Lecture 2B: Graph Theory II

UC Berkeley EECS 70
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Tarang Srivastava

Announcements!

- Read the Weekly Post
- We have caught academic misconduct cases
- **HW 2** and **Vitamin 2** have been released, due **Thu** (grace period Fri)
- Throughout this lecture <u>**definitions**</u> will be underlined

Minimum Edges for Connectivity

Theorem: Any connected graph with n vertices must have at least n-1 edges

Complete Graphs

A graph G is <u>**complete**</u> if it contains the maximum number of edges possible.

Correction: K is for mathematician Kazimierz Kuratowski

Examples:

Trees

The following definitions are all equivalent to show that a graph G is a **tree**.

1. G is connected and contains no cycles
2. G is connected and has n-1 edges (where $n = |V|$)
3. G is connected, and the remove of any single edge disconnects G
4. G has no cycles, and the addition of any single edge creates a cycle

Tree Definitions are Equivalent

Theorem: For a connected graph G it contains no cycles iff it has n-1 edges.
Proof:

Tree Definitions are Equivalent (cont.)

Theorem: For a connected graph G it contains no cycles iff it has n-1 edges.

Bipartite Graphs

A graph G is **bipartite** if the vertices can be split in two groups (L or R) and edges only go between groups.

G is bipartite iff G is two colorable

Examples:

Planar Graphs

A graph is called **planar** if it can be drawn in the plane without any edges crossing.

Examples:

Euler's Formula: $v - e + f = 2$

Theorem: If G is a connected planar graph, then $v - e + f = 2$.

Proof:

Euler's Formula Corollary: $e \leq 3v - 6$

Corollary: For a connected planar graph with $v \geq 3$, we have $e \leq 3v - 6$
Proof:

K_5 is non-planar

Proof:

$K_{3,3}$ is non-planar

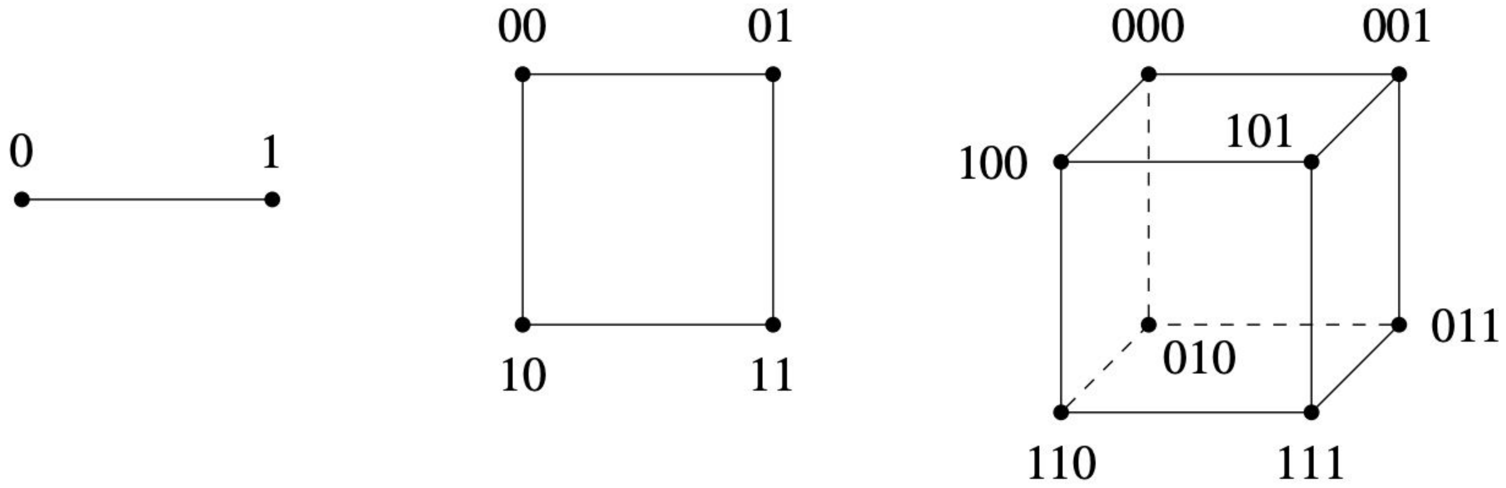
Proof:

Kuratowski's Theorem

Theorem: A graph is **non-planar** iff it contains K_5 or $K_{3,3}$
Example:

Hypercubes

The vertex set of a n-dimensional **hypercube** G=(V, E) is given by V = $\{0, 1\}^n$
i.e. the vertices are n-bit strings.



Number of Edges in Hypercubes

Lemma: The total number of edges in an n-dimensional hypercube is $n2^{n-1}$

Proof: