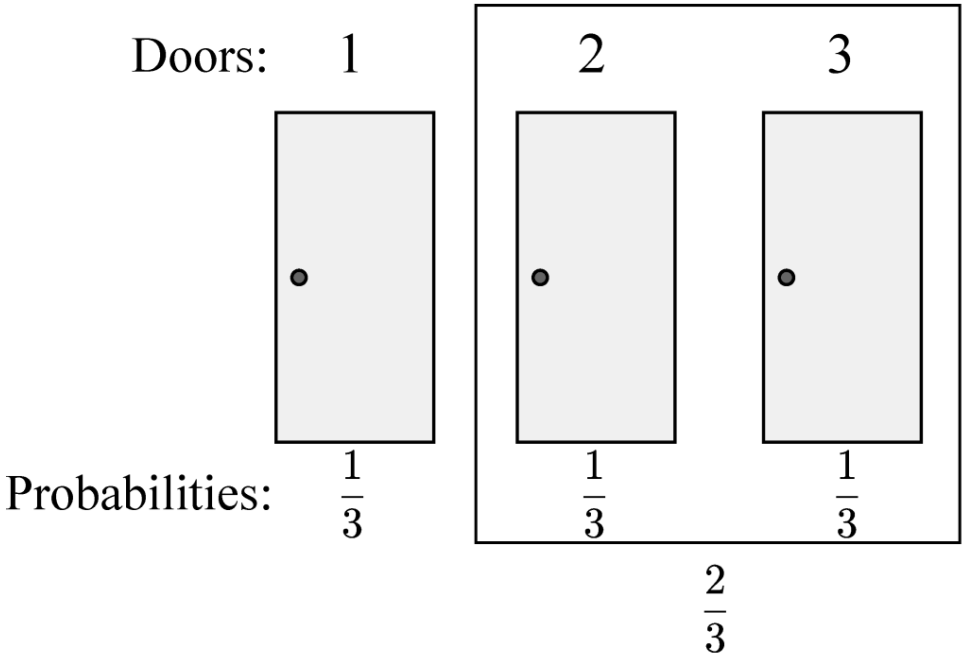
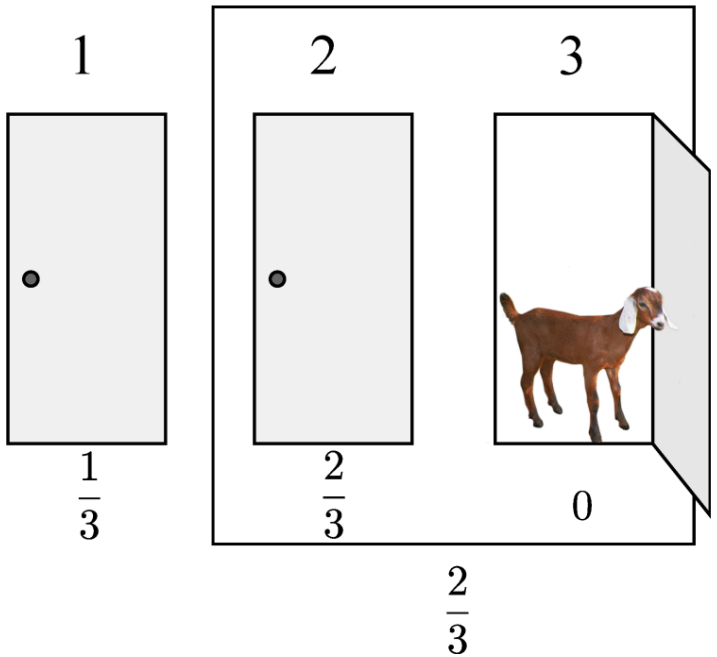
Explanation (4/4)

Suppose the host opens door 3, how do the probabilities change?

- Chance that your original choice is correct is unchanged. It's still 1/3.
- Chance that door 3 has the car is now zero.
- Chance that door 2 has the car is now 2/3 (since door 3 can't have the car).





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Explanation Summary

The chance that you were correct with your first guess is 1/3.

That is, only two possibilities exist before the door is opened:

- You are right (with probability 1/3).
- You are wrong (with probability 2/3).

After opening a door, the chance you were right is STILL 1/3.

- So switching (to the other binary choice) must have a probability of 2/3.

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The Monty Hall Problem

When this problem was answered in Parade magazine by columnist Marilyn vos Savant in 1990, the answer drew huge amounts of angry mail including from folks holding PhDs.

Let's take a walk down memory lane: https://web.archive.org/web/20130121183432/http://marilynvossavant.com/game-show-problem/

Bayes Rule and the Total Probability Rule (TPR)

Lecture 17, CS70 Fall 2026

Conditional Probabilities

- Conditional Probability of an Outcome
- Conditional Probability of an Event
- The Monty Hall Problem

Bayesian Inference

- **Bayes Rule and the Total Probability Rule (TPR)**
- Applications of Bayes and TPR
- Generalized Bayes and TPR

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Bayesian Inference

Bayesian Inference: A technique for updating knowledge based on observation.

- Conditional probability is an important concept for making this possible.

In this context:

- $\boldsymbol{P}(A)$: Assessment of the likelihood of A without any current observation, also known as a **prior probability**.
- $\boldsymbol{P}(A|B)$: Assessment of the likelihood of A given a current observation B, also known as a **posterior probability**. Includes the knowledge we gained from the observation.

Note: In this section, we will stop coloring our events A and B.

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Motivating Example: Flu Testing

Let A be the event where a person is infected with the flu, and B be the event where a person tests positive on some test. We want to know $\boldsymbol{P}(A|B)$.

Suppose we know the following about the test:

- When taken by a person who is infected, comes up positive 90% of the time, and negative 10% of the time (false negatives).

If you test positive, what is the chance you are actually infected?

- Not enough information!

A: Infected
B: Tests Positive

CC BY NC SA

In Terms of Conditional Probability

Let A be the event where a person is infected with the flu, and B be the event where a person tests positive on some test. We want to know $\boldsymbol{P}(A|B)$.

Suppose we know the following about the test:

- $\boldsymbol{P}(B|A)$: When taken by a person who is infected, comes up positive 90% of the time, and negative 10% of the time (false negatives).

If you test positive, what is the chance you are actually infected? $\boldsymbol{P}(A|B)$

- Not enough information! No reason that $\boldsymbol{P}(A|B) = \boldsymbol{P}(B|A)$

$$\boldsymbol{P}(B|A) = 0.9$$

A: Infected
B: Tests Positive

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What If We Know the False Positive Rate?

Suppose we have a test for the flu where we know:

- $\boldsymbol{P}(B|A)$: When taken by a person who is infected, comes up positive 90% of the time, and negative 10% of the time (false negatives).
- $\boldsymbol{P}(B|\overline{A})$: When taken by a person who is not infected, comes up positive 20% of the time (false positives), and negative 80% of the time.

If you test positive, what is the chance you are actually infected?

- Not enough information!

$$\boldsymbol{P}(B|A) = 0.9 \qquad \boldsymbol{P}(B|\overline{A}) = 0.2$$

A: Infected
B: Tests Positive

CC BY NC SA

The Last Piece of Information We Need: Our Prior $P(A)$

Suppose we have a test for the flu where we know:

- $\boldsymbol{P}(B|A)$: When taken by a person who is infected, comes up positive 90% of the time, and negative 10% of the time (false negatives).
- $\boldsymbol{P}(B|\overline{A})$: When taken by a person who is not infected, comes up positive 20% of the time (false positives), and negative 80% of the time.

$\boldsymbol{P}(A)$: Suppose that 1% of the people in a region currently have the flu.

If you (in that region) test positive, what is the chance you are actually infected?

- Now we finally know enough! The answer, it may surprise you, is 4.4%. Let's talk about how I computed that answer.

$\boldsymbol{P}(A) = 0.01$ $\boldsymbol{P}(B|A) = 0.9$ $\boldsymbol{P}(B|\overline{A}) = 0.2$

A: Infected
B: Tests Positive

CC BY NC SA

Example: Flu Testing

Let A be the event where a person is infected and B be the event where a person tests positive. Then we know:

$$\boldsymbol{P}(A) = 0.01 \qquad \boldsymbol{P}(B|A) = 0.9 \qquad \boldsymbol{P}(B|\overline{A}) = 0.2$$

Using this information, we want to compute $\boldsymbol{P}(A|B)$, i.e. the chance that if a person tests positive that they have the flu.

We know that $\boldsymbol{P}(A|B) = \frac{\boldsymbol{P}(A \cap B)}{\boldsymbol{P}(B)}$.

- Need to somehow compute these two quantities: $\boldsymbol{P}(A \cap B)$ and $\boldsymbol{P}(B)$!

Let's do this first.

CC BY NC SA

Example: Flu Testing. Computing $\boldsymbol{P}(A \cap B)$

Let A be the event where a person is infected and B be the event where a person tests positive. Then we know:

$$\boldsymbol{P}(A) = 0.01 \qquad \boldsymbol{P}(B|A) = 0.9 \qquad \boldsymbol{P}(B|\overline{A}) = 0.2$$

Using this information, we want to compute $\boldsymbol{P}(A|B)$, i.e. the chance that if a person tests positive that they have the flu.

We know that $\boldsymbol{P}(A|B) = \frac{\boldsymbol{P}(A \cap B)}{\boldsymbol{P}(B)}$. Computing $\boldsymbol{P}(A \cap B)$ is easy:

Example: Flu Testing. Computing $\boldsymbol{P}(A \cap B)$

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We know that $\boldsymbol{P}(A|B) = \frac{\boldsymbol{P}(A \cap B)}{\boldsymbol{P}(B)}$. Computing $\boldsymbol{P}(A \cap B)$ is easy:

- We know that $\boldsymbol{P}(B|A) = \frac{\boldsymbol{P}(A \cap B)}{\boldsymbol{P}(A)}$
- Therefore, $\boldsymbol{P}(A \cap B) = \boldsymbol{P}(B|A)\boldsymbol{P}(A) = 0.9 \times 0.01 = 0.009$

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Example: Flu Testing

Let A be the event where a person is infected and B be the event where a person tests positive. Then we know:

$$\boldsymbol{P}(A) = 0.01 \qquad \boldsymbol{P}(B|A) = 0.9 \qquad \boldsymbol{P}(B|\overline{A}) = 0.2$$

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We know that $\boldsymbol{P}(A|B) = \frac{\boldsymbol{P}(A \cap B)}{\boldsymbol{P}(B)} = \frac{0.009}{\boldsymbol{P}(B)}$.

- Need to somehow compute these two quantities: $\boldsymbol{P}(A \cap B)$ and $\boldsymbol{P}(B)$!

We just saw this was 0.009.

So let's compute this next (the probability that you test positive).

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Example: Flu Testing

Let A be the event where a person is infected and B be the event where a person tests positive. Then we know:

$$\boldsymbol{P}(A) = 0.01 \qquad \boldsymbol{P}(B|A) = 0.9 \qquad \boldsymbol{P}(B|\overline{A}) = 0.2$$

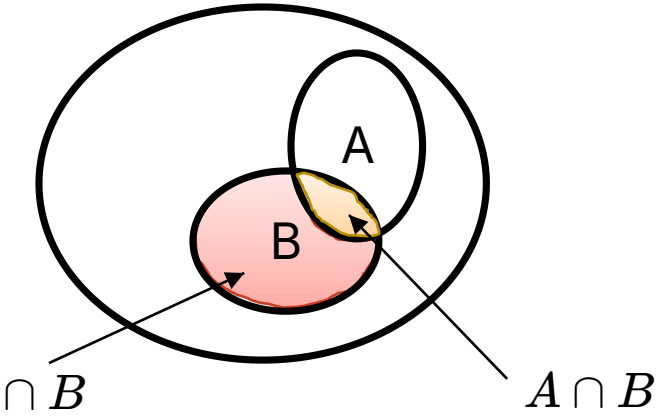
Using this information, we want to compute $\boldsymbol{P}(A|B)$, i.e. the chance that if a person tests positive that they have the flu.

