

Countability review

Computability

The theory of computation

Michael Psenka

Logical paradoxes

“This statement is false”

Turing machines

Church-Turing Thesis: “any $f: \mathbb{N} \rightarrow \mathbb{N}$ computable by an effective method is computable by a Turing machine”

Computable sets

How big is the collection of computable sets?

Quines, recursion theorem

The Halting Problem

TestHalt(P, x):

 if (program P halts on input x):

 return 1

 else

 return 0

The Halting Problem

Turing(P):

```
    if TestHalt(P,P):  
        # loop forever  
        while 1:  
            print('I am looping')  
    else:  
        # exit  
        return 1
```

“Easy” Halting Problem

How many computable sets?

Theorem. *The collection of computable sets is countable.*

Corollary. There are uncountably many uncomputable sets.

How many computable things?

The following sets are countable:

- 1. The set of computable functions*
- 2. The set of computable numbers*

Gödel's Incompleteness Theorem

Gödel numbering

Gödel's Incompleteness Theorem

Unprovable statement

Gödel's Incompleteness Theorem