1 Truth Tables

Determine whether the following equivalences hold, by writing out truth tables. Clearly state whether or not each pair is equivalent.

(a) \( P \land (Q \lor P) \equiv P \land Q \)
(b) \( (P \lor Q) \land R \equiv (P \land R) \lor (Q \land R) \)
(c) \( (P \land Q) \lor R \equiv (P \lor R) \land (Q \lor R) \)

2 Propositional Practice

Convert the following English sentences into propositional logic and the following propositions into English. State whether or not each statement is true with brief justification.

(a) There is a real number which is not rational.
(b) All integers are natural numbers or are negative, but not both.
(c) If a natural number is divisible by 6, it is divisible by 2 or it is divisible by 3.
(d) \( \forall x \in \mathbb{Z} \) \( (x \in \mathbb{Q}) \)
(e) \( \forall x \in \mathbb{Z} \) \((((2 \mid x) \lor (3 \mid x)) \implies (6 \mid x)) \)
(f) \( \forall x \in \mathbb{N} \) \(((x > 7) \implies ((\exists a,b \in \mathbb{N}) (a+b = x))) \)
3 Converse and Contrapositive

Consider the statement "if a natural number is divisible by 4, it is divisible by 2".

(a) Write the statement in propositional logic. Prove that it is true or give a counterexample.

(b) Write the inverse of the implication in English and in propositional logic. Prove that it is true or give a counterexample. (The inverse of an implication $P \implies Q$ is $\neg P \implies \neg Q$.)

(c) Write the converse of the implication in English and in propositional logic. Prove that it is true or give a counterexample.

(d) Write the contrapositive of the implication in English and in propositional logic. Prove that it is true or give a counterexample.

4 Equivalences with Quantifiers

Evaluate whether the expressions on the left and right sides are equivalent in each part, and briefly justify your answers.

\[
\begin{array}{c|c|c}
(a) & \forall x \left( (\exists y Q(x,y)) \implies P(x) \right) & \forall x \exists y \left( Q(x,y) \implies P(x) \right) \\
(b) & \neg \exists y \forall x \left( P(x,y) \implies \neg Q(x,y) \right) & \forall x \left( (\exists y P(x,y)) \land (\exists y Q(x,y)) \right) \\
(c) & \forall x \exists y \left( P(x) \implies Q(x,y) \right) & \forall x \left( P(x) \implies (\exists y Q(x,y)) \right)
\end{array}
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