We know that $\boldsymbol{P}(A|B) = \frac{\boldsymbol{P}(A \cap B)}{\boldsymbol{P}(B)} = \frac{0.009}{\boldsymbol{P}(B)}$. Computing $\boldsymbol{P}(B)$ is a little trickier:

- $\boldsymbol{P}(B) = \boldsymbol{P}(A \cap B) + \boldsymbol{P}(\overline{A} \cap B)$

Because

$\Omega =$



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Example: Flu Testing

Let A be the event where a person is infected and B be the event where a person tests positive. Then we know:

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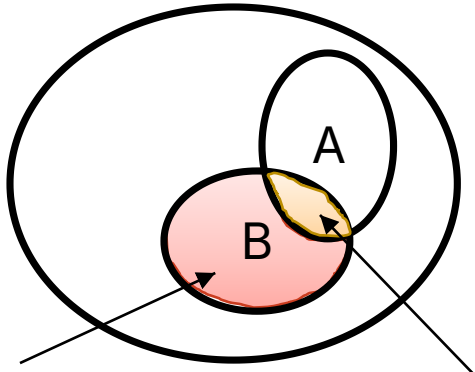
- $\boldsymbol{P}(B) = \boldsymbol{P}(A \cap B) + \boldsymbol{P}(\overline{A} \cap B)$
- $\boldsymbol{P}(\overline{A} \cap B) = \boldsymbol{P}(B|\overline{A})\boldsymbol{P}(\overline{A})$

Because

Since $\boldsymbol{P}(B|\overline{A}) = \frac{\boldsymbol{P}(\overline{A} \cap B)}{\boldsymbol{P}(\overline{A})}$

$\Omega =$



$\overline{A} \cap B$ $\qquad$ $A \cap B$

CC BY NC SA

Example: Flu Testing

Let A be the event where a person is infected and B be the event where a person tests positive. Then we know:

$$\boldsymbol{P}(A) = 0.01 \qquad \boldsymbol{P}(B|A) = 0.9 \qquad \boldsymbol{P}(B|\overline{A}) = 0.2$$

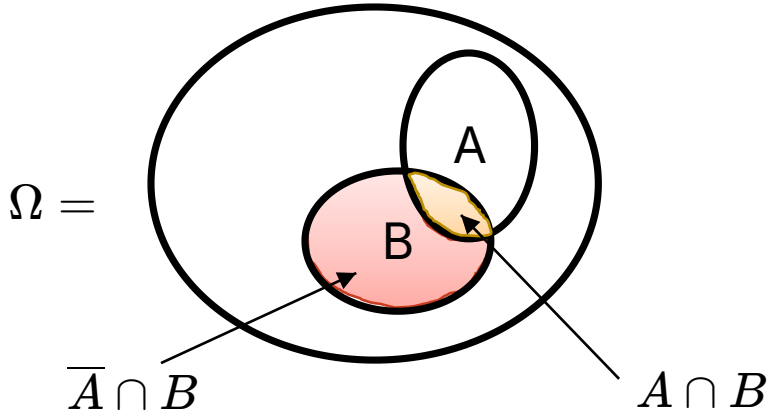
Using this information, we want to compute $\boldsymbol{P}(A|B)$, i.e. the chance that if a person tests positive that they have the flu.

We know that $\boldsymbol{P}(A|B) = \frac{\boldsymbol{P}(A \cap B)}{\boldsymbol{P}(B)} = \frac{0.009}{\boldsymbol{P}(B)}$. Computing $\boldsymbol{P}(B)$ is a little trickier:

- $\boldsymbol{P}(B) = \boldsymbol{P}(A \cap B) + \boldsymbol{P}(\overline{A} \cap B)$
- $\boldsymbol{P}(\overline{A} \cap B) = \boldsymbol{P}(B|\overline{A})\boldsymbol{P}(\overline{A})$

$$= \boldsymbol{P}(B|\overline{A})(1 - \boldsymbol{P}(A))$$

$$= 0.2(1 - 0.01) = 0.198$$



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Example: Flu Testing

Let A be the event where a person is infected and B be the event where a person tests positive. Then we know:

$$\boldsymbol{P}(A) = 0.01 \qquad \boldsymbol{P}(B|A) = 0.9 \qquad \boldsymbol{P}(B|\overline{A}) = 0.2$$

Using this information, we want to compute $\boldsymbol{P}(A|B)$, i.e. the chance that if a person tests positive that they have the flu.

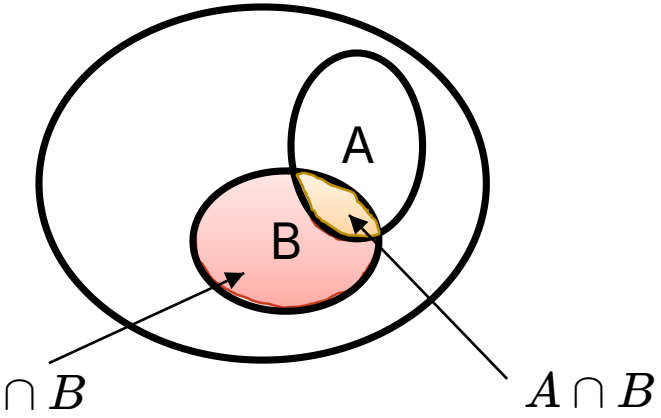
We know that $\boldsymbol{P}(A|B) = \frac{\boldsymbol{P}(A \cap B)}{\boldsymbol{P}(B)} = \frac{0.009}{\boldsymbol{P}(B)}$. Computing $\boldsymbol{P}(B)$ is a little trickier:

- $\boldsymbol{P}(B) = \boldsymbol{P}(A \cap B) + \boldsymbol{P}(\overline{A} \cap B) = 0.009 + 0.198 = 0.207$
- $\boldsymbol{P}(\overline{A} \cap B) = \boldsymbol{P}(B|\overline{A})\boldsymbol{P}(\overline{A})$

  $= \boldsymbol{P}(B|\overline{A})(1 - \boldsymbol{P}(A))$

  $= 0.2(1 - 0.01) = 0.198$

$\Omega =$



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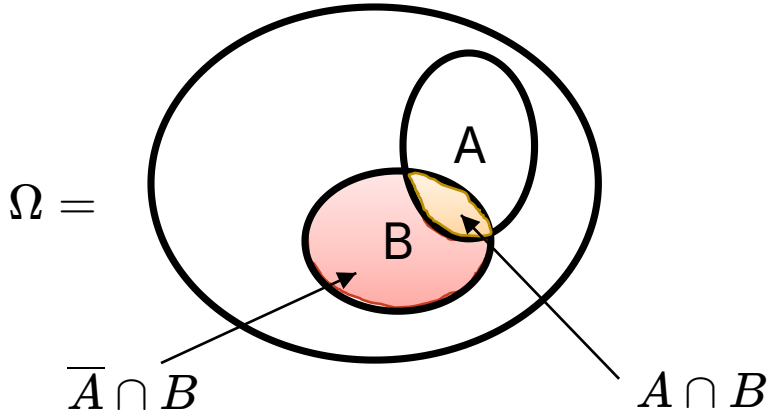
We know that $\boldsymbol{P}(A|B) = \frac{\boldsymbol{P}(A \cap B)}{\boldsymbol{P}(B)} = \frac{0.009}{0.207} = 0.044$, i.e. 4.4%.



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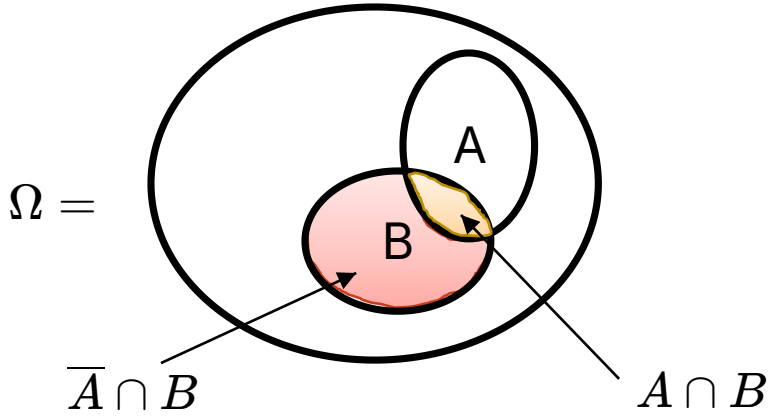
In other words, even though the test has a 90% true positive rate and only a 20% false positive rate, if we get a positive result, we only have a 4.4% chance of having the flu.



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...Or Entirely Symbolically

We can also write what we've just done symbolically. Given $\boldsymbol{P}(A)$, $\boldsymbol{P}(B|A)$, $\boldsymbol{P}(B|\overline{A})$, we can compute $\boldsymbol{P}(A|B)$ as follows:

We know that $\boldsymbol{P}(A|B) = \dfrac{\boldsymbol{P}(A \cap B)}{\boldsymbol{P}(B)} = \dfrac{\boldsymbol{P}(B|A)\boldsymbol{P}(A)}{\boldsymbol{P}(B)}$

$$= \frac{P(B|A)\,P(A)}{P(A \cap B) + P(\bar{A} \cap B)}$$

$$= \frac{P(B|A)\,P(A)}{P(B|A)\,P(A) + P(B|\bar{A})\,P(\bar{A})}$$

$$= \frac{P(B|A)\,P(A)}{P(B|A)\,P(A) + P(B|\bar{A})(1-P(A))}$$

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$$
\begin{aligned}
\text{We know that } \boldsymbol{P}(A|B) = \frac{\boldsymbol{P}(A \cap B)}{\boldsymbol{P}(B)} &= \frac{\boldsymbol{P}(B|A)\boldsymbol{P}(A)}{\boldsymbol{P}(B)} \\
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\end{aligned}
$$

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Example:

$$\boldsymbol{P}(A) = 0.01 \qquad \boldsymbol{P}(B|A) = 0.9 \qquad \boldsymbol{P}(B|\overline{A}) = 0.2$$

$$\boldsymbol{P}(A|B) = \frac{0.9 \times 0.01}{0.9 \times 0.01 + 0.2 \times (1 - 0.01)} = 0.044$$

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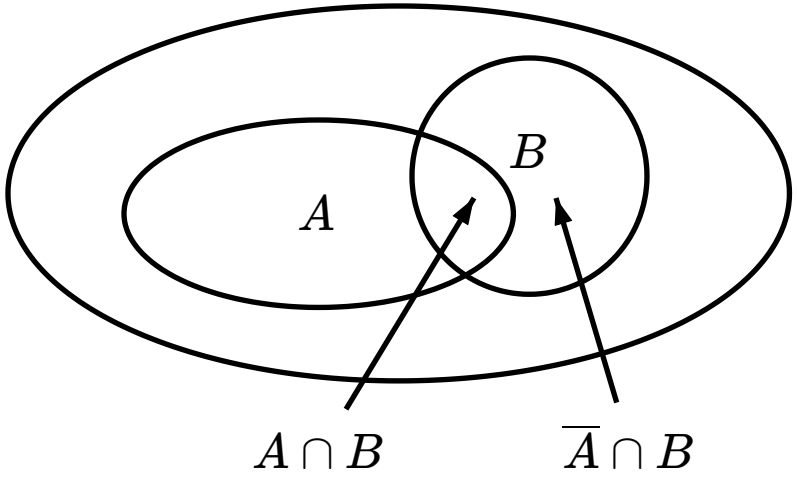
Bayes Rule and the Total Probability Rule

We've just derived two commonly used formulas:

Bayes Rule: $$\boldsymbol{P}(A|B) = \frac{\boldsymbol{P}(B|A)\boldsymbol{P}(A)}{\boldsymbol{P}(B)}$$

Total Probability Rule: $$\boldsymbol{P}(B) = \boldsymbol{P}(B|A)\boldsymbol{P}(A) + \boldsymbol{P}(B|\overline{A})(1 - \boldsymbol{P}(A))$$



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Applications of Bayes and TPR

Lecture 17, CS70 Fall 2026

Conditional Probabilities

- Conditional Probability of an Outcome
- Conditional Probability of an Event
- The Monty Hall Problem

Bayesian Inference

- Bayes Rule and the Total Probability Rule (TPR)
- **Applications of Bayes and TPR**
- Generalized Bayes and TPR

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Example: Applying the Total Probability Rule

Suppose we have two shopping centers.

Shopping center 1:

3 Indian restaurants
2 Moroccan restaurants

Shopping center 2:

1 Indian restaurant
1 Moroccan restaurant

Suppose we pick a center randomly with probability 50%, and then pick a restaurant from that center with equal probability.

- What is the probability $\boldsymbol{P}(\bullet)$ that we eat at a Moroccan restaurant?

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- What is the probability $\boldsymbol{P}(\bullet)$ that we eat at a Moroccan restaurant?

Total probability rule:

$$\boldsymbol{P}(\bullet) = P(\bullet|C_1)P(C_1) + P(\bullet|C_2)P(C_2)$$

$$= 0.4 \times 0.5 + 0.5 \times 0.5 = 0.45$$

TPR using symbols earlier: $\boldsymbol{P}(B) = \boldsymbol{P}(B|A)\boldsymbol{P}(A) + \boldsymbol{P}(B|\overline{A})(1 - \boldsymbol{P}(A))$

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Shopping Center Puzzle Using Bayes Rule

Suppose we have two shopping centers.

Shopping center 1:

3 Indian restaurants
2 Moroccan restaurants

Shopping center 2:

1 Indian restaurant
1 Moroccan restaurant

Suppose we pick a center randomly with probability 50%, and then pick a restaurant from that center with equal probability.

- If we eat at a Moroccan restaurant, what's the chance we picked center 1?

$$P(C_1|\bullet) = \frac{P(\bullet|C_1)P(C_1)}{P(\bullet)}$$

$$\boldsymbol{P}(\bullet) = 0.45$$ (By TPR)

Solve using Bayes Rule.

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Shopping Center Puzzle Using Bayes Rule

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Suppose we pick a center randomly with probability 50%, and then pick a restaurant from that center with equal probability.

- If we eat at a Moroccan restaurant, what's the chance we picked center 1?

$$P(C_1|\bullet) = \frac{P(\bullet|C_1)P(C_1)}{P(\bullet)}$$

$$= \frac{0.4 \times 0.5}{0.45} = \frac{0.2}{0.45} = 4/9$$

$$\boldsymbol{P}(\bullet) = 0.45$$ (By TPR)

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Generalized Bayes and TPR

Lecture 17, CS70 Fall 2026

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- Bayes Rule and the Total Probability Rule (TPR)
- Applications of Bayes and TPR
- **Generalized Bayes and TPR**

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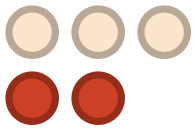
Multiple Events

Suppose we have more than 2 shopping centers:

Shopping center 1:

3 Indian restaurants
2 Moroccan restaurants

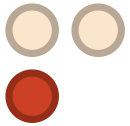


Shopping center 2:

1 Indian restaurant
1 Moroccan restaurant

Shopping center 3:

2 Indian restaurants
1 Moroccan restaurant



To handle such situations, let's define a "partition of an event".

We say that an event A is partitioned into n events $A_1, \ldots, A_n$ if:

- $A = A_1 \cup A_2 \cup \ldots \cup A_n$
- $A_i \cap A_j = \emptyset$ for all $i \neq j$. That is, the events are mutually exclusive.

For the example above, we have three events, one for each shopping center.

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Total Probability Rule

Our TPR (total probability rule) for events A and B was:

$$P(B) = P(B|A)P(A) + P(B|\overline{A})(1 - P(A))$$

When considering a partition $A_1, \ldots, A_n$ of the sample space Ω, the TPR for an event B is:

$$P(B) = \sum_{i=1}^{n} P(B \cap A_i)$$

Question: Why are these two formulas basically saying the same thing?

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Total Probability Rule

Our TPR (total probability rule) for events A and B was:

Partitions Ω into two events, $A_1 = A$ and $A_2 = \overline{A}$

$$P(B) = \underbrace{P(B|A)P(A)}_{P(B \cap A)} + \underbrace{P(B|\overline{A})(1 - P(A))}_{P(B \cap \overline{A})}$$

When considering a partition $A_1, \ldots, A_n$ of the sample space Ω, the TPR for an event B is:

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$$P(B) = \sum_{i=1}^{n} P(B \cap A_i)$$

Can also write this as (using the definition of conditional probability):

$$P(B) = \sum_{i=1}^{n} P(B|A_i)P(A_i)$$

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Bayes Rule

For two events A and B, Bayes Rule was:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

When considering a partition $A_1, \ldots, A_n$ of the sample space Ω, Bayes Rule is:

$$P(A_i|B) = \frac{P(B|A_i)P(A_i)}{P(B)}$$

In both cases, you can naturally replace $P(B)$ by the TPR.

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In both cases, you can naturally replace $P(B)$ by the TPR.

- For partitions of space into A and $\overline{A}$, $P(B) = P(B|A)P(A) + P(B|\overline{A})(1 - P(A))$
- For partitions into $A_1, \ldots, A_n$, $P(B) = \sum_{i=1}^{n} P(B|A_i)P(A_i)$

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