

1 Natural Induction on Inequality

Prove that if $n \in \mathbb{N}$ and $x > 0$, then $(1+x)^n \geq 1+nx$.

2 Strengthen Induction

Show by induction that $\sum_{i=1}^n \frac{1}{i^3} \leq 2$.

3 Binary Numbers

Prove that every positive integer n can be written in binary. In other words, prove that we can write

$$n = c_k \cdot 2^k + c_{k-1} \cdot 2^{k-1} + \cdots + c_1 \cdot 2^1 + c_0 \cdot 2^0,$$

where $k \in \mathbb{N}$ and $c_i \in \{0, 1\}$ for all $i \leq k$.