Lecture 1C: Induction

UC Berkeley EECS 70
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Announcements!

- Lecture is posted under “Media Gallery” in bCourses
- **HW 1** and **Vitamin 1** have been released, due **Today** (grace period Friday)

What is induction?

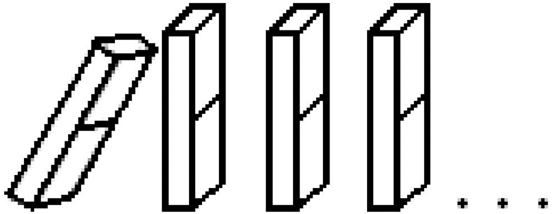
Goal in induction is to prove some statement for all natural numbers $(\forall n \in \mathbb{N}), P(n)$

Principle of Induction

- Base Case: **Prove P(0)**
- Inductive Hypothesis: **Assume P(n)**
- Inductive Step: **Prove P(n) ⇒ P(n+1)**

Visual Analogy

Prove all the dominos fall down



- $P(0)$ = "First domino falls"
- $P(k) \Rightarrow P(k+1)$ "kth domino falls implies that $k+1st$ domino falls"

Even if you had infinite dominos lined up, this method would prove all of them will fall down (More on this Week 4).

Simple Induction (Example 1)

Theorem: For all natural numbers n, $0 + 1 + 2 + ... + n = \frac{n(n+1)}{2}$
Proof:

Simple Induction (Example 2)

Theorem: For all $n \in \mathbb{N}$, $3|(n^3 - n)$

Proof:

Simple Induction (Example 3)

Theorem: Any map formed by dividing the plain into regions by drawing *straight* lines can be properly colored with two colors
Proof:

Improving Induction Hypothesis (Example 1)

Theorem: The sum of the first n odd numbers is a perfect square
Improved:
Proof:

Improving Induction Hypothesis (Example 2)

Theorem: For all $n \geq 1$, $\sum_{i=1}^{n} \frac{1}{i^2} \leq 2$

Improved:

Proof:

What is Strong Induction?

Principle of Strong Induction

- Base Case: **Prove P(0)**
- Inductive Hypothesis: **Assume P(0) and P(1) and … and P(n)**
- Inductive Step: **Prove P(0) and … and P(n) ⇒ P(n+1)**

Strong Induction (Example 1)

Theorem: Every natural number greater than 1 can be written as a product of one or more primes
Proof:

Strong Induction with Multiple Base Cases (Example 2)

Theorem: For every natural number $n \geq 12$, it holds that $n = 4x + 5y$ for some $x, y \in \mathbb{N}$

Proof:

Why ever use weak induction?

Weak Induction $\Rightarrow$ Strong Induction

If you wanted to you could always use strong induction

It is nicer to only use weak induction if strong induction is not needed.

Well-Ordering Principle

The Well-Ordering Principle states that for any non-empty subset of the natural numbers there will be a least element.

Theorem: Every natural number greater than 1 can be written as a product of one or more primes
Proof using WOP:

Summary

- Simple Induction
  - P(0) and show P(n) ⇒ P(n+1)
- Multiple Base Cases
  - You may need multiple base cases to prove a statement
- Improve the Inductive Hypothesis
  - Sometimes proving a “stronger” statement is easier
- Strong Induction
  - P(0) and show P(0) and … and P(n) ⇒ P(n+1)
- Well Ordering Principle
  - For any subset of the naturals there is a least element