Lecture 1C: Induction

UC Berkeley EECS 70
Summer 2022
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Announcements!

- Lecture is posted under “Media Gallery” in bCourses
- HW 1 and Vitamin 1 have been released, due Today (grace period Friday)
What is induction?

Goal in induction is to prove some statement for all natural numbers $(\forall n \in \mathbb{N}), P(n)$

Principle of Induction

- **Base Case:** Prove $P(0)$
- **Inductive Hypothesis:** Assume $P(n)$
- **Inductive Step:** Prove $P(n) \Rightarrow P(n+1)$
Visual Analogy

Prove all the dominos fall down

- $P(0) = \text{“First domino falls”}$
- $P(k) \implies P(k+1) \text{ “}k\text{th domino falls implies that }k+1\text{st domino falls”}$

Even if you had infinite dominos lined up, this method would prove all of them will fall down (More on this Week 4).
Simple Induction (Example 1)

Theorem: For all natural numbers $n$, $0 + 1 + 2 + ... + n = \frac{n(n+1)}{2}$

Proof:
Simple Induction (Example 2)

Theorem: For all $n \in \mathbb{N}$, $3|(n^3 - n)$

Proof:
Simple Induction (Example 3)

Theorem: Any map formed by dividing the plain into regions by drawing straight lines can be properly colored with two colors

Proof:
Improving Induction Hypothesis (Example 1)

Theorem: The sum of the first $n$ odd numbers is a perfect square
Improved:
Proof:
Improving Induction Hypothesis (Example 2)

Theorem: For all \( n \geq 1 \), \( \sum_{i=1}^{n} \frac{1}{i^2} \leq 2 \)

Improved:
Proof:
What is Strong Induction?

Principle of Strong Induction

- **Base Case:** Prove $P(0)$
- **Inductive Hypothesis:** Assume $P(0)$ and $P(1)$ and ... and $P(n)$
- **Inductive Step:** Prove $P(0)$ and ... and $P(n) \Rightarrow P(n+1)$
Strong Induction (Example 1)

Theorem: Every natural number greater than 1 can be written as a product of one or more primes

Proof:
Strong Induction with Multiple Base Cases (Example 2)

Theorem: For every natural number $n \geq 12$, it holds that $n = 4x + 5y$ for some $x, y \in \mathbb{N}$

Proof:
Why ever use weak induction?

Weak Induction $\Rightarrow$ Strong Induction

If you wanted to you could always use strong induction

It is nicer to only use weak induction if strong induction is not needed.
Well-Ordering Principle

The Well-Ordering Principle states that for any non-empty subset of the natural numbers there will be a least element.

Theorem: Every natural number greater than 1 can be written as a product of one or more primes
Proof using WOP:
Summary

- **Simple Induction**
  - $P(0)$ and show $P(n) \Rightarrow P(n+1)$

- **Multiple Base Cases**
  - You may need multiple base cases to prove a statement

- **Improve the Inductive Hypothesis**
  - Sometimes proving a “stronger” statement is easier

- **Strong Induction**
  - $P(0)$ and show $P(0)$ and ... and $P(n) \Rightarrow P(n+1)$

- **Well Ordering Principle**
  - For any subset of the naturals there is a least element