1 Eulerian Tour and Eulerian Walk

(a) Is there an Eulerian tour in the graph above? If no, give justification. If yes, provide an example.

(b) Is there an Eulerian walk in the graph above? An Eulerian walk is a walk that uses each edge exactly once. If no, give justification. If yes, provide an example.

(c) What is the condition that there is an Eulerian walk in an undirected graph? Briefly justify your answer.
2 Coloring Trees

Prove that all trees with at least 2 vertices are bipartite: the vertices can be partitioned into two groups so that every edge goes between the two groups.

[Hint: Use induction on the number of vertices.]

3 Not everything is normal: Odd-Degree Vertices

Claim: Let \( G = (V, E) \) be an undirected graph. The number of vertices of \( G \) that have odd degree is even.

Prove the claim above using:

(i) Direct proof (e.g., counting the number of edges in \( G \)). Hint: in lecture, we proved that \( \sum_{v \in V} \deg v = 2|E| \).
(ii) Induction on \( m = |E| \) (number of edges)

(iii) Induction on \( n = |V| \) (number of vertices)
4 Trees and Components

(a) Bob removed a degree 3 node from an \( n \)-vertex tree. How many connected components are there in the resulting graph? Please provide an explanation.

(b) Given an \( n \)-vertex tree, Bob added 10 edges to it and then Alice removed 5 edges. If the resulting graph has 3 connected components, how many edges must be removed in order to remove all cycles from the resulting graph? Please provide an explanation.