CS 70 Discrete Mathematics and Probability Theory
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1 Squaring Fives

Note 1 Consider these equations:

$$5^2 = 25$$
$$15^2 = 225$$
$$25^2 = 625$$
$$35^2 = 1225$$
$$45^2 = 2025$$

Do you see a pattern? Do you see more patterns? Can you write down a conjecture to describe the patterns you see in natural language? Can you formalize your conjecture and prove it?

Solution: Looking at the equations, we can see that when we square a number starting with k and ending with 5, we get a number starting with $k(k+1)$ and ending with 25.

We can formalize this by noticing that $5 = 0+5$, $15 = 10+5$, and $25 = 20+5$. In general, we can write a number starting with k and ending with 5 as $k \cdot 10 + 5$. A number starting with l and ending with 25 can be written as $l \cdot 100 + 25$. With this in mind, we can translate the statement as

$$(k \cdot 10 + 5)^2 = k(k+1) \cdot 100 + 25.$$

Finally, we can prove this with some algebra

$$\begin{aligned}(k \cdot 10 + 5)^2 &= k^2 \cdot 100 + k \cdot 50 + k \cdot 50 + 25 \\ &= k^2 \cdot 100 + k \cdot 100 + 25 \\ &= k(k+1) \cdot 100 + 25\end{aligned}$$

which shows that $(k \cdot 10 + 5)^2 = k(k+1) \cdot 100 + 25$ as desired.

2 Perfect Square

Note 1 Let n be a natural number. Prove that if n is odd, then n^2 can be written in the form $8k+1$ for some integer k.

Solution: Since n is odd, $n = 2m+1$ for some integer m. It follows that

$$n^2 = (2m+1)^2 = 4m^2 + 4m + 1 = 4m(m+1) + 1.$$

One of the consecutive integers m and $m+1$ is even, so $m(m+1) = 2k$ for some integer k. Therefore,

$$n^2 = 4(2k)+1 = 8k+1,$$

as desired.

3 Multiplying Division

Note 1

Recall that $a \mid b$ means a divides b. Let $m,n \in \mathbb{Z}$. Prove that if $n \mid m$ and $(n+1) \mid m$, then

$$n(n+1) \mid m.$$

Solution: We can rewrite $n \mid m$ and $(n+1) \mid m$ as

$$\begin{aligned} m &= nk \\ m &= (n+1)k' \end{aligned}$$

for some $k, k' \in \mathbb{Z}$. This implies that

$$\begin{aligned} nk &= (n+1)k' \\ nk &= nk' + k' \\ nk - nk' &= k' \\ n(k-k') &= k'. \end{aligned}$$

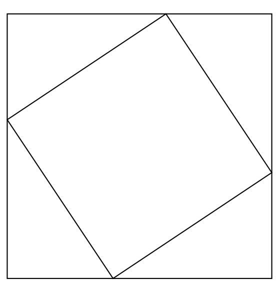
Substituting for k' in $m = (n+1)k'$ gives

$$m = (n+1)n(k-k')$$

so $n(n+1) \mid m$ as desired.

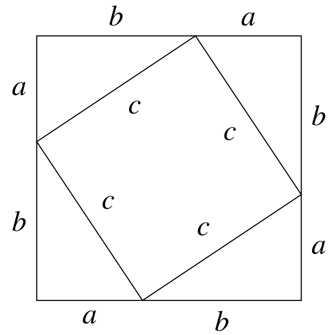
4 Pythagorean Theorem

Note 1



Using the above diagram, prove the Pythagorean Theorem: if a and b are the lengths of the legs of a right triangle and c is the length of its hypotenuse, then $a^2+b^2=c^2$.
Hint: Look for right triangles in the diagram and label them with a, b, and c.

Solution:



Note that the large square consists of four equally sized right triangles and a small square. The area of the large square is $(a+b)^2$, the area of each right triangle is $ab/2$ and the area of the small square is c^2, so we have

$$\begin{aligned}(a+b)^2 &= 4\frac{ab}{2}+c^2\\ a^2+2ab+b^2 &= 2ab+c^2\\ a^2+b^2 &= c^2\end{aligned}$$