Conditional probability cont., Independence

Modeling (lack of) causality

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What we aim to model

Independence

Definition. *Given a random variable* $X = (\Omega, \mathbb{P})$, *two events* $A, B \subset \Omega$ *are said to be independent if the following equation holds:*

$$\mathbb{P}(A \cap B) = \mathbb{P}(A)\mathbb{P}(B).$$

Equivalent definition

Examples

Independence in joint distributions

Mutual independence

Definition. *Given a random variable* $X = (\Omega, \mathbb{P})$, *a collection of events* $A_1, \dots, A_n \subset \Omega$ *are said to be mutually independent if the following equation holds for every subset* $I \subset \{1, \dots, n\}$ *such that* $|I| \geq 2$:

$$\mathbb{P}(\cap_{i \in I} A_i) = \Pi_{i \in I} \mathbb{P}(A_i).$$

Why mutual independence?

General product rule

Theorem. *Given a random variable* $X = (\Omega, \mathbb{P})$, *and a collection of events* $A_1, \ldots, A_n \subset \Omega$, *the following equation holds:*

$$\mathbb{P}(A_1 \cap A_2 \cap \cdots \cap A_n) = \mathbb{P}(A_1)\mathbb{P}(A_2 \mid A_1)\mathbb{P}(A_3 \mid A_1 \cap A_2) \cdots \mathbb{P}(A_n \mid A_1 \cap \cdots \cap A_{n-1}).$$

General product rule

General product rule: equivalent forms

Monty Hall revisited