Lecture 2B: Graph Theory II

UC Berkeley EECS 70
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Tarang Srivastava
Announcements!

- Read the Weekly Post
- We have caught academic misconduct cases
- HW 2 and Vitamin 2 have been released, due Thu (grace period Fri)
- Throughout this lecture definitions will be underlined
Minimum Edges for Connectivity

Theorem: Any connected graph with $n$ vertices must have at least $n-1$ edges
Complete Graphs

A graph $G$ is **complete** if it contains the maximum number of edges possible.

Correction: $K$ is for mathematician Kazimierz Kuratowski

Examples:
Trees

The following definitions are all equivalent to show that a graph $G$ is a **tree**.

1. $G$ is connected and contains no cycles
2. $G$ is connected and has $n-1$ edges (where $n = |V|$)
3. $G$ is connected, and the removal of any single edge disconnects $G$
4. $G$ has no cycles, and the addition of any single edge creates a cycle
Tree Definitions are Equivalent

Theorem: For a connected graph \( G \) it contains no cycles iff it has \( n-1 \) edges.

Proof:
Theorem: For a connected graph $G$ it contains no cycles iff it has $n-1$ edges.
Bipartite Graphs

A graph $G$ is **bipartite** if the vertices can be split in two groups (L or R) and edges only go between groups.

$G$ is bipartite iff $G$ is two colorable

Examples:
Planar Graphs

A graph is called \textit{planar} if it can be drawn in the plane without any edges crossing.

Examples:
Euler’s Formula: $v - e + f = 2$

Theorem: If $G$ is a connected planar graph, then $v - e + f = 2$.

Proof:
Euler’s Formula Corollary: $e \leq 3v - 6$

Corollary: For a connected planar graph with $v \geq 3$, we have $e \leq 3v - 6$

Proof:
$K_5$ is non-planar

Proof:
$K_{3,3}$ is non-planar

Proof:
Kuratowski’s Theorem

Theorem: A graph is non-planar iff it contains $K_5$ or $K_{3,3}$

Example:
Hypercubes

The vertex set of a $n$-dimensional hypercube $G=(V, E)$ is given by $V = \{0, 1\}^n$ i.e. the vertices are $n$-bit strings.
Number of Edges in Hypercubes

Lemma: The total number of edges in an $n$-dimensional hypercube is $n2^{n-1}$

Proof: