Conditional probability

Modeling knowledge of uncertainty

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What we aim to model

Conditional probability

Definition. *The conditional probability* $\mathbb{P}(A \mid B)$ *(read "probability of A given B") is defined as the following:*

$$\mathbb{P}(A \mid B) \coloneqq \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)}$$

Bayes' Theorem

Theorem. *For a distribution* $\mathbb{P}$, *the following equation holds:*

$$\mathbb{P}(A \mid B) = \frac{\mathbb{P}(B \mid A)\mathbb{P}(A)}{\mathbb{P}(B)}$$

Why Bayes'?

Total probability rule

Theorem. *Let* $(\Omega, \mathbb{P})$ *be a random variable. If we have a collection of sets* $\{A_i\}_{i=1}^n$ *that "partition" the sample space* Ω*, i.e.:*

1. $A_i \cap A_j = \emptyset$ *for all* $i, j \in \{1, \dots, n\}$,
2. $\cup_{i=1}^n A_i = \Omega$,

Then the following equation holds for any event $B \subset \Omega$*:*

$$\mathbb{P}(B) = \sum_{i=1}^{n} \mathbb{P}(B \mid A_i)\mathbb{P}(A_i).$$

Total probability rule

Joint distributions

Applying Bayes